Optimal Fiscal Policy under Endogenous Disaster Risk: How to Avoid Wars? *

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Abstract

How should governments finance defense spending? We answer this question through the lens of an optimal fiscal policy framework that features endogenous disaster risk, where war is the disaster. Besides standard policy tools, such as distortionary taxes and non-contingent debt, the planner can invest in defense stock, which alleviates financing needs during the war and minimizes disaster risk. We find that standard policy prescriptions to increase taxes and accumulate savings are greatly affected when defense spending affects the disaster probability. The optimal policy prescribes increased borrowing and reduced current taxes in anticipation of war, as a result of a balance between tax smoothing across states and over time.

JEL Classification: E62, D52, E60.

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1 Introduction

How should fiscal policy optimally manage disaster risks? In this paper, we answer this question by means of a Ramsey problem where the planner faces the risk of economic disaster. The planner can preemptively invest in defense stock to mitigate both the disaster probability and its damage. We find the question interesting from both theoretical and policy points of view. Theoretically, disaster risk models have demonstrated major success in explaining stock market moments (e.g., Rietz, 1988; Barro, 2006) and the business cycles Gourio (2012). Yet, there is little theoretical guidance on how disaster risk affects optimal policy and how policymakers should manage these risks. Policy-wise, Western policymakers need to navigate an environment of increasing geopolitical risk arising from Russia's or China's revisionist policy stance. Advocates of greater defense spending typically argue that more spending contributes to deterrence. Yet, the mainstream discussion on how to finance it does not account for the fact that more spending mitigates these risks and, therefore, affects the borrowing costs. In this paper, we ask whether risk mitigation is quantitatively important and how defense spending should be optimally financed when it affects the disaster probability.

To answer these questions, we study a Ramsey problem where the planner can issue non-state contingent debt and levy distortionary taxes. Besides standard stochastic expenditure shocks, the planner faces time-varying disaster risk in the form of exceedingly high expenditure needs and large falls in productivity. In addition to these standard policy instruments, the planner can invest in defense spending to replenish an additional stock variable – namely, defense stock– that allows to mitigate the disaster risks. Defense spending is valuable for two purposes. First, it mitigates the disaster probability. Second, defense spending contributes to defense stock, which can be used to absorb some of the spending needs in the disaster state. We refer to the former as the risk-management motive and to the latter as an insurance motive. The benefits of defense spending confer to defense stock similar properties to those of a state-contingent asset. The key novelty of the paper is to study the optimal policy when the planner can affect the disaster probability, along with the use of standard policy tools. The analysis proceeds in two steps.

First, after laying out the full quantitative model, we study the two-period sub-case of the model to gain analytical insights in section 4. Such simplification allows to highlight the key trade-offs and to isolate the role of each channel separately. We show that borrowing to finance defense spending has different implications for bond prices than borrowing to finance standard government expenditure. In particular, both risk-management and insurance motives exert a negative pressure on bond prices. Abstracting from these price effects, we then study what defense spending implies for tax smoothing. By accumulating assets, the planner builds a cushion in case a disaster state occurs in the future. Such reserves enable the planner to smooth taxes in all future states but require to increase taxes in the current period. Hence, such policy implies tax smoothing across states. In contrast, investment in defense, if financed through borrowing, does not require to increase current taxes and allows to mitigate the disaster risk. Yet, the debt needs to be repaid independently of the state realizing in the future, which means that smoothing of distortions in the disaster state needs to be sacrificed. We show that optimal financing of defense spending sacrifices smoothing over states to favor smoothing over time. In other words, defense spending through borrowing minimizes the disaster probability but makes the disaster more severe for the households. Finally, we show that the optimal financing mix of defense spending involves more borrowing than financing standard government expenditure. The intuition comes from the optimality condition that equates the excess burden of taxation today to the expected excess burden of taxation tomorrow. Debt issuance increases the expected excess burden of future taxation, as higher future taxes will be needed to repay debt. However, because of the risk-management channel, defense spending makes the disaster state less likely, and the expected excess burden of taxation increases by less than when debt is issued to finance other government spending. Consequently, through the optimality condition, the current excess burden of taxation also needs to increase by less, meaning that taxes increase by less and debt increases by more than when debt is used to finance other government spending.

Second, we analyze optimal defense spending dynamics in an infinite-horizon model solved nonlinearly and globally using a neural network-based algorithm proposed in Valaitis and Villa (2024). Specifically, in section 5, we study our baseline model dynamics comparing it to counterfactual models, where we first switch off the insurance channel and then, in addition, we also switch off the risk-management channel. In this case, the problem collapses to a standard policy problem under incomplete asset markets as in Aiyagari, Marcet, Sargent, and Seppala (2002), with the only difference that our setting contains uncertainty shocks, in addition to the standard government expenditure shocks. The exercise allows us to test whether the analytical predictions from the two-period model survive in the infinite-horizon setting as well as to gain additional insights.

Even though the planner can invest in defense to mitigate disaster risk, ex-ante, it is not obvious if it is optimal to do so and which of the channels is more relevant. Instead, the planner may simply accumulate assets to be able to smooth tax distortions arising from spending needs in the disaster state. Long-run averages show that, in our baseline calibration, it is optimal to invest significant amounts of output into defense. Once we switch off the insurance channel, defense spending remains almost the same, suggesting that it is the risk-management motive that drives investment in defense. In contrast, both channels are relevant when determining the optimal levels of debt. When we switch off the insurance channel, average debt goes down from 15% of output to close to 0, and when none of these channels are present, the planner accumulates assets up to 20% of output.

We then present generalized impulse responses to both uncertainty and government spending shocks. Response to the spending shock allows us to ask if it is optimal to cut defense spending when other spending needs arise and why. Results show that it is indeed optimal to cut investment in defense. Moreover, such cuts reduce the total spending needs, but falling defense stocks increase household precautionary saving motives, making debt financing cheaper. Consequently, debt responds more strongly to government spending shocks than in the baseline model. Lastly, we study the dynamics in response to uncertainty shocks. In the standard model, the planner responds by accumulating assets, which is accompanied by a modest increase in taxes and a fall in consumption. In our baseline model and the counterfactual without the insurance motive, the planner responds by issuing large amounts of debt to finance defense investment. This entails a large fall in household consumption and utility, which the planner finds optimal as the policy allows to minimize risks of even larger shocks.

Related Literature

This paper builds on the optimal fiscal policy literature (Lucas and Stokey, 1983; Barro, 1979). Specifically, it considers the Ramsey problem without state-contingent debt and under Full Commitment, following Aiyagari, Marcet, Sargent, and Seppala (2002). We contribute to the literature by merging the optimal fiscal policy approach with the disaster risk literature (Rietz, 1988; Barro, 2006, 2009) by allowing the Ramsey planner to invest in the stock variable that mitigate the disaster risk. Our framework nests Aiyagari, Marcet, Sargent, and Seppala (2002) as a special case, when the planner is unable to affect the disaster probability.

The paper is not the first to study optimal policies when the planner uses policy tools to affect the actual (or perceived) event probabilities. A series of papers have considered optimal fiscal policy design under ambiguity-averse agents (Karantounias, 2013; Ferriere and Karantounias, 2019; Karantounias, 2023; Michelacci and Paciello, 2019; Benigno and Paciello, 2014), in which case the planner has incentives to use policy tools to affect the perceived worst-case belief of the agents. The latter two papers (Michelacci and Paciello, 2019; Benigno and Paciello, 2014) study how monetary policy is affected by ambiguity-averse agents who endogenously form worst-case beliefs. Our focus is on fiscal policy. Karantounias (2013) considers a setting where agents have doubts about the probability model of government expenditures and a planner who trusts the model and acts paternalistically using contingent taxes to manage the endogenous probability of a particular state to affect prices of contingent claims. Karantounias (2023) considers a more general setting where both the agents and the planner have doubts about the true model. Ferriere and Karantounias (2019) instead consider a setting with ambiguity-averse agents and endogenous government expenditure. They uncover a crucial role of the elasticity of intertemporal substitution. The planner uses state-contingent taxes to affect the agents' perceived probability distribution and, consequently, the prices of state-contingent bonds. The correlation between government expenditure, taxes, and prices depends crucially on whether income and substitution effects dominate, as determined by the elasticity of intertemporal substitution. These papers consider how the planner can use standard policy tools to manipulate agents' beliefs in a setting with state-contingent debt, building up on Lucas and Stokey (1983). We, instead, consider the setting with non-state contingent debt following Aiyagari, Marcet, Sargent, and Seppala (2002) and ask whether the planner should resort to the standard policy tools or invest in the stock variable that affects the disaster probability.

Another closely related paper is Niemann and Pichler (2011), who consider optimal fiscal policies under disaster risk and when the planner issues non-state contingent debt. They compare and contrast policies under full commitment and no commitment to future policies. They show that, under full commitment, the planner mainly uses debt while, under no commitment, an increase in debt leads to rising inflation expectations, rendering debt issuance costly. Consequently, the planner issues little debt and resorts to using distortionary taxes. In their paper, disaster risk is completely exogenous, and the planner has no policy tools to affect its probability. Hence, our setting nests the full commitment case of Niemann and Pichler (2011) as a limiting case.

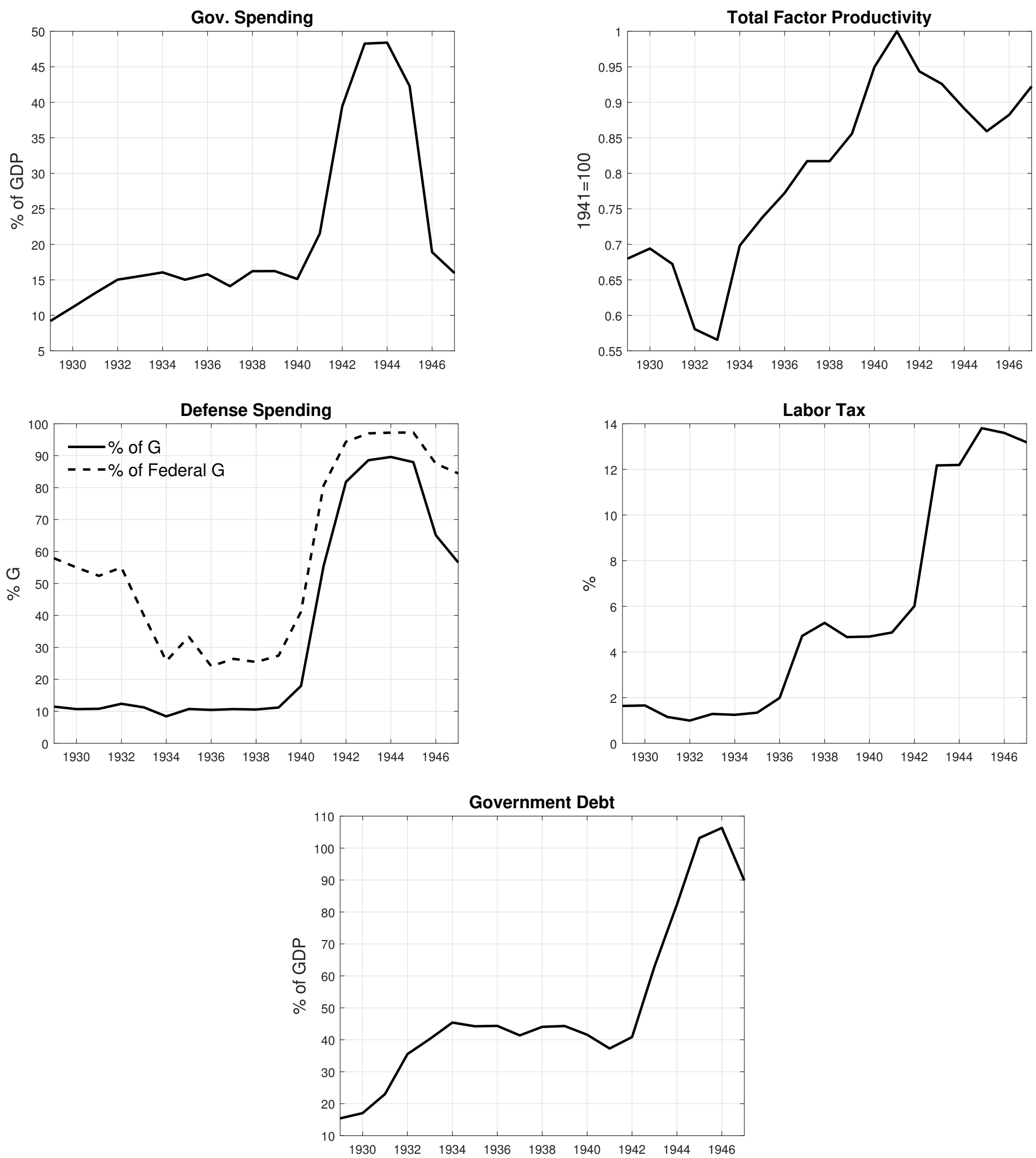
The analysis of policies where the planner's choices endogenously affect the future size of the economy and the financing needs shares similarities to the carbon taxation literature, which uses a seminal DICE Nordhaus (2008) framework. There, carbon taxation affects the private sector incentives to use energy, leading to lower emissions, lower stock of carbon dioxide, and lower damage to future output. Additionally, higher emissions increase the likelihood of reaching climate tipping points, which refer to a critical threshold at which a tiny perturbation can qualitatively alter the state of a climate system. A typical approach is to consider Pigouvian taxes that correct for this climate externality, as in Golosov, Hassler, Krusell, and Tsyvinski (2014) or Cai and Lontzek (2019). Notable exceptions of the application of the Ramsey taxation to DICE economy include Barrage (2019) and Douenne, Hummel, and Pedroni (2022). Barrage (2019) asks how carbon taxes affect the use of other distortionary taxes, namely labor and capital. Douenne, Hummel, and Pedroni (2022) extend Barrage (2019)'s analysis to include household heterogeneity and study whether inequalities and redistributive taxation call for more or less ambitious environmental policies. Both papers consider deterministic settings, while our paper is about how the planner should manage risks in an incomplete market setting.

