

Self-Imposed Debt Ceilings as Partial Commitment Devices^{*}

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Abstract

This paper shows that self-imposed debt ceilings can serve as intermediate commitment devices that alter sovereign bond pricing by disciplining future borrowing. In a sovereign default model with long-term debt, each government inherits a previously announced ceiling but may revise it by paying a political or institutional deviation cost. This friction generates a state-dependent form of partial commitment. The ceiling mitigates debt dilution at the expense of fiscal flexibility, leading the government to optimally limit the discretion of its future selves. Calibrating the model to Argentina, we show that moderate ceilings reduce sovereign spreads and raise welfare by limiting dilution, while excessively tight ceilings increase default risk by constraining fiscal flexibility. Governments therefore voluntarily choose ceilings that are costly to revise yet flexible enough to adjust when needed, meaningfully disciplining future borrowing and improving macroeconomic outcomes, even without an external enforcer.

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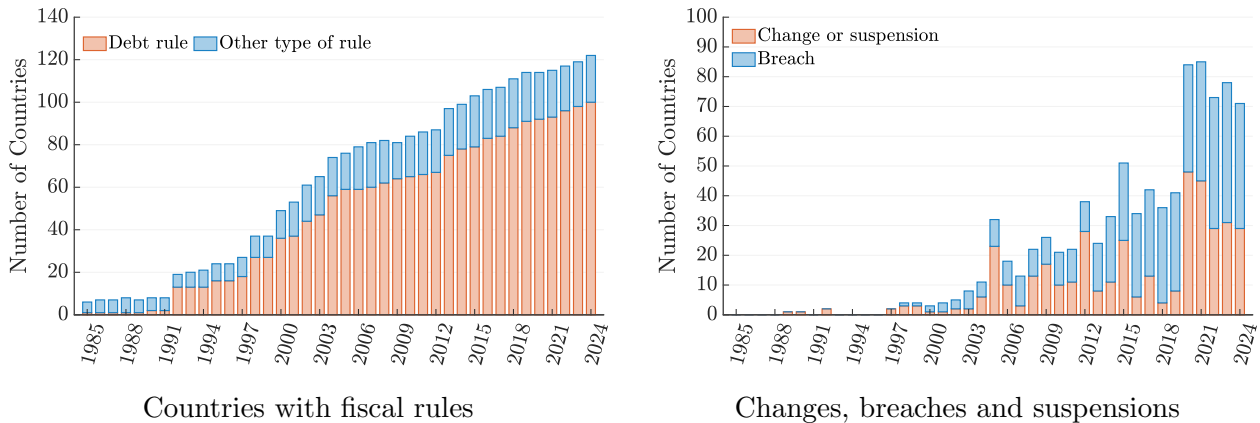
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1 Introduction

Sovereign bond markets price not only the fundamentals that determine a government’s capacity to repay, but also its incentives to dilute existing creditors by issuing additional debt. Such dilution incentives create a classic dynamic tension: governments borrow more in the future than is ex-ante optimal, raising default risk and current borrowing costs. To manage these frictions, many countries have adopted fiscal rules—including debt ceilings, deficit limits, and expenditure caps—that function as policy anchors for fiscal behavior and have proliferated over the past four decades (left panel of Figure 1). Over the same period, changes, suspensions, and breaches of these rules have also become more frequent (right panel of Figure 1), suggesting that governments value the discipline fiscal rules provide but also face incentives to revise them when conditions deteriorate. Why do governments adopt fiscal rules they may later overturn, and what are the implications for sovereign spreads and macroeconomic outcomes?

Figure 1: Trends in the Adoption and Revision of Fiscal Rules



Note: The figure documents the rise in fiscal rules and in subsequent changes, suspensions, and breaches. Data through 2021 come from the IMF Fiscal Rules Database. Data for 2022–2024 were compiled using IMF Article IV reports, the Inter-American Development Bank’s macroeconomic country reports, and country-specific sources. The updated series currently covers the 106 countries included in the 2021 IMF database.