The paper is organized as follows. Section 2 provides empirical context. Section 3 presents the full infinite-horizon model. Section 4 provides analytical results from a two-period model. Section 5 contains the quantitative results and section 6 concludes.

2 Context

We start by providing the empirical context of the U.S. experience in the 20$^{\text{th}}$ century.

Figure 1: Disaster state dynamics.



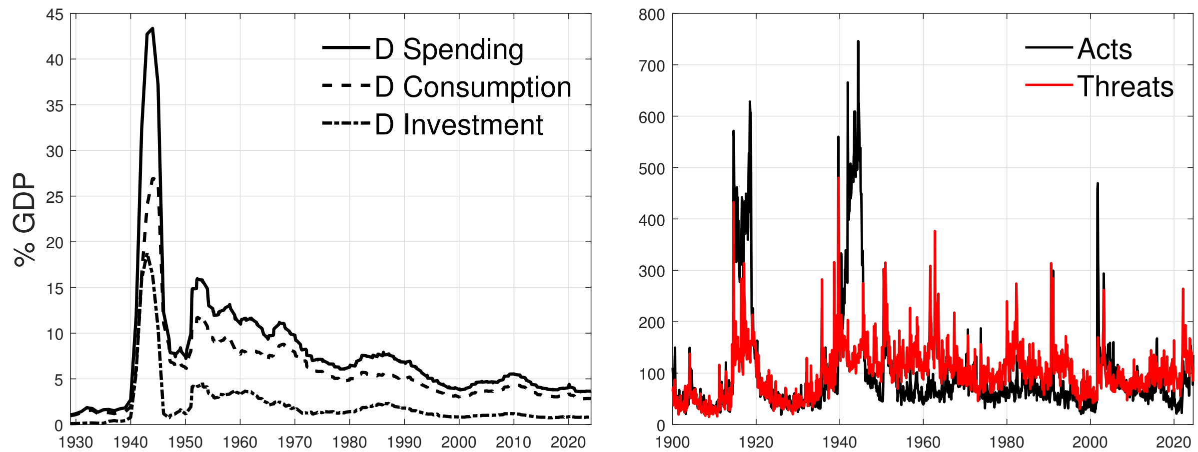
Notes: The figure shows the dynamics of U.S. government spending, total factor productivity, debt-to-GDP ratio, and labor tax rate around World War II. Government spending, debt, GDP, and labor tax rate come from NIPA tables. A detailed explanation about data construction can be found in Clymo, Lanteri, and Villa (2023). Total factor productivity is the series estimated in Field (2023).

In particular, we are interested in the macroeconomic dynamics during World War II, which we consider a representative definition of a disaster considered in this paper. Figure 1 shows the dynamics of government spending, total factor productivity, government debt, labor tax rate, and defense spending.

Top panels show that disaster manifests as a simultaneous rise in government expenditure and a drop in total factor productivity. In this particular case, government spending increased by 30% of the GDP up from 15% of the GDP and total factor productivity fell by around 15% relative to the initial level in 1941. As Field (2023) argues, this was due wartime supply chain disruptions, capital and manpower shortages, and the need to adapt production plants for the production of wartime goods. As is natural during wartime, most of the government spending increase was going to defense. The middle-left panel shows that defense spending – expressed as share of total government spending – went up from 10% to 90% and almost to a 100% share of the federal government spending. The last two panels show that this increase was financed with a mix of taxes and debt. While we present this for contextual purposes and do not attempt to achieve a perfect data match, we use this data as guidance to motivate certain modeling choices.

Next, we examine the relationship between defense spending and disaster risk. The left panel of figure 2 shows the U.S. defense spending as a share of GDP and splits it into consumption and investment components. One can think of the consumption component as salaries and other operating expenditures, and the investment component as the purchase of structures and equipment. We view the investment component as a contributing factor to the overall stock of defense capabilities, an asset that takes years to build, and that depreciates over time absent new investment. This stock also gets depleted during war episodes. The right panel shows the geopolitical risk (GPR) index, constructed in Caldara and Iacoviello (2022). The index attempts to capture geopolitical risk, defined as *the threat, realization, and escalation of adverse events associated with wars, terrorism, and any tensions among states and political actors that affect the peaceful course of international relations.* The index is constructed using a machine learning algorithm that computes the share of articles mentioning adverse geopolitical events in leading newspapers published in the United States, the United Kingdom, and Canada. While the aggregate index interprets risk as both the threat and the realization events such as wars, authors provide separate series of threats (red line) and acts (black line) indices. We can see that the acts index peaks during the two world wars and the September 11's episode. The threats index tends to be more stable. Notably, it steadily increased in the 1930s leading to World War II, and it was above the threats index throughout the Cold War period. Simultaneously, U.S. defense spending was meager in the 1930s, which potentially could have led to the eruption of the war. Conversely, during the Cold War, the low value of the acts index and the simultaneously high values of the threats index are associated with elevated U.S. defense spending. This is often viewed as a period when high U.S. defense capabilities acted as a credible deterrence mechanism. While this does not allow for a causal interpretation, we interpret it as a suggestive evidence that investment in defense helps to mitigate disaster risks. In this paper, we ask what is the optimal defense investment policy and what is the optimal financing mix when defense investment mitigates disaster risks. Surprisingly, the extant literature provides little guidance in this regard.

Figure 2: Geopolitical Risk and Defense Spending.



(a) US Defense Spending, % of GDP

(b) Geopolitical Risk Index

Notes: The left panel shows various U.S. annual defense spending measures, all expressed as a share of GDP. The solid line shows total defense spending. The dashed line shows defense spending that goes to consumption. The dot-dashed line shows the defense spending that goes to investment in equipment and structures as a. Data are sourced from the NIPA table 395. The right panel shows the historical geopolitical risk index constructed by Caldara and Iacoviello (2022). The black line shows the geopolitical acts index. The red line shows the geopolitical threats index.

3 Model

In this section, we describe an infinite-horizon model where the Ramsey planner levies distortionary taxes and issues non-state contingent bonds to finance an exogenous stream of government expenditures, which does not provide utility. Additionally, it invests in defense capabilities that are useful for risk-management and insurance purposes. This setting allows us to understand how the optimal defense financing mix differs from financing government expenditures and enables direct comparison with the classic Aiyagari, Marcet, Sargent, and Seppala (2002) results.

Environment

We consider a closed economy populated by a continuum of identical households, a continuum of identical firms, and a government. Time is discrete and infinite, $t = 0, 1, 2, ..\infty$. The economy is driven by two stochastic processes: (i) the government expenditure and (ii) the disaster risk. Allocations and policy variables are determined by the Ramsey planner, who is constrained by the competitive equilibrium in choosing policy variables. Both households and the planner discount the future with the same rate β.

Preferences. Households rank streams of consumption c_t and leisure l_t according to the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t)\right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, and $u(.)$ and $v(.)$ are differentiable functions such that $u_c > 0$, $u_{cc} < 0$, $v_l > 0$, $v_{ll} < 0$.

Technology. Output is produced by a continuum of measure one of competitive firms with a linear production function $F(h)$, where hours worked are the only input. Hence, aggregate output is given by $Y_t = A_t \cdot h_t$ and A_t is the exogenous labor productivity.

Shocks. There are two types of shocks. First, there are government expenditure shocks denoted as g_t, which we assume to be a continuously distributed $AR(1)$ process in logs $\ln g_t = \mu_g + \rho_g \ln g_t + \epsilon_t^g$ and are simply consumed by the government. Second, there are disaster risk shocks that contribute to drive the disaster probability. We label it as ξ_t and, as well, we assume that it is a continuously distributed AR(1) process in logs $\ln \xi_t = \mu_\xi + \rho_\xi \ln \xi_t + \epsilon_t^\xi$.[1] We denote by $g^t \equiv \{g_0, g_1, ..., g_t\}$ and $\xi^t \equiv \{\xi_0, \xi_1, ..., \xi_t\}$ the histories of the shock realizations up until time t. To simplify notation, we avoid explicitly denoting allocations as functions of histories g^t and ξ^t but it is understood that c_t, l_t and other allocations are measurable with respect to g^t and ξ^t.

Wars. Wars are extreme events that come in a form of a productivity drop and an additional expenditure need. We assume that during the war, g_t follows the same continuously distributed exogenous process $\ln g_t = \mu_g^W + \rho_g \ln g_t + \epsilon_t^g$ with $\mu_g^W >> \mu_g$. We denote the realization of government expenditure during the war state as g_t^W. Moreover, we use g^e to denote the difference between expenditure realizations in the war and normal states. Additionally, wartime productivity falls to a level A^W with $A^W << A$, where the productivity A during the normal state is constant. For ease of notation we use an indicator variable $\mathcal{I}_t = \{0, 1\}$ to denote the disaster state.

Risk-Management. The planner can choose to invest in the stock variable that together with the exogenous process ξ_t determines the disaster probability. We call this a defense stock and

[1] We view this shock as a reduced-form way to capture the policy stance of expansionist foreign governments.

denote it by DS_t. More formally, probability of a war occuring at time t is given by:

$$P(\mathcal{I}_t = 1) = P(DS_{t-1}, \xi_{t-1}),$$

where we assume that $\frac{\partial P(DS_{t-1},\xi_{t-1})}{\partial DS_{t-1}} < 0$ and $\frac{\partial P(DS_{t-1},\xi_{t-1})}{\partial \xi_{t-1}} > 0$. We label the planners incentive to invest in DS_t to affect the probability of the disaster state as the *Risk-Management* channel.

Insurance. We assume that a fraction ϕ of expenditure needed during war can be met by depleting the defense stock, which we denote as the *Insurance* channel. Specifically, let g^e denote the difference in spending in the war and normal states, i.e. $g^W - g^N$. Then the spending need during the war state is related to the undepreciated defense stock through a generic function $\mathcal{S}$, such that the financing needs in the war state are equal to $g_t^W - \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e)$. This channel has a natural interpretation. Typically, a large share of wartime expenditure is defense-related and can be purchased in advance, e.g. ammunition stockpiles. If accumulated in advance, it can be used during the wartime without incurring additional costs. At the same time, the stockpiles become useless if the war state does not realize.

Defense Stock. Defense stock is an endogenous state variable that depreciates at a rate δ and is built up through defense investment (D). The stock may also get depleted during the war episodes if the insurance channel is strong, in the sense that ϕ is close to 1. More formally, defense stock evolves according to the following law of motion:

$$DS_t = DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e). \tag{2}$$

Resources. The resource constraint of the economy is given by

$$c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e) = Y_t = A_t h_t. \tag{3}$$

We normalize the household's time endowment to one, therefore $h_t = 1 - l_t$.