In this paper, we show that self-imposed debt ceilings can serve as intermediate commitment devices that discipline future borrowing while balancing the loss of fiscal flexibility: ceilings that are too loose provide little commitment, whereas ceilings that are too tight excessively constrain policy. By “self-imposed,” we mean ceilings that governments voluntarily adopt and optimally choose period by period but whose revision entails political or institutional deviation costs.¹ When borrowing above a previously announced ceiling triggers such

¹For example, violating a ceiling may lower the incumbent party’s probability of reelection.

a deviation cost, incumbents can limit the discretion of their future selves and commit them to a more disciplined borrowing path. This partial-commitment mechanism reduces expected dilution and lowers spreads, even without an external enforcer. Governments therefore have incentives to choose ceilings that are costly to revise yet flexible enough to adjust when needed, meaningfully disciplining future borrowing and improving macroeconomic outcomes.

In this context, we develop a sovereign default model with long-term debt in which each government strategically announces a debt ceiling for the following period, and the successor inherits this promise while retaining the option to borrow above it by paying a deviation cost. Within this environment, governments voluntarily choose self-imposed debt ceilings that are costly to revise yet flexible enough to adjust when necessary. These ceilings meaningfully discipline future borrowing, thereby reducing sovereign spreads and improving macroeconomic outcomes, even in the absence of an external enforcer.

The underlying mechanism hinges on a simple trade-off that balances two economic forces. On the one hand, carrying the debt ceiling forward as a binding promise gives the government partial commitment not to issue excessive long-term bonds in the future, thereby mitigating the classic “debt dilution” bias in [Eaton and Gersovitz \(1981\)](#)-type models with long-term debt: when additional issuance today raises the probability of default tomorrow, investors anticipate legacy-holder ex-post dilution incentives and demand higher yields, lowering bond prices ([Hatchondo and Martinez, 2009](#); [Chatterjee and Eyigungor, 2012](#)). On the other hand, the same ceiling restricts the government’s ability to respond to adverse shocks by issuing additional debt, which can make default more attractive in some states and thus raise the risk of actual repayment failure. The overall welfare impact reflects the trade-off between lower borrowing costs through enhanced commitment and higher default risk due to reduced fiscal flexibility. Naturally, endogenous intermediate commitment choices emerge from the balance between preserving fiscal flexibility and restraining the successor in order to keep borrowing costs low.

Hence, a central implication of the mechanism is that credible self-imposed constraints reduce dilution risk and, in turn, lower sovereign spreads. To provide simple motivating evidence for this prediction, we run the following cross-country regressions in the spirit of [Cruces and Trebesch \(2013\)](#):²

$$\text{EMBI Spread}_{i,t} = \alpha_i + \gamma_t + \beta \text{FiscalRule}_{i,t} + Z'_{i,t}\delta + \varepsilon_{i,t},$$

²We use a sample of 57 countries up to 2019, augmenting their dataset with the fiscal-rule indicator constructed from the IMF Fiscal Rules Database (the same source underlying [Figure 1](#)). We extend the original Cruces–Trebesch dataset using the updated series in [Arce and Fourakis \(2025\)](#).

where the dependent variable is the EMBI Global sovereign spread (a monthly measure of hard-currency borrowing costs for emerging economies), $\text{FiscalRule}_{i,t}$ is a dummy equal to one when country i has an active fiscal rule at time t , and $Z_{i,t}$ collects the lagged restructuring and haircut variables, global risk proxies, and standard macro controls. Appendix Table 2 shows that the fiscal-rule coefficient β is consistently negative and statistically significant across several specifications, implying that countries with a fiscal rule exhibit EMBIG spreads roughly 40–60 basis points lower than otherwise similar economies. Moreover, Appendix Table 3 documents a parallel result for market volatility: fiscal-rule adopters display significantly lower monthly EMBIG volatility (a reduction of about 4–5 basis points in the standard deviation of daily returns). This empirical pattern aligns with the mechanism in our model, in which self-imposed fiscal constraints act as partial commitment devices that reduce perceived sovereign risk.