Household Optimality

Households demand consumption goods, supply labor, and trade real non-contingent government bonds denoted as b_t, respectively. To simplify notation, we avoid explicitly denoting bonds as functions of histories s^{t-1} where $s^t \equiv \{g^{t-1}, \xi^{t-1}\}$, but it is understood that b_t is measurable with respect to s^{t-1}. The household budget constraint reads

$$q_t b_{t+1} + c_t \leq w_t h_t (1-\tau_t) + b_t,$$

where w_t is the wage rate, τ_t is the proportional labor tax, and q_t is the bond's price.

Household optimization yields the following private sector optimality conditions:

$$q_t = \beta \mathbb{E}_t \frac{u_c(c_{t+1})}{u_c(c_t)}, \tag{4}$$

$$\tau_t = 1 - \frac{v_l(l_t)}{A_t u_c(c_t)}. \tag{5}$$

Government

The government needs to finance the exogenous stream of government spending g_t and the endogenously chosen defense spending D_t using labor income tax and bonds, subject to the following constraint:

$$g_t + D_t + b_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e) = \tau_t w_t h_t + q_t b_{t+1}. \tag{6}$$

At date t, the government chooses current tax rate τ_t, D_t, and current bonds b_{t+1}, which are measurable with respect to $\{g^t, \xi^t\}$.

Implementability Constraints

We now derive the implementability constraint of the government problem and follow Lucas and Stokey (1983) by taking the primal approach, which allows to substitute away bond prices and taxes with policy instruments.

The government budget constraint (6) can be combined with the private sector's first-order conditions (4) and (5) to obtain a sequence of recursive implementability constraints for $t = 0, 1, ...$ that read:

$$\forall t : b_t = s_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot b_{t+1} \right], \tag{7}$$

where $u_c(c_t)s_t \equiv u_c(c_t)c_t - v_l(l_t)h_t$ denotes the government's surplus in marginal utility terms, and wage w_t is equal to A_t. Besides, we substitute out leisure and labor everywhere using the resource constraint (3). Also, the notation is such that $b > 0$ indicates a positive amount of government debt and $b < 0$ corresponds to government lending to households. We follow the literature on optimal fiscal policy under incomplete markets (e.g., Aiyagari, Marcet, Sargent, and Seppala, 2002; Faraglia, Marcet, Oikonomou, and Scott, 2019) and we assume that exogenous debt limits $b_t \in [\underline{b}, \overline{b}]$ are in place to prevent Ponzi schemes. Note that we set these limits to be sufficiently loose such that they never bind in equilibrium.

Alternatively, the implementability constraint (7) can be formulated to express the government's liabilities b_t as an expected net present value of surpluses.

$$\forall t: \; b_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right], \tag{8}$$

provided that a tangentiality condition $\lim_{t\to\infty} b_{t+1} = 0$ is in place.

3.1 The Ramsey Problem under Incomplete Markets

In this subsection, we solve for the time-inconsistent Ramsey plan under incomplete debt markets, following Aiyagari, Marcet, Sargent, and Seppala (2002). Such a problem is nonrecursive as the planner needs to keep track of all the past promises made when deciding on policies at time t. To make the problem recursive, we follow Marcet and Marimon (2019) by introducing an additional co-state variable that summarizes the previous commitments made by the planner. The Ramsey planner seeks to maximize household utility (1) subject to the implementability constraint (7) with multiplier μ_t, law of motion for the defense stock (2) with multiplier μ_t^D and taking into account that defense stock affects the probability $P(DS, \xi)$ of a war occurring. More formally, the recursive Lagrangian of the planner reads:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ & u(c_t) + v(l_t) + \mu_t(\Omega_t + \beta \mathbb{E}_t u_c(c_{t+1}) b_{t+1} - u_c(c_t) b_t) + \\ & \mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e) - DS_t) + \\ & + \lambda_t (c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e) - A_t h_t) + \lambda_t^D D_t \Big\},\end{aligned}$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t) c_t - v_l(l_t) h_t$.

Inspection of the optimality conditions highlights the key trade-offs.[2] The optimality condition for bonds (9)

$$\mu_t = \mathbb{E}_t(n_{t+1} \mu_{t+1}), \quad \text{where} \quad n_t \equiv \frac{u_{c,t}}{\mathbb{E}_{t-1}(u_{c,t})}, \tag{9}$$

corresponds to the standard result of Aiyagari, Marcet, Sargent, and Seppala (2002) that the recursive multiplier μ_t is a risk-adjusted martingale sequence. This is because of the non-state contingent nature of government debt, past promises matter for current policies, which introduces persistence in the tax rates and debt. μ_t can also be interpreted as the excess burden of taxation. Condition (9) captures that the planner uses tax and debt policies to smooth distortions on average. Using recursive notation we can expand the expression to highlight the role of risk

[2] In appendix A.2 we report all optimality conditions.

management in smoothing tax distortions through DS, as shown in equation (10):[3]

$$\begin{aligned}\mu =&P(DS,\xi)\mathbb{E}_{g'|g}[\mathbb{E}_{\xi'|\xi}[n(g',\xi',\mu,b,DS,1)\mu(g',\xi',\mu,b,DS,1)]]+\\ &(1-P(DS,\xi))\mathbb{E}_{g'|g}[\mathbb{E}_{\xi'|\xi}[n(g',\xi',\mu,b,DS,0)\mu(g',\xi',\mu,b,DS,0)]].\end{aligned} \tag{10}$$

In the standard model, the planner has no control over the probabilities of future states and uses tax and debt policies to influence the value of the excess burden of taxation (μ_{t+1}) state by state so that, in expectation, the excess burden of taxation at $t+1$ is the same as at t. We define this as *smoothing across states*. Risk management through DS introduces an additional channel to achieve the same tax smoothing properties by influencing the probabilities of certain states realizing at $t+1$. We define this policy as *smoothing over time*. While ex-ante it is not obvious which type of smoothing is preferred, in section 4 we investigate analytically how investment in DS affects the excess burden of taxation in various states.

The optimality conditions with respect to D_t is

$$\underbrace{\mu_t^D}_{\text{Marginal Benefit}} = \underbrace{-v_l(l_t)\frac{\partial l_t}{\partial D_t} - \mu_t\frac{\partial \Omega_t}{\partial D_t}}_{\text{Marginal Cost}}, \tag{11}$$

and the optimality condition with respect to DS_t is

$$\begin{aligned}\mu_t^D =& \underbrace{\beta\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}\mathbb{E}_t^x\left(U(c_{t+1}^W,l_{t+1}^W)-U(c_{t+1}^N,l_{t+1}^N)\right)}_{\text{Risk Management}} + \underbrace{\beta\mathbb{E}_t\left(\mu_{t+1}\frac{\partial s_{t+1}u_c(c_{t+1})}{\partial DS_t}\right)}_{\text{Insurance}} +\\ &\underbrace{\beta\mathbb{E}_t\left(\mu_{t+1}^D(1-\delta)-\frac{\mathcal{I}_t\partial\mathcal{S}(DS_{t-1},g_t^e\phi))}{\partial DS_{t-1}}\right)}_{\text{Future Terms}} + \lambda_t^D.\end{aligned} \tag{12}$$

Naturally, these conditions highlight that the optimal investment in DS weights in marginal costs and marginal benefits. Marginal costs are contemporaneous and come from the fact that higher D_t decreases the government primary surpluses and makes the implementability constraint more binding. Additionally, higher D_t, ceteris paribus, means that more labor hours are needed.

The marginal benefits are shifted in the future, as shown by equation (12). The first is the *risk management* term, which captures the idea that higher DS makes the disaster state less likely and, consequently, helps to smooth household's consumption. The quantitative importance of this term depends on the degree to which DS can affect the disaster probability, as captured

[3] The vector of state variables X_t at time t is $X_t = \{g_t, \xi_t, \mu_{t-1}, b_{t-1}, DS_{t-1}, \mathcal{I}_t\}$, where $\mathcal{I}_t \in \{0,1\}$ indicates whether the economy is in the state of war at time t. $\mathcal{I}_t = 0$ stands for the normal state and $\mathcal{I}_t = 1$ stands for the war state.

by the gradient of P with respect to DS. Additionally, the term becomes more important if household cannot insure against the disaster, captured by the difference in their utility in normal and disaster states. The second term captures the *insurance* channel, which captures the idea that higher DS can help to alleviate some of the spending needs in the disaster state and, therefore, it also help to reduce the future excess burden of taxation. The last term captures the benefits from the same two motives beyond $t+1$. Note that *risk management* becomes irrelevant if we make P invariant to DS and the *insurance* term becomes irrelevant if we set ϕ to 0.

As explained in appendix A.1.1, by combining the optimality conditions for consumption and leisure, we can express the optimal tax rate as a function of elasticities and, importantly, multipliers and debt levels:

$$\tau_t = \frac{\mu_t(\epsilon_{cc} + \epsilon_{hh})}{1 + \mu_t(1 + \epsilon_{hh})} - \frac{b_t}{c_t}\epsilon_{cc}\frac{(\mu_t - \mu_{t-1})}{1 + \mu_t(1 + \epsilon_{hh})}. \tag{13}$$

Equation (13) highlights how taxes depend on past multipliers and the levels of outstanding debt. While it is the same as in the standard model with exogenous disasters, it still offers insights on how the optimal optimal debt management relates to taxes. Under complete markets, $\mu_t = \mu_{t-1} = \mu$ and debt levels become irrelevant. In the standard incomplete markets setting, as in (Aiyagari, Marcet, Sargent, and Seppala, 2002), the planner is issuing debt to achieve smoothing across states and, therefore, μ_t is a near random walk meaning that μ_t and μ_{t-1} are typically close. This means that tax volatility is not impacted by outstanding debt levels. Under endogenous disaster management, the planner may opt for tax smoothing *over time*, which would aim to reduce the probability of bad states, while allowing the multiplier in those states to be higher than otherwise. Such policy would allow for occasional large differences between μ_t and μ_{t-1}. If the planner simultaneously issues large levels of debt, the policy of smoothing *over time* then allows for large changes in taxes in some periods.

4 Analysis

To investigate the underlying economic mechanism, we consider a two-period version of the model, with dates denoted as t = 0, 1, along with other simplifying assumptions, as discussed below. This streamlined setting allows to analytically characterize the behavior multipliers, taxes and debt in the models with and without disaster risk management.

Assumptions. We make the following four assumptions. 1. The economy consists of two periods, denoted as $t = 0, 1$. At $t = 0$, the economy is in a normal state, while the state at $t = 1$ is uncertain. 2. We assume that $\sigma(\epsilon^g) = \sigma(\epsilon^\xi) = 0$, such that $\xi_t = \exp(\mu_\xi/(1-\rho_\xi))$ and g_t is either $\exp(\mu_g/(1-\rho_g))$ or $\exp(\mu_g^W/(1-\rho_g))$ in the normal and the war state, respectively. 3. Household preferences are time-separable with constant Frisch elasticity of labor supply. 4. The economy does not experience a productivity drop in the disaster state, thus $z_t = 1$ for $t \in \{0,1\}$. Relaxing any of these assumptions still allows for an analytical characterization of the planner's trade-offs but the expressions become too involved thereby offering little additional insights.