Our analysis proceeds in two steps. First, we provide analytical results using a three-period model in which all debt issued in periods 1 and 2 is repaid (or defaulted on) in period 3, which isolates the core mechanism. Without commitment, the period-2 government overborrows relative to the commitment allocation, because it fails to internalize how additional issuance dilutes legacy bondholders and raises future default risk. Anticipating this behavior, the period-1 government strategically announces a debt ceiling for period-2 borrowing, backed by a cost of violating the ceiling, in order to discipline its successor. Depending on the size of the violation cost, the resulting allocation lies strictly between the no-commitment and full-commitment benchmarks. Once the cost exceeds a simple threshold—which we also characterize analytically—the ceiling fully sustains the commitment allocation, and further tightening has no effect. Moreover, we show that this threshold structure is robust to alternative cost functions, under which breaches may or may not occur in equilibrium but the basic mechanism disciplining period-2 borrowing remains the same.

Second, we develop a stochastic infinite-horizon model in which a sequence of Markov governments make borrowing and default decisions and announce a debt ceiling for the following period. The successor inherits this ceiling but may issue above it by paying a deviation cost, adapting the partial-commitment mechanism of Clymo, Lanteri and Villa (2023) to sovereign debt limits. Embedding these announcements in a sovereign default environment is essential: dilution incentives, endogenous default risk, and bond pricing jointly determine the value of commitment. We calibrate the model to Argentina—where no formal debt ceiling exists—and conduct counterfactual experiments that evaluate the welfare consequences of introducing an optimally announced ceiling. The results show that the government voluntarily imposes a costly ceiling on itself in order to harness the benefits of partial commitment. By disciplining its successor, the self-imposed rule lowers borrowing

costs and raises welfare, even in the absence of any external enforcer. The magnitude of these gains depends on the structure of the cost function and on the strength of the deviation penalty.

The results highlight a broad message for sovereign debt markets: credible but flexible fiscal rules can serve as valuable commitment devices, reducing spreads and improving welfare by mitigating dilution, while fully rigid rules may backfire by increasing default risk.

Related literature. This paper contributes to two strands of the literature: (i) sovereign default and fiscal rules, and (ii) partial commitment in optimal fiscal policy. The sovereign-default literature studies dilution and fiscal rules under exogenously given commitment frictions, while the partial-commitment literature develops costly-deviation mechanisms outside default environments. We bring the two together by modeling debt-ceiling announcements as an endogenous source of partial commitment within a sovereign-default framework, and we use the framework to show how governments can benefit from voluntarily restricting their successors’ choices.

Sovereign Default and Fiscal Rules. In sovereign-debt models à la [Eaton and Gersovitz \(1981\)](#) with long-term debt, the “debt-dilution” time inconsistency highlighted by [Hatchondo and Martinez \(2009\)](#) and [Chatterjee and Eyigungor \(2012\)](#) generates persistent deficits: new bond issues dilute legacy claims, raising required yields and lowering bond prices. Subsequent work has quantified the welfare losses from dilution ([Aguiar et al., 2020](#)) and proposed both state-contingent rules that eliminate dilution ([Hatchondo et al., 2016](#)) and simpler rules that mitigate its impact ([Hatchondo et al. 2022](#); [Roch and Roldán 2023](#)), evaluating gains by comparing commitment versus no-commitment equilibria. Relatedly, [Mateos-Planas et al. \(2025\)](#) examine how different exogenously imposed forms of commitment—such as commitment to default thresholds or to continuation prices—alter default outcomes in standard sovereign-debt models. Their commitment structures are taken as given and operate directly on default conditions. In our framework, commitment instead arises endogenously: governments choose debt-ceiling announcements period by period to constrain their successors, who may revise them only by incurring a cost. This mechanism generates state-dependent degrees of partial commitment as an equilibrium outcome and shows how governments can benefit from voluntarily restricting the discretion of their future selves. It thereby complements the normative “rules versus flexibility” literature initiated by [Amador et al. \(2006\)](#) and developed in [Halac and Yared \(2014a, 2017, 2020, 2022\)](#) by showing how fiscal rules operate when the government is endowed with an endogenous, state-dependent choice of rule—a debt

ceiling—that it strategically announces each period and uses in equilibrium to discipline its future selves, while allowing successors to breach it at a political and institutional cost.³