Notation. Under these assumptions, there are two states of the world in period 1, namely, disaster and normal. We use superscripts W and N to denote period 1 variables in the disaster and normal states, respectively. The planner's implementability constraints then read

$$\begin{aligned}
&\tau_0 h_0 + Q_0 b_1 = g_0 + D_1 + b_0, \text{ at } t = 0,\\
&\tau^W h^W = g^W + b_1 - \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)), \text{ at } t = 1 \text{ and war},\\
&\tau^N h^N = g^N + b_1, \text{ at } t = 1 \text{ and peace},
\end{aligned}$$

and the bond's optimality condition (9) simplifies to

$$\mu_0 = P(D_1)\mu^W + (1 - P(D_1))\mu^N \tag{14}$$

and, similarly, the bond's price is

$$Q_0 = \beta\left(P(D_1)\frac{u_c(c^W)}{u_c(c_0)} + (1 - P(D_1))\frac{u_c(c^U)}{u_c(c_0)}\right).$$

We begin by analyzing the planner's trade-offs. We consider hypothetical scenarios where all the planner's constraints hold – i.e., we are in a feasible competitive equilibrium – but the economy is not necessarily at the Ramsey equilibrium. First, it is instructive to compare and contrast how defense spending affects bond prices. For a benchmark, consider an increase in g_0 financed with period 0's debt issuance that is to be repaid in period 1. Equation (15) decomposes the effect on bond prices:

$$\frac{\partial Q_0}{\partial g_0} = \underbrace{\frac{\partial Q_0}{\partial c_0}\frac{\partial c_0}{\partial g_0}}_{\text{GE effect}} + \underbrace{P(D_1)\beta\frac{u_{cc}(c^W)}{u_c(c_0)}\left(\frac{\partial c^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial g_0}\right) + (1-P(D_1))\beta\frac{u_{cc}(c^N)}{u_c(c_0)}\left(\frac{\partial c^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial g_0}\right)}_{\text{Higher } \tau_1}. \tag{15}$$

The first term captures the general equilibrium effect through consumption and labor supply. Through the resource constraint, higher government expenditure requires either a fall in consumption, or higher hours worked, or both. To the extent that both consumption and leisure are normal goods, this term is negative, as lower current consumption means higher marginal utility and hence lower price. The second term captures the effect of higher taxes in period 1. Higher taxes are associated with lower consumption; hence, higher future marginal utility and higher prices.

Now consider an analogous debt-financed increase in defense spending D_1. Equation (16) decomposes the effect on bond prices:

$$\frac{\partial Q_0}{\partial D_1} = \underbrace{\frac{\partial Q_0}{\partial c_0}\frac{\partial c_0}{\partial D_1}}_{\text{GE effect}} + \underbrace{P(D_1)\beta\frac{u_{cc}(c^W)}{u_c(c_0)}\left(\frac{\partial c^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial D_1}\right) + (1-P(D_1))\beta\frac{u_{cc}(c^N)}{u_c(c_0)}\left(\frac{\partial c^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial D_1}\right)}_{\text{Higher } \tau_1} +$$

$$\underbrace{\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)}}_{\text{Risk Management}} + \underbrace{\beta P(D_1)u_{cc}(c^W)\frac{\partial c^W}{\partial D_1}}_{\text{Insurance}}. \tag{16}$$

In addition to the terms in equation (15), equation (16) contains both the *risk management* and *insurance* channels contributing to affect the bond's price.

The *risk management* channel contained in equation (16) has a natural interpretation. D_1 makes the disaster state less likely, i.e. $P'(D_1) < 0$. Indeed, investment in defense reduces the household's precautionary saving motives and lowers the bond's price. The importance of this term depends on the gradient of disaster probability with respect to D_1 and the difference between marginal utilities of consumption in W and N states. The last term captures the *insurance* channel. Investing in D_1 alleviates spending needs in the war state. This, in turn, lowers τ^W resulting in a larger c^W and a lower future marginal utility, hence, a lower price. These considerations lead us to formulate the following proposition, which formalizes the differential effect of debt-financed spending on bond prices.

Proposition 1 ***Defense Spending and the Bond's Price***
Assume that the planner decides to finance an increase spending by issuing debt. Debt-financed defense spending D_1 exerts a higher negative pressure on the bond's price compared to debt-financed government spending g_0; i.e., $\frac{\partial Q_0}{\partial D_1} < \frac{\partial Q_0}{\partial g_0}$.

Proof. See Appendix A.1.2. ■

Next, we turn to analyze the model behavior when both debt and taxes adjust optimally. The planner can respond to disaster risks by either accumulating assets or by investing in D_1. If the planner chooses to invest in D_1, it can be done by either issuing debt or using current taxes. Financing through current taxes front-loads tax distortions and, importantly, allows to avoid the risk of exceedingly high tax distortions in the disaster state. In this sense, the policy allows cross-state tax smoothing in period 1 by sacrificing the smoothing between period 0 and period 1. Debt financing does the opposite. Since debt needs to be repaid in period 1 regardless of the state of the world, such financing sacrifices cross-state tax smoothing, while allowing to smooth tax distortions over time. In such a case, there is little change in tax distortions in period

0, while the expected distortions in period 1 move in an ambiguous way as higher D_1 reduces the disaster probability. The other option for the planner is to ignore the *risk management* and accumulate assets that are to be used to smooth tax distortions in the disaster state. Note that if the *risk management* motive is absent, the planner always insures by accumulating assets rather than investing in D_1 for insurance. The reason is that assets pay off in both states of the world, while D_1 only in one, hence it has a lower expected return as an investment.
Proposition 2 states that, in absence of insurance motives, the optimal mix of defense financing is such that cross-state smoothing of distortions is sacrificed. In other words, it is optimal to finance D_1 with a mix of taxes and debt. The fact that D_1 is financed with debt also means that simultaneous risk-management through D_1 and accumulation of assets are not optimal.

Proposition 2 ***Optimal Financing of Defense Spending***
Assume quasilinear preferences in consumption and no insurance motive, i.e. $\phi = 0$. Optimal financing of defense spending is such that following the increase in D_1, excess burden of taxation increases by more in the disaster state than in the normal state, i.e. $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$. Optimal financing of defense spending sacrifices smoothing across states to smoothing over time.

Proof. See Appendix A.1.3. ■

Proposition 2 assumes no insurance motive. In this case, cross-state smoothing of distortions is sacrificed whenever D_1 is financed by any mix of taxes and positive debt issuance. The same would hold for financing government expenditure g_0. Proposition 3 then highlights that the optimal mix of D_1 financing involves a larger share of borrowing and more backloading of tax distortions. The intuition is the following. Both debt-financed g_0 and D_1 increase the expected excess burden of taxation in period 1, $\mathbb{E}_0(\mu_1)$. However, because risk management through D_1 makes the disaster state less likely, $\mathbb{E}_0(\mu_1)$ increases by less in response to debt-financed D_1 than debt-financed g_0. Consequently, through the bond optimality condition, optimal μ_0 also increases by less, meaning there is a smaller increase in current taxes and a larger increase in debt. In this sense, proposition 3 states that, in the absence of insurance motives, the optimal financing mix of D_1 sacrifices cross-state tax smoothing by more than the optimal financing mix of government expenditure g_0.

Proposition 3 ***Debt Levels and Defense Spending***
Assume quasilinear preferences in consumption and no insurance motive, i.e. $\phi = 0$. Optimal level of debt responds more strongly to changes in defense spending than to changes in government expenditure; i.e., $\frac{\partial b_1}{\partial D_1} > \frac{\partial b_1}{\partial g_0}$.

Proof. See Appendix A.1.4. ■

5 Quantitative Results

This section presents the quantitative results using the calibrated infinite-horizon model. We show that the qualitative results from the two-period model of section 4 hold in the infinite-horizon model, in absence of the assumptions of the two-period model. In addition to that, quantitative results allow us to gain insights that are inacessible analytically. We first discuss the model calibration and then turn to analyzing the results.

5.1 Calibration

The model frequency is annual with $\beta = 0.96$. We parameterize the utility function as follows: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $v(l) = B\frac{l^{1-\eta}}{1-\eta}$, with $\eta = 1.8$ to match the unitary Frisch elasticity of labor supply and $B = 5.71$ to match an average hours worked of 1/3 of the time endowment in the first-best case of the N state. Note that with this preference specification, Frisch elasticity is not constant but co-moves with the labor supply. The production function is linear $F(h) = Ah$, where A^N is normalized to a unit value.

We calibrate the peacetime government expenditure process using the U.S. government expenditure data for the years 1947-2023. Our definition of g_t is the federal government consumption and investment net of federal defense spending, both reported in the NIPA table 3.9.5. ρ^g and σ^g are estimated using a linearly detrended and deflated data series. We then set μ_g so that the model implied government share of output is equal to 13%, consistent with the data.

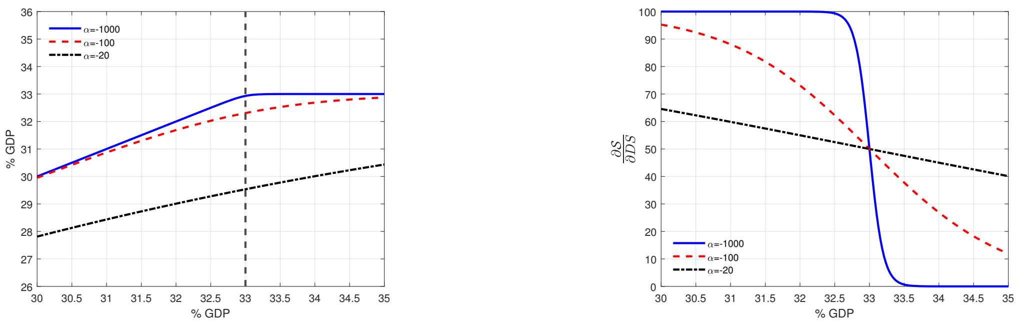
It is less obvious how to calibrate the war-related parameters. To get an estimate for δ, we estimate the depreciation rate of the main battle tanks using a sequence of sales of German Leopard 2A4 tanks over a decade. There, we observe the unit prices and have information on the production year.[4] This gives a value of 8.8% annual depreciation. We then set g^e to 0.13 so that, during war, government expenditure increases by 30% of the normal state GDP, consistent with the Ukrainian experience in 2022. Similarly, we assume that the war state productivity is 15% lower. ξ_t should be understood as a latent process driving the disaster risk. We set ρ_ξ to 0.97 to reflect the persistence in the foreign policy of potential aggressor countries.

Insurance. To discipline the insurance motive of DS we set ϕ to 0.5 implying that 50% of additional wartime expenditure needs can be met by depleting the DS stock. Additionally, the model solution through the system of optimality conditions requires taking the derivative of the function $\mathcal{S}(DS, g^e)$, which, in principle, is a nondifferentiable max function. We proceed by approximating it with a LogSumExp function of the form $\mathcal{S}(DS, g^e) = \frac{1}{\alpha}\log(e^{\alpha DS} + e^{\alpha\phi g^e})$, where $\lim_{\alpha\to\infty}\mathcal{S}(DS, g^e) = \max(DS, \phi g^e)$ and $\lim_{\alpha\to 0}\mathcal{S}(DS, g^e) = (DS + \phi g^e)/2$. A major benefit of LogSumExp over other smooth maximum operators is that it provides monotonic first deriva-

[4] The data can be found at https://www.army-guide.com/eng/product1645.html.

tives, which is needed for numerical work. Figure 3 illustrates the function and its derivative for different values of α.

Figure 3: LogSumExp at different values of α.



(a) $\mathcal{S}(DS, g^e)$. (b) $\frac{\partial \mathcal{S}(DS, g^e)}{\partial DS}$.

Risk-Management. We parameterize $P(DS, \xi)$ with a logistic function of the form

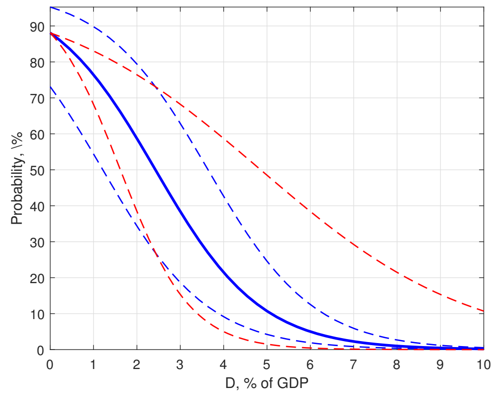
$$P(DS, \xi) = \frac{1}{1 + e^{-\beta_1 - \beta_2 \frac{DS}{\xi}}}, \tag{17}$$

where β_1 controls the level of risk when $DS = 0$ and β_2 controls the gradient with respect to DS. In the current calibration we set $\beta_1 = 2$ and $\beta_2 = -2$. This yields a disaster probability of 10% when annual defense spending is equal to 5% of the GDP.[5] Figure 4 illustrates $P(DS, \xi)$ at different values of β_1 and β_2, where the solid blue line shows our baseline specification. Table 1 summarizes all parameter values.

We solve the model using an algorithm similar in spirit to the Parameterized Expectations Algorithm proposed by den Haan and Marcet (1990). We provide details on the solution method in appendix A.3. The method relies on stochastic simulation and uses an artificial neural network to approximate forward-looking terms in the optimality conditions as functions of the state vector. We use stochastic simulation as the model has 6 state variables making the state-space too large for grid based methods. The presence of disasters requires approximating highly nonlinear dynamics with large deviations from the average values, rendering the nonparametric nature of the artificial neural network particularly handy.

[5] In the next version of the paper we aim to estimate β_1 and β_2 using the Geopolitical Risk Index from Caldara and Iacoviello (2022).

Figure 4: $P(DS, \xi)$ for different values of β_1 and β_2.



Notes: The figure shows the $P(DS, \xi)$ function for different values of β_1 and β_2 evaluated at $\xi = \mathbb{E}(\xi_t)$. The solid blue line shows the baseline specification. The dashed blue lines illustrate the effect of changing β_1 and the dashed red lines show the effect of changing β_2.

Table 1: Parameter values.

Parameter	Value	Target
β	0.96	Annual frequency
γ	2	Standard value
η	1.8	Average Frisch elasticity 1
B	5.71	Hours 1/3 of time endowment
β_1, β_2	$2, -2$	
δ	0.088	Depreciation rate of MBT[6]
$A^W - A^N$	0.15	
$g^W - g^N$	0.13	
μ_g, ρ_g, σ_g	$-0.145, 0.954, 0.032$	U.S. Federal non-defense expenditure
$\mu_\xi, \rho_\xi, \sigma_\xi$	$-0.05, 0.977, 0.010$	
ϕ	0.5	
α	100	

5.2 Quantitative Analysis

We now turn to the quantitative analysis, which consists of three main steps. First, we present long-run dynamics obtained over multiple realizations of exogenous processes. Our main focus is on the long-run distributions of endogenous variables. Second, we analyze generalized impulse

[6]Main Battle Tank.

responses to both g_t and ξ_t shocks. Third, we analyze the dynamics during war episodes, again, obtained through long stochastic simulations over multiple realizations of exogenous processes.

In all these experiments, we compare the baseline model against two benchmarks. In the first benchmark, we switch off the *insurance* motive by setting $\phi = 0$ and label this as the *No Insurance* benchmark. The purpose is to identify which of the two motives drives the results and to enable direct comparison with propositions 2 and 3, that also assume $\phi = 0$. In the second benchmark, we switch off both channels to render investment in D ineffective and label this as the *No D* benchmark. In this case, the economy still experiences ξ_t shocks but the planner can only respond to such uncertainty shocks using standard policy tools, such as taxes and debt. In this benchmark, we readjust the mean of the ξ_t process so that the average disaster probability is the same as in the baseline.

We show that the analytical results from section 4 hold in the infinite-horizon model. Specifically, regarding proposition 1, we show in subsection 5.2.2 that an increase in D_t is indeed related to a lower bond's price compared to the second benchmark. Regarding proposition 3, we show in subsection 5.2.1 that the baseline model has higher levels and more volatile debt. In 5.2.2 we also show that debt responds more strongly to both shocks. In subsection 5.3, we perform an additional exercise, where we do not readjust the disaster probability in the benchmark where neither channel is operative. This allows us to understand how investment in D_t affects the smoothing of distortions over time and across states. Quantitative results from this exercise are consistent with proposition 2.

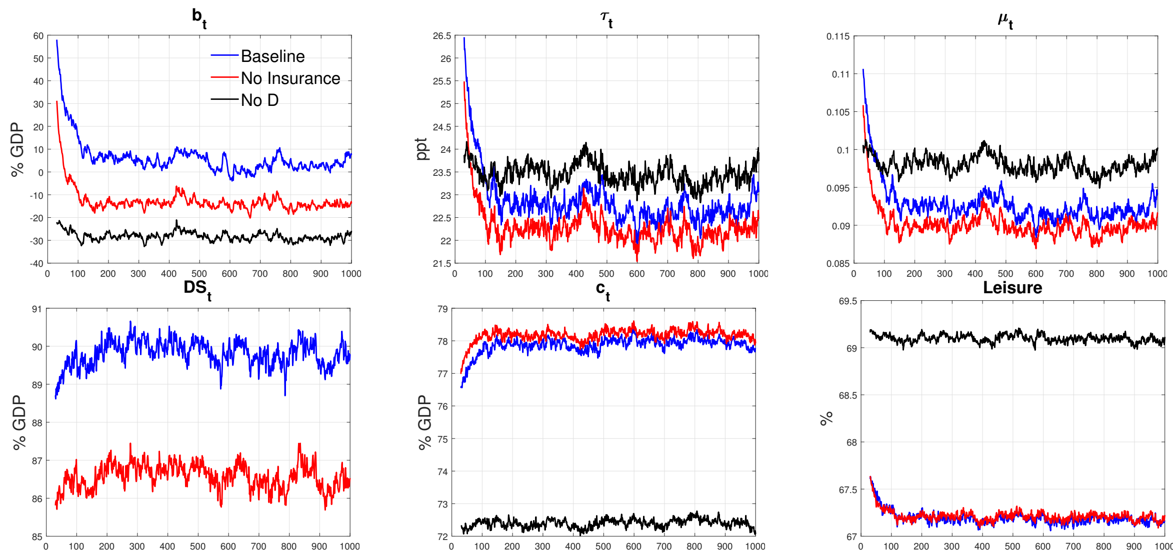
Besides confirming the analytical results, the quantitative results offer additional insights. The main takeaway is that most of the investment in DS is due to risk-management motives. This channel is also the main force driving the differential effects of bond prices in response to both g_t and ξ_t shocks. We proceed by discussing these results in greater detail.

5.2.1 Long-Run Moments

We start by comparing long-run moments across the three models. To allow for a fair comparison, in the *No D* benchmark, we set μ^g so that the peace time's average government spending $\mathbb{E}(g_t^N)$ matches the average total spending $\mathbb{E}(g_t^N + D_t)$ of the baseline model. Since g^N is linear in logs, we also adjust σ^g so that the standard deviation of g^N does not change. Furthermore, we tune μ^ϵ so that the average disaster probability in the *No D* benchmark is the same as in the baseline specification.

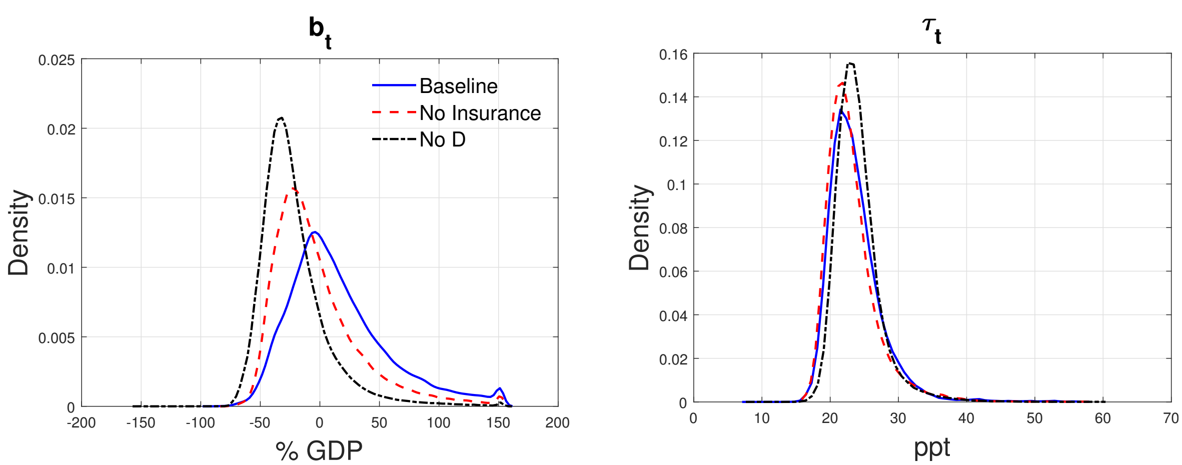
Figure 5 shows the average time paths. The first point to note is the striking difference in the average levels of debt. While in the baseline model it is around 10% of output, in the benchmark models, the planner accumulates assets up to 30% of output. Such differences are due to both *risk management* and *insurance* motives, since the path of debt in the benchmark without *insurance* motives lies in the middle between the other two models. While the *insurance* motive is an important determinant for the level of debt, the bottom left panel shows that investment in DS is mainly driven by *risk management* motives. In the baseline model the stock of DS tends to fluctuate around 90% of output, while in the case without *insurance* motives it is at 86%, just 3% lower.

Figure 5: Average time paths.



Notes: The figure shows average time series paths over N shock realizations. Specifically, series represent $x_t = \sum_{i=1}^{N} x_{i,t}$ with $N = 200$.

Figure 6: Long-run distributions of debt and taxes.



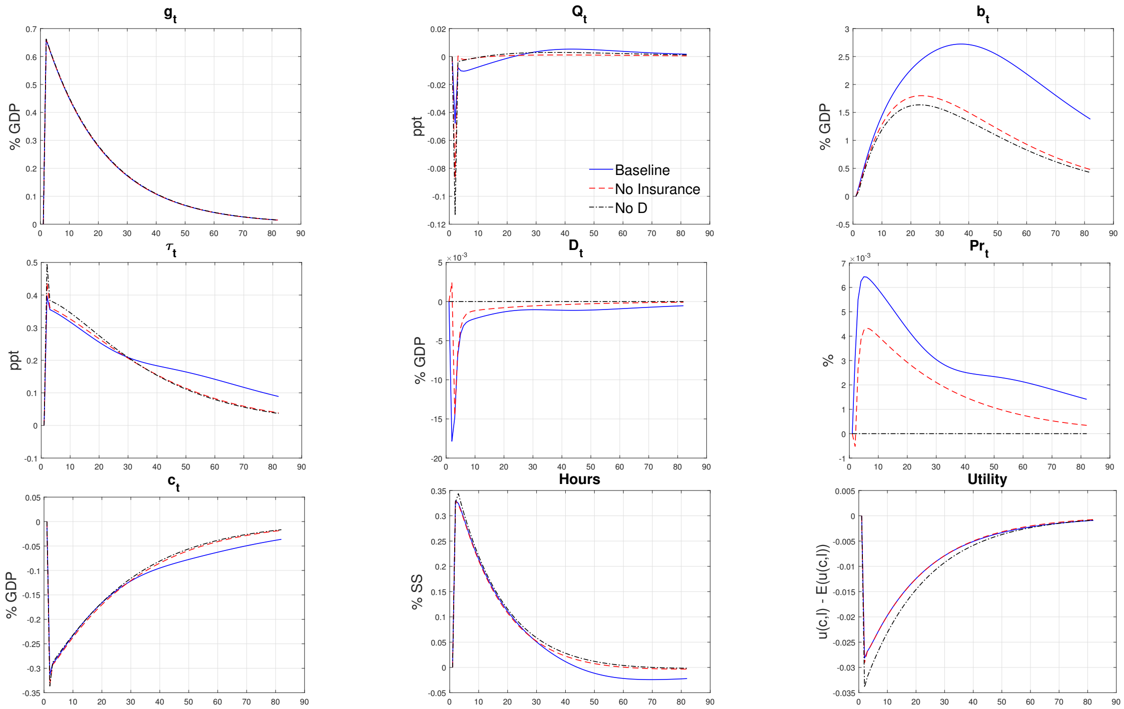
Notes: The figure shows long-run distributions of taxes and debt. Specifically, it shows fitted kernel densities using 100000 simulated data points that come from 200 simulations of 5000 periods each.

Figure 6 shows the long-run distributions of debt and taxes. The left panel shows that debt is not just on average higher in the baseline model but it is also more volatile, as indicated by higher tails of the distribution, suggesting that it is more responsive to shocks.

5.2.2 Generalized Impulse-Responses to g_t and ξ_t shocks

We now investigate the model responses to a one standard deviation positive shock to ϵ^g and to ϵ^ξ. The first exercise allows us to ask whether it is optimal to decrease investment in DS when other spending needs arise and what are the effects on prices and debt issuance. This allows for a neat comparison as the responses in the *No D* benchmark are the same as in Aiyagari, Marcet, Sargent, and Seppala (2002), which we refer to as "standard model." Figure 7 shows that the response in the standard model (dot-dashed black line) is to finance an increase in spending needs with a mix of taxes and debt. This is accompanied by an immediate drop in the bond's price as falling consumption in the shock period reduces household inter-temporal motives to save. The responses of the baseline model in the blue line show that it is optimal to cut defense spending. On impact, it falls by 1.5% of output or by 17% of the average spending on D. Such a cut has two effects. First, it allows to redirect labor tax income to financing g_t, easing the pressure to increase taxes. Second, it leads to falling defense stocks and, consequently, rising disaster probabilities. Such endogenous rise in risk increases the household's precautionary saving motives and alleviates a fall in bond prices relative to the standard model. As bond prices fall by less, the planner finds it optimal to issue more debt than in the standard model. Both of these channels contribute to a smaller increase in prices and a smaller drop in consumption in the baseline model. As the planner responds by issuing more debt, the tax response in the baseline model is more persistent. Analyzing the *No insurance* model, we can see that debt, risk, and price dynamics closely mimic those of the baseline model.

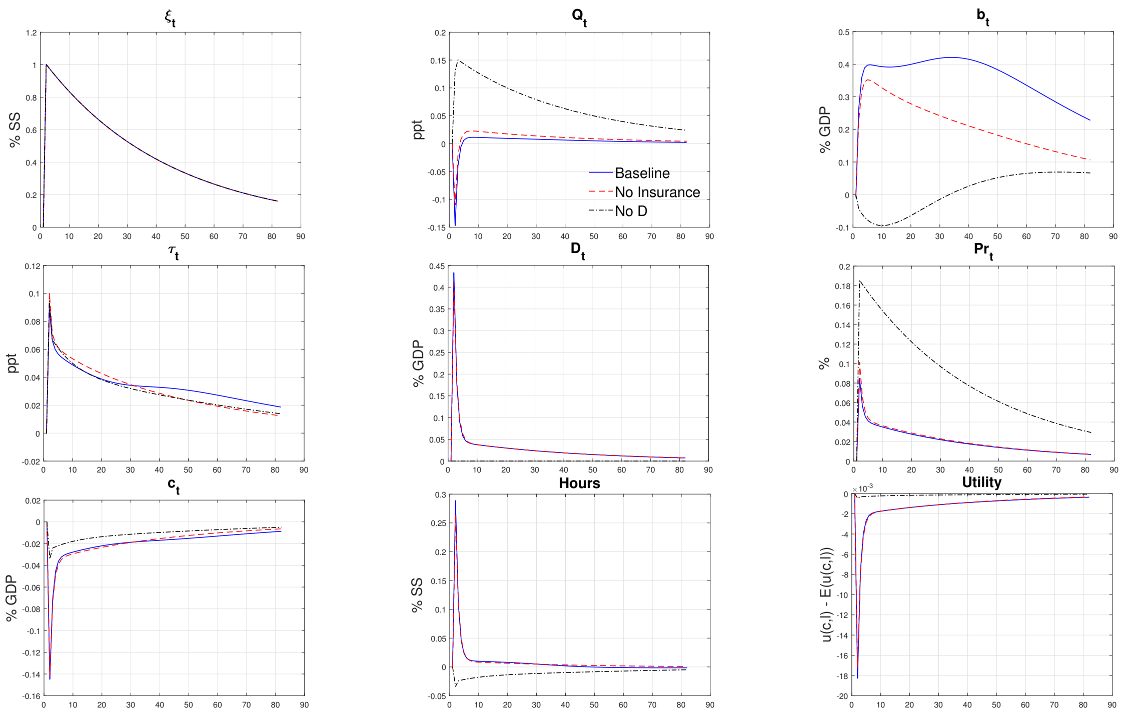
Figure 7: Generalized impulse-response to a government expenditure shock.



Notes: The figure shows the generalized impulse responses to a one standard deviation shock to ϵ_t^g. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line reports the benchmark where D is ineffective.

Next, we consider a one standard deviation shock to ϵ_t^ξ. Keeping DS fixed, a higher ξ increases the disaster probability. For this reason, we interpret ϵ_t^ξ as an uncertainty shock. In the standard model, where D is ineffective, the planner responds to an increased risk by accumulating assets financed by an increase in taxes. Essentially, the planner uses non-contingent assets to create insurance against the disaster state. Such asset purchases are associated with a minor fall in consumption and an increase in the bond's price, as the household's precautionary motives also increase in the presence of higher uncertainty. In our baseline model, as well as in the *No insurance* benchmark, the *risk management* motives are quantitatively strong and the planner forgoes insurance motives and responds by investing in DS, which is financed by debt issuance. Such policy entails a larger increase in taxes and a disproportionately larger fall in the household's consumption, which is needed to absorb the debt issuance. Increased investment in DS mitigates the rise in uncertainty and the household's precautionary saving motives relative to the standard model. For this reason, together with a falling consumption, the household's inter-temporal saving motive creates a downward pressure on the bond's price and in fact, the bond's price moves in the opposite direction compared to the standard model. Overall, in responding to uncertainty shocks, the planner weighs the benefits of building up reserves to meet the disaster state versus investing in DS to mitigate the disaster's risk, at the expense of a large fall in current consumption. Finally, the analysis shows that a fall in current consumption is optimal.

Figure 8: Generalized impulse-response to a ξ shock.

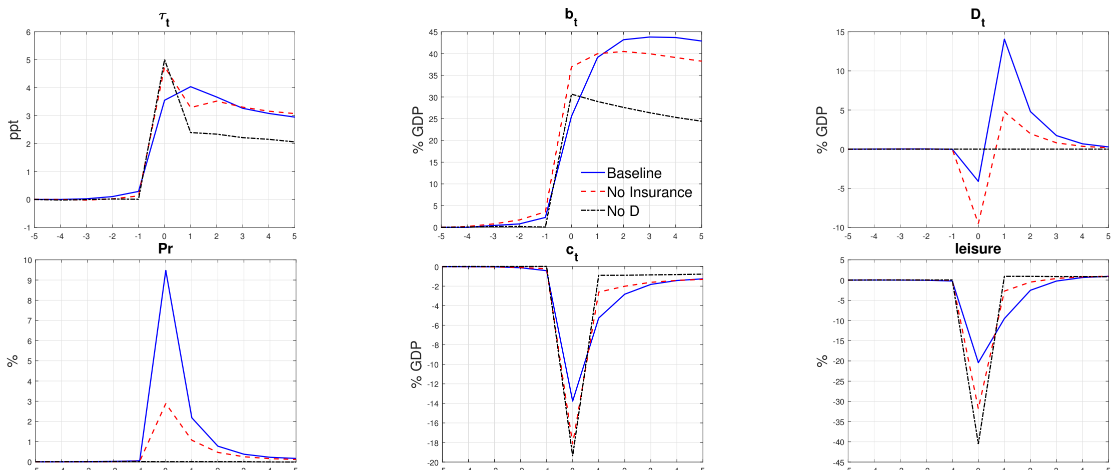


Notes: The figure shows the generalized impulse responses to a one standard deviation shock to ϵ_t^g. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line the benchmark model where D is ineffective.

5.2.3 War Episodes

As shown above, in the baseline model it is optimal to engage in risk management by building up defense stock, while simultaneously accumulating debt. Next, we look at what this policy implies for the dynamics during the war episode, compared to the standard model where it is optimal to accumulate assets. Figure 9 shows median dynamics around war episodes in all three models. The dashed black line shows that, in the standard model, the planner depletes asset reserves and even issues debt together with raising taxes. Unsurprisingly, consumption and leisure both fall. Next, consider the *No Insurance* benchmark. Even though in this economy average debt is higher at the beginning of war episodes, the planner issues even more debt and charges lower taxes compared to the standard model. The reason is that the planner cuts investment in DS causing the future war probability to increase. This, in turn, creates an upward pressure on the bond's price, enabling the planner to borrow more. These dynamics are further amplified in the baseline model, where the DS stock is depleted through the insurance channel, leading to an even larger increase in the future disaster's risk. This translates into an even higher upward pressure on the bond's prices and further borrowing. This mechanism, together with the *insurance* channel, allows for a significantly smoother consumption and leisure compared to the standard model. Since the DS stock gets depleted through the *insurance* channel, it is optimal to cut it by less during the war episode, requiring higher post-war investments to rebuild it. These dynamics partially rely on a counterfactual finding that defense spending falls during the war, which will be fixed in the future by allowing the insurance channel to depend on both defense stock DS_{t-1} and defense spending D_t.

Figure 9: Median dynamics during war episodes.



Notes: The figure shows median model dynamics around war events. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line reports the benchmark where D is ineffective. Simulated data comes from 200 shock realizations of 5000 periods. In total, this gives 7485 war episodes.

5.3 Smoothing taxes across states or over time?

In the last exercise, we are interested to validate the results from proposition 2 using the infinite-horizon model. To this end, we consider a benchmark specification, where the planner cannot invest in defense and the disaster probability is given by $P(war = 1) = P(0, \xi)$. Such specification makes the disaster probability constant and equal to 18.24%.[7] This is a specification analogous to the *No D* benchmark in the previous sections except that we do not readjust μ^ϵ and μ^g to make disaster probability and total spending the same as in the baseline. We compare it to the model with only the *risk management* channel, consistent with the assumption in proposition 2. Tables 2 and 3 show a few selected long-run moments. When disaster risk is endogenous, the planner

[7]For this exercise we use $\beta_1 = -1.5$ instead of $\beta_1 = 2$.

optimally invests in defense to reduce the probability of disaster from 18.24% to 3.24%. Such policy entails higher average peacetime spending and taxes, but fewer disaster events when the planner needs to finance the steep increase in government spending. Columns two and three show that, under disaster risk management, average spending and taxes are lower by 6% of GDP and 6 percentage points, respectively. The average level of debt is determined by two forces. On the one hand, according to proposition 3, peacetime investment in D_t entails more borrowing than financing of g_t. Hence, one would expect the risk-management model to have higher levels of debt. On the other hand, we have shown that it is optimal to borrow extensively during disaster episodes, which suggests that an economy with more frequent disasters would have higher levels of debt. Quantitatively, the second force dominates and the economy with risk-management has lower levels of debt. The last two columns of table 2 reports the average debt's level in the war and peace states. Consistently with proposition 3, the risk-management model is characterized by higher peace-time debt levels and large increases in debt during wars. As wars are less frequent in this model, the average tax burden is lower leading to lower average debt levels.

Table 2: Selected Moments.

	$\mathbb{E}(P(war=1))$ %	$\mathbb{E}(g+D)$ % of GDP	$\mathbb{E}(\tau)$ % of GDP	$\mathbb{E}(b)$ % of GDP	$\mathbb{E}(b^W)$ % of GDP	$\mathbb{E}(b^N)$ %
No D	18.24	20.54	23.93	-10.38	15.84	-16.54
No Insurance	3.24	14.39	17.84	-13.91	30.64	-15.66

Notes: The table shows selected long-run moments from the baseline and the counterfactual specification, where DS has no role and $P(war=1) = P(0,\xi)$. Simulated data comes from 200 shock realizations of 5000 periods.

Table 3 shows implication for tax smoothing by analyzing long-run moments of recursive multiplier μ_t, which measures the shadow value of relaxing the implementability constraint. Note that μ_t is an increasing transformation of the tax rate, therefore, understanding the dynamics of μ_t is analogous to understanding the dynamics of taxes. Under risk-management, by minimizing the occurrence of wars, the planner achieves lower values of μ_t and greater time smoothing of distortions, indicated by the lower variance of the multiplier in column two. The last two columns show the average values of μ_t in war and peace states. It shows that while the planner achieves greater smoothing of distortions on average, the difference between $\mathbb{E}\mu^W$ and $\mathbb{E}\mu^N$ is larger in the risk-management model. That is, consistently with proposition 2, it sacrifices the smoothing of distortions across states to smoothing over time.

Table 3: Selected Moments: multipliers.

	$\mathbb{E}(\mu)$	$\sigma(\mu)$	$\mathbb{E}(\mu^W)$	$\mathbb{E}(\mu^N)$
No D	0.099	0.0296	0.1109	0.0955
No Insurance	0.071	0.0208	0.0884	0.0701

Notes: The table shows selected long-run moments from the baseline and the counterfactual specification, where DS has no role and $P(war = 1) = P(0, \xi)$. Data comes from 200 shock realizations of 5000 periods.

6 Conclusion

In light of the rising geopolitical risks, it seems that Western governments are reaching a consensus that defense capabilities need to be increased. Yet, thus far there is little theoretical guidance on what is the optimal amount of defense spending and how these expenditures should be financed. In this paper, we fill this gap by studying the optimal policy problem where the Ramsey planner can respond to an increase in disaster's risk by investing in defense stock, which is important for *risk management* and *insurance* purposes. We show that *risk management* motives are quantitatively more important. Indeed, it is optimal to finance defense spending by borrowing and giving up tax smoothing across states to favor tax smoothing over time. In fact, the model where disasters are endogenous not only features higher levels of debt but also more responsive debt issuance in response to both government expenditure and uncertainty shocks. Looking further, it would be interesting to explore settings where the planner cannot commit to future policies and to solve for the optimal time-consistent policy, following Klein, Krusell, and Rios-Rull (2008). We speculate that the negative pressure of defense investment on the bond's price could counteract the current planners temptation to postpone tax distortions but we leave this for future work.

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A Appendix

A.1 Derivations and Proofs

A.1.1 Optimal Tax Rate

Start by laying out the optimality conditions for consumption and leisure:

$$
\begin{aligned}
& u_{c,t} + \mu_t \Omega_{c,t} - b_t u_{cc,t}(\mu_t - \mu_{t-1}) = \lambda_t, \\
& -v_{l_t} + \mu_t \Omega_{h,t} = -\lambda_t z_t.
\end{aligned}
$$

Divide through to eliminate the multiplier λ_t to get an expression for z_t that reads

$$
z_t = \frac{v_{l,t}(1 - \mu_t \Omega_{h,t}/v_{l,t})}{u_{c,t}(1 + \mu_t \Omega_{c,t}/u_{c,t} - u_{cc,t} b_t (\mu_t - \mu_{t-1})/u_{c,t})}.
$$

Note that $\tau_t = 1 - \frac{v_{l,t}}{z_t u_{c,t}}$. Also, define $\epsilon_{cc} \equiv -\frac{u_{cc} c}{u_c}$, $\epsilon_{hh} \equiv -\frac{v_{ll} h}{v_l}$, and $\epsilon_{ch} \equiv \frac{v_{cl} h}{u_c}$. This allows to express primary surpluses in terms of elasticities:

$$
\frac{\Omega_{c,t}}{u_{c,t}} = \frac{u_{cc,t}}{u_{c,t}} + u_{c,t}/u_{c,t} - \frac{v_{lc,t}}{u_{c,t}} = 1 - \epsilon_{cc} - \epsilon_{ch},
$$

$$
\frac{\Omega_h}{v_l} = -\frac{u_{cl} c}{v_l} - \frac{v_l}{v_l} + \frac{v_{ll} h}{v_l} = -\epsilon_{hc} - 1 - \epsilon_{hh}.
$$

Substitute in taxes and elasticities to get the following expression for z_t

$$
z_t = (1 - \tau_t) z_t \frac{1 + \mu_t (1 + \epsilon_{hh})}{1 + \mu_t (1 - \epsilon_{cc}) + \epsilon_{cc} b_t / c_t (\mu_t - \mu_{t-1})},
$$

and rearrange the equation above for τ_t to finally get

$$
\tau = \frac{\mu_t(\epsilon_{hh} + \epsilon_{cc}) - \epsilon_{cc} b_t / c_t (\mu_t - \mu_{t-1})}{1 + \mu_t (1 + \epsilon_{hh})}.
$$

A.1.2 Proof of Proposition 1

Proof.

Start by substituting out labor supply in terms of c, D, and g using the three aggregate resource constraints:

$$
\begin{aligned}
& c_0 + g_0 + D_1 = h_0 \rightarrow h_0 = h(c_0, g_0, D_1), \\
& c^N + g^N = h^N \rightarrow h^N = h(c^N, g^N), \\
& c^W + g^W - \mathcal{S}((1-\delta) D_1, \phi(g^W - g^N)) = h^W \rightarrow h^W = h(c^W, g^W, g^N, D_1).
\end{aligned}
$$

Hence, substitute out labor supply from the household's intra-temporal optimality condition

$\tau_t = 1 - \frac{v_l(1-h_t)}{c_t}$ and express consumption as a function of D, τ, and g:

$$\tau_0 = 1 - \frac{v_l(1-h(c_0,g_0,D_0))}{u_c(c_0)} \rightarrow c_0 = c_0(\tau_0, g_0, D_1),$$
$$\tau_0 = 1 - \frac{v_l(1-h(c^N,g_0))}{u_c(c^N)} \rightarrow c^N = c^N(\tau^N, g^N),$$
$$\tau^W = 1 - \frac{v_l(1-h(c^W,g^W,D_1))}{u_c(c^W)} \rightarrow c^W = c^W(\tau^W, g^W, g^N, D_1).$$

The bond's price $Q_0 = P(D_1)\frac{u_c(c^W)}{u_c(c_0)} + (1-P(D_1))\frac{u_c(c^N)}{u_c(c_0)}$ is then a function of $(g_0, g^N, g^W, D_1, \tau_0, \tau^N, \tau^W)$. These substitutions also allow to express the government's revenue in marginal utility terms $\Omega \equiv h\tau u_c(c)$ as a function of $(g_0, g^N, g^W, D_1, \tau_0, \tau^N, \tau^W)$. Finally, the planner's implementability constraints define a system that relates $(\tau_0, \tau^N, \tau^W, b_1)$ to $(D_1, g_0, g^N, g^W, b_0)$ as follows:

$$(g_0 + b_0)u_c(F(\tau_0, g_0, D_1)) = \Omega_0 + b_1 P(D_1)u_c(c^W) + (1-P(D^W))u_c(c^N), \tag{18}$$
$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N, \tag{19}$$
$$(g^W + b_1 - D_1(1-\delta))u_c(F(\tau^W, g^W, D_1)) = \Omega^W. \tag{20}$$

This can be simplified further by substituting out b_1 using the period 0's implementability constraint $b_1 = \frac{(g_0+b_0)u_c(F(\tau_0,g_0,D_1))-\Omega_0}{Q_0}$, which yields $b_1 = b_1(g_0, b_0, \tau_0, D_1, g^N, \tau^N, g^W, \tau^W)$. This gives the following system of two equations in two endogenous variables τ^N and τ^W:

$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N,$$
$$(g^W + b_1 - \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)))u_c(F(\tau^W, g^W, D_1)) = \Omega^W.$$

Implicitly defining $f(\tau^W, \tau^N, D_1, g_0)$ and assuming that τ_0 is constant, one can use the implicit function theorem to calculate $\left(\frac{\partial \tau^W}{\partial D_1}, \frac{\partial \tau^N}{\partial D_1}\right)$ and $\left(\frac{\partial \tau^W}{\partial g_0}, \frac{\partial \tau^N}{\partial g_0}\right)$. These objects then allow to get $\frac{\partial Q_0}{\partial D_1}$ and $\frac{\partial Q_0}{\partial g_0}$. It remains to be shown that $\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)} < 0$ and $\beta P(D_1)u_{cc}(c^W)\frac{\partial c^W}{\partial D_1} < 0$.

For what regards $\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)}$, $P'(D_1) < 0$ by assumption and $c^W \leq c^N$ as long as $g^W \geq g^N$. For what regards $\beta P(D_1)u_{cc}(c^W)\frac{\partial c^W}{\partial D_1}$, $u_{cc}(c^W) < 0$, we need to show the sign of $\frac{\partial c^W}{\partial D_1}$ using the household's intra-temporal condition:

$$u_c(c^W)(\tau^W - 1) + v_l(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N))) = 0.$$

Finally, the implicit function theorem yields:

$$\frac{\partial c^W}{\partial D_1} = -\frac{(1-\delta)v_{ll}(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1\phi, \phi(g^W - g^N)))\frac{\partial \mathcal{S}}{\partial D_1}}{u_{cc}(c^W)(\tau^W - 1) - v_{ll}(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)))} < 0.$$

■

A.1.3 Proof of Proposition 2

Proof. Using the optimal tax formula and assuming quasilinear preferences, so that $\epsilon_{cc} = 0$ one can express the multiplier in terms of tax rates:

$$\mu^W = \frac{\tau^W}{\epsilon_{hh} - \tau^W(1 + \epsilon_{hh})}. \tag{21}$$

Hence, the model can be characterized by the following system of equations:

$$\begin{aligned}
&\frac{\tau_0}{\epsilon_{hh} - \tau_0(1 + \epsilon_{hh})} = P(D_0)\frac{\tau^W}{\epsilon_{hh} - \tau^W(1 + \epsilon_{hh})} + (1 - P(D_0))\frac{\tau^N}{\epsilon_{hh} - \tau^N(1 + \epsilon_{hh})}, \\
&g^W + b_1 = h(\tau^W)\tau^W, \\
&g^N + b_1 = h(\tau^N)\tau^N, \\
&g_0 + b_0 + D_1 = h(\tau_0)\tau_0 + \beta b_1,
\end{aligned}$$

where we have used the household's optimality condition $\tau_t = 1 - v_l(1 - h_t)$ to express h as functions of the tax rate.

To simplify the system further, first express τ in terms of the multiplier: $\tau^x = \frac{\mu^x \epsilon_{hh}}{1+\mu^x(1+\epsilon_{hh})}$. Substitute out μ_0 using $\mu_0 = P(D_1)\mu^W + (1 - P(D_1))\mu^N$. Also substitute b_1 using $b_1 = 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0)$, where it is understood that h^x is a function of τ^x, and τ^x is a function of μ^x. μ_0 is a function of μ^W, μ^N and D_1. Then the model can be summarized by the following system:

$$\begin{aligned}
h^W\tau^W - g^W - 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0) = 0, \\
h^N\tau^N - g^N - 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0) = 0.
\end{aligned}$$

This system can be thought as an implicit function $f(\mu^W, \mu^N, D_1) = 0$. Apply the implicit function theorem to compute:

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -f_\mu^{-1} f_D, \tag{22}$$

where, after defining $H^x \equiv \frac{\partial h^x \tau^x}{\partial \mu^x}$ for ease of notation,

$$f_\mu = \begin{pmatrix} H^W + 1/\beta H_0 P(D_1) & 1/\beta H_0(1 - P(D_1)) \\ 1/\beta H_0 P(D_1) & H^N + 1/\beta H_0(1 - P(D_1)) \end{pmatrix},$$

and

$$f_D = \begin{pmatrix} -1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) \\ -1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) \end{pmatrix}.$$

In order to compute f_u^{-1} in equation (22), we need to compute the determinant $det(f_\mu)$ and the adjugate $adj(f_\mu)$. Respectively, these are:

$$\begin{aligned} det(f_\mu) &= [H^W + 1/\beta H_0 P(D_1)][H^N + 1/\beta H_0(1 - P(D_1))] - 1/\beta H_0 P(D_1) 1/\beta H_0(1 - P(D_1)) = \\ &= H^W H^N + 1/\beta H^W H_0(1 - P(D_1)) + 1/\beta H^N H_0 P(D_1), \end{aligned}$$

and

$$adj(f_\mu) = \begin{pmatrix} H^N + 1/\beta H_0(1 - P(D_1)) & -H_0 1/\beta(1 - P(D_1)) \\ -H_0 1/\beta P(D_1) & H^W + H_0 1/\beta P(D_1) \end{pmatrix}.$$

Finally, multiply $f_\mu^{-1} = adj(f_\mu)/det(f_\mu)$ by f_D to rewrite (22) as:

$$\frac{\partial \mu}{\partial D_1} = -\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)} \begin{pmatrix} H^N + 1/\beta H_0(1 - P(D_1)) - 1/\beta H_0(1 - P(D_1)) \\ -H_0 1/\beta P(D_1) + H^W + H_0 1/\beta P(D_1) \end{pmatrix},$$

which further simplifies to

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -\underbrace{\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)}}_{\equiv \mathcal{Z}} \begin{pmatrix} H^N \\ H^W \end{pmatrix}, \tag{23}$$

and where H^x can be written as:

$$H^x = \frac{\partial h^x \tau^x}{\mu^x} = \frac{\partial h^x \tau^x}{\partial \tau^x} \frac{\partial \tau^x}{\partial \mu^x} = \underbrace{\frac{\partial h^x \tau^x}{\partial \tau^x}}_{\text{Laffer slope}} \frac{\epsilon_{hh}}{1 + \mu^x(1 + \epsilon_{hh})}.$$

Assuming the economy is on the left-hand-side of the Laffer curve, then $H^W > 0$, $H^N > 0$, and $H_0 > 0$. Hence, $det(f_\mu) > 0$. On the left-hand-side of the Laffer curve $\tau^W \geq \tau^N$. According to equation (21), this implies that $\mu^W > \mu^N$, hence $-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) < 0$ and $\frac{\epsilon_{hh}}{1+\mu^W(1+\epsilon_{hh})} \leq \frac{\epsilon_{hh}}{1+\mu^N(1+\epsilon_{hh})}$. The fact that $det(f_u) > 0$ and that $-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) < 0$ imply that $\mathcal{Z} > 0$. Hence, in order to establish whether $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$, we need to investigate whether or not $H^N > H^W$. For this purpose, we are left with the task to study the terms $\frac{\partial h^N \tau^N}{\partial \tau^N}$

and $\frac{\partial h^W \tau^W}{\partial \tau^W}$.Given that we assumed that the economy is on the left-hand-side of the Laffer curve and that the Laffer curve is single peaked and, given differentiability, this also means that the Laffer curve is concave in τ. When the Laffer curve is concave in τ, we have $\frac{\partial h^N \tau^N}{\partial \tau^N} > \frac{\partial h^W \tau^W}{\partial \tau^W}$. Hence, $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$. ■

A.1.4 Proof of Proposition 3

Proof. Using the period 0's budget constraint, the response of debt to g_0 and D_1 is given by

$$\frac{\partial b_1}{\partial g_0} = 1/\beta \left(1 - \frac{\partial h_0 \tau_0}{\partial g_0}\right),$$
$$\frac{\partial b_1}{\partial D_1} = 1/\beta \left(1 - \frac{\partial h_0 \tau_0}{\partial D_1}\right).$$

Debt is more responsive to D_1 when $\frac{\partial h_0 \tau_0}{\partial D_1} < \frac{\partial h_0 \tau_0}{\partial g_0}$. Following the notation from the proof above

$$\frac{\partial h_0 \tau_0}{\partial g_0} = \frac{\partial h_0 \tau_0}{\partial \mu_0}\frac{\partial \mu_0}{\partial g_0} = H^0 \frac{\partial \mu_0}{\partial g_0},$$
$$\frac{\partial h_0 \tau_0}{\partial D_1} = \frac{\partial h_0 \tau_0}{\partial \mu_0}\frac{\partial \mu_0}{\partial D_1} = H^0 \frac{\partial \mu_0}{\partial D_1}.$$

One needs to compare $\frac{\partial \mu_0}{\partial g_0}$ and $\frac{\partial \mu_0}{\partial D_1}$

$$\frac{\partial \mu_0}{\partial g_0} = P(D_1)\frac{\partial \mu^W}{\partial g_0} + (1 - P(D_1))\frac{\partial \mu^N}{\partial g_0},$$
$$\frac{\partial \mu_0}{\partial D_1} = P(D_1)\frac{\partial \mu^W}{\partial D_1} + (1 - P(D_1))\frac{\partial \mu^N}{\partial D_1} + P'(D_1)(\mu^W - \mu^N),$$

where, through the bond's optimality condition, μ_0 is a function of D_1, μ^W, and μ^N. Use the implicit function theorem to get the effect on μ^W and μ^N. In the proof above we have shown that

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)} \begin{pmatrix} H^N \\ H^W \end{pmatrix},$$

which was equation (23). Similarly, the marginal effect of g_0 is

$$\frac{\partial \boldsymbol{\mu}}{\partial g_0} = -\frac{-1/\beta}{det(f_\mu)} \begin{pmatrix} H^N \\ H^W \end{pmatrix}. \tag{24}$$

Assuming the economy is on the left-hand side of the Laffer curve, $\frac{\partial \mu^x}{\partial D_1} > \frac{\partial \mu^x}{\partial g_0}$ for $x \in \{N, W\}$. The debt choice b_1 responds more to D_1 than to g_0, iff $\frac{\partial \mu_0}{\partial D_1} < \frac{\partial \mu_0}{\partial g_0}$. Equivalently, using the bond optimality condition (14), $\frac{\partial \mathbb{E}_0(\mu_1)}{\partial D_1} < \frac{\partial \mathbb{E}_0(\mu_1)}{\partial g_0}$

Expanding and rearranging terms gives

$$P(D_1)\left(\frac{\partial\mu^W}{\partial D_1}-\frac{\partial\mu^W}{\partial g_0}\right)+(1-P(D_1))\left(\frac{\partial\mu^N}{\partial D_1}-\frac{\partial\mu^N}{\partial g_0}\right)<P'(D_1)(\mu^N-\mu^W). \tag{25}$$

Using (23) and (24) it is easy to show that

$$\begin{aligned}\frac{\partial\mu^W}{\partial D_1}-\frac{\partial\mu^W}{\partial g_0}&=\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0 P'(D_1)(\mu^N-\mu^W),\\ \frac{\partial\mu^N}{\partial D_1}-\frac{\partial\mu^N}{\partial g_0}&=\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0 P'(D_1)(\mu^N-\mu^W).\end{aligned}$$

Using these expressions, the left-hand side of (25) is

$$P(D_1)\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0 P'(D_1)(\mu^N-\mu^W)+(1-P(D_1))\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0 P'(D_1)(\mu^N-\mu^W)=$$
$$\underbrace{\left(P(D_1)\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0+(1-P(D_1))\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0\right)}_{\mathcal{K}}P'(D_1)(\mu^N-\mu^W).$$

Given this, $\frac{\partial\mu_0}{\partial D_1}<\frac{\partial\mu_0}{\partial g_0}$ is equivalent to

$$\underbrace{\left(P(D_1)\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0+(1-P(D_1))\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0\right)}_{\mathcal{K}}P'(D_1)(\mu^N-\mu^W)<\frac{\partial P^W(D_1)}{\partial D_1}(\mu^N-\mu^W)$$

It remains to show that the $\mathcal{K}$ is less than one:

$$\mathcal{K}=\frac{1/\beta H_0 H^N P(D_1)+(1-P(D_1))H_0 H^W 1/\beta}{1/\beta H_0 H^N P(D_1)+(1-P(D_1))H_0 H^W 1/\beta+H^W H^N}<1,$$

since both the numerator and the denominator are positive and $H^W H^N>0$. Hence, $\frac{\partial b_1}{\partial D_1}>\frac{\partial b_1}{\partial g_0}$.

■

A.2 Optimal Policy under Full Commitment

We consider a full commitment approach to optimal debt and disaster management with incomplete bond markets.

Incomplete Markets

In this subsection, we solve for the time-inconsistent Ramsey plan under incomplete debt markets. The Ramsey planner seeks to maximize the household's utility (1) subject to a series of implementability constraints

$$b_t = \mathbb{E}_0 \left[\sum_{j=t}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right],$$

with multiplier μ and the law of motion for the defense stock

$$DS_t = DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e), \tag{26}$$

with multiplier μ_t^D. The Ramsey planner also needs to take into account that defense stock affects the disaster probability, i.e. $P(DS, \xi)$, and needs to take into account the $D_t > 0$ constraint, to which we assign multiplier λ_t^D. Additionally, the planner needs to respect the aggregate resource constraint

$$c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e) = A_t h_t.$$

More formally, the recursive Lagrangian of the planner reads:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) + v(l_t) + \mu_t(\Omega_t + \beta \mathbb{E}_t u_c(c_{t+1}) b_{t+1} - u_c(c_t) b_t) +$$
$$\mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e) - DS_t) +$$
$$+ \lambda_t (c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta), \phi g^e) - A_t h_t) + \lambda_t^D D_t \Big\},$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t) c_t - v_l(l_t) h_t$.

We list all optimality conditions in the following bullet points.

- c_t:

$$0 = u_c(c_t) + v_l(l_t)\frac{\partial l_t}{\partial c_t} + \mu_t \left(\frac{\partial (s_t u_c(c_t))}{\partial c_t} \right) - u_{cc}(c_t) b_t (\mu_t - \mu_{t-1}). \tag{27}$$

- b_{t+1}:

$$\mu_t = \frac{E_t(u_c(c_{t+1})\mu_{t+1})}{E_t(u_c(c_{t+1}))}. \tag{28}$$

- D_t:

$$0 = v_l(l_t)\frac{\partial l_t}{\partial D_t} + \mu_t \left(\frac{\partial s_t u_c(c_t)}{\partial D_t} \right) + \mu_t^D. \tag{29}$$

- DS_t:

$$\begin{aligned} \mu_t^D =& \beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^x \left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N) \right) + \beta \mathbb{E}_t \left(\mu_{t+1} \frac{\partial s_{t+1} u_c(c_{t+1})}{\partial DS_t} \right) + \\ & \beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_{t+1} \partial \mathcal{S}(DS_t, g_{t+1}^e \phi))}{\partial DS_t} \right). \end{aligned} \tag{30}$$

where:

$$\begin{aligned} \frac{\partial s_t u_c(c_t)}{\partial D_t} &= -\frac{v_l(l_t)}{z_t} - v_{ll}(l_t)\frac{\partial l_t}{\partial D_t} \frac{c_t + g_t + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}, g_t^e \phi))}{z_t}, \\ \frac{\partial s_t u_c(c_t)}{\partial c_t} &= u_{cc}(c_t)c_t + u_c(c_t) - \frac{v_l(l_t)}{z_t} - v_{ll}(l_t)\frac{\partial l_t}{\partial c_t} \frac{c_t + g_t + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}, g_t^e \phi))}{z_t}, \\ \frac{\partial s_t u_c(c_t)}{\partial DS_{t-1}} &= -\frac{v_l(l_t)}{z_t} \frac{\mathcal{I}_t \partial \mathcal{S}(DS_{t-1}, g_t^e \phi))}{\partial DS_{t-1}} - \frac{c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}, g_t^e \phi)}{z_t} v_{ll}(l_t) \frac{\partial l_t}{\partial DS_{t-1}}, \\ \frac{\partial l_t}{\partial D_t} &= -\frac{1}{z_t}, \\ \frac{\partial l_t}{\partial c_t} &= -\frac{1}{z_t}, \\ \frac{\partial l_t}{\partial DS_{t-1}} &= \frac{1}{z_t} \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}, g_t^e \phi))}{\partial DS_{t-1}}. \end{aligned}$$

Note that we did not explicitly take the optimality condition with respect to leisure. Instead, we used the aggregate resource constraint to substitute out leisure in terms of consumption. Also note that $\mathbb{E}^x$ denotes the expectation operator over g_{t+1} and ξ_{t+1} after integrating out uncertainty over the disaster state. These four optimality conditions together with the implemantability constraints

$$\Omega_t + \beta \mathbb{E}_t u_c(c_{t+1}) b_{t+1} - u_c(c_t) b_t = 0, \tag{31}$$

and the law of motion for DS_t equation (26) characterize the model equilibrium dynamics.

A.3 Solution Algorithm

Here we provide a brief summary of the algorithm. More implementation details along with the sample code can be found in Valaitis and Villa (2024). PEA algorithm requires making a projection of expected value terms on the state variables. We do this by projecting the integrands in the expected value terms in the system of equations (27),(28), (29), (30), (26), (31) onto the state variables using an artificial neural network. The we use Gaussian quadrature to approximate the expected value terms. Solution algorithm:

1. Generate a sequence of shocks $\{g_t, \xi_t\}_{t=1}^T$. Given an educated guess, initialize the neural network $\mathcal{ANN}(g_t, \xi_t, \mu_{t-1}, b_{t-1}, DS_{t-1}, \mathcal{I}_t)$, where $\mathcal{I}_t$ indicates whether economy is n the disaster state.

2. Given this guess, simulate the model by solving the system of equations (27),(28), (29), (30), (26), (31) at every t to obtain sequences of endogenous variables.

3. Given the simulated sequence train the neural network and update network weights.

4. Check if the $\mathcal{ANN}$ predictions are consistent with the simulated data and the network weights do not change across iterations. If not, go back to step 2 and simulate the model again using the updated neural network.