Optimal Fiscal Policy with Partial Commitment and Fiscal Announcement. Our framework is related to the literature on optimal fiscal policy under limited commitment and on fiscal announcements. First, on intermediate commitment (“partial commitment”), we build on the idea that governments make non-contingent announcements but face costs when deviating from them. Papers in this literature that study partial commitment typically model re-optimization as arriving exogenously (e.g. [Debortoli and Nunes, 2010, 2013](#)), treating deviations as opportunities that arise independently of the state. Unlike these approaches, we adopt the costly-deviation mechanism of [Clymo et al. \(2023\)](#) and adapt it to sovereign debt ceilings and default, so that the extent of reneging—and thus the degree of commitment—is determined endogenously by the model’s state variables. This state-dependent slack then shapes strategic debt-ceiling announcements. Relatedly, [Farhi \(2010\)](#), [Klein et al. \(2008\)](#), and [Karantounias \(2019\)](#) use generalized Euler–equation methods to explore time consistency and default, while [Clymo and Lanteri \(2020\)](#) show that even short-horizon commitment can sustain first-best outcomes. We extend their approach by introducing costly, state-contingent reneging of debt ceilings in a stochastic economy with sovereign default, where dilution incentives, endogenous default risk, and bond pricing jointly determine the value of commitment.

Second, on fiscal announcements per se, we bridge optimal-policy theory with the empirical and quantitative work that treats announcements as exogenous drivers of expectations. Empirical papers such as [Mertens and Ravn \(2012\)](#) and [Alesina et al. \(2015\)](#) document the macro effects of announced plans, and quantitative studies like [Mertens and Ravn \(2011\)](#) and [Fernández-Villaverde et al. \(2015\)](#) embed announcement “shocks” in DSGE settings. By distinguishing between announced and implemented policies—and by endogenizing the cost of deviating from announcements—we embed insights from the empirical literature on fiscal announcements in an optimal-policy framework. Our approach also relates to the fiscal-rules literature (e.g., [King et al. 1988](#); [Schmitt-Grohe and Uribe 1997](#); [Athey et al. 2005](#); [Halac and Yared 2014b](#)), which shows that limits on state contingency can amplify fluctuations. We demonstrate that costly, partial state contingency—driven by political constraints on commitment and strategic debt-ceiling announcements in a sovereign-default environment—generates rich dynamics with meaningful policy implications for the design of fiscal rules and debt-management frameworks.

³See also [Espino et al. \(2022\)](#) for an analysis of fiscal-rule suspensions during the COVID-19 crisis.

Structure of the paper. The remainder of the paper is organized as follows. Section 2 presents a three-period version of the model and delivers analytical results that clarify the mechanism. Section 3 develops the quantitative sovereign default model with endogenous debt-ceiling announcements, showing that self-imposed ceilings improve borrowing terms by mitigating debt dilution but may also constrain fiscal flexibility and raise default risk; the resulting strategic interactions generate intermediate, history-dependent fiscal rules that enhance welfare and reduce spreads. Section 4 concludes.

2 Stylized Model

This section presents a tractable three-period benchmark that isolates the core mechanism behind our debt-ceiling results.

Environment. Time is discrete with three periods, $t \in \{1, 2, 3\}$. The economy has no output in the first two periods, $y_1 = y_2 = 0$, and produces a deterministic endowment $\bar{y} > 0$ in the final period. The representative government values consumption according to

$$\sum_{t=1}^3 \beta^{t-1} u(c_t), \quad \text{where} \quad u(c) = -\frac{1}{c},$$

corresponding to CRRA preferences with risk aversion $\sigma = 2$.

The government issues debt in periods 1 and 2, both maturing at $t = 3$. Let b_1 denote the face value issued in period 1 (long-term debt) and b_2 the face value issued in period 2 (short-term debt). Total obligations due at maturity are therefore $B \equiv b_1 + b_2$.

For analytical convenience, define the *repayment slack*

$$x \equiv \frac{\bar{y} - B}{\bar{y}} \in (0, 1], \quad \text{so that} \quad B = \bar{y}(1 - x). \quad (1)$$

The variable x measures the fraction of endowment remaining after meeting debt obligations, or equivalently, the economy's fiscal space at repayment. Higher x indicates lower debt and a greater capacity to repay.

Default, pricing, and period-3 utility. At $t = 3$, the government decides whether to repay or default. Output is $\bar{y} > 0$, and default entails an output loss governed by a random cost parameter θ . We assume θ follows a Pareto distribution on $[1, \infty)$ with shape parameter $\alpha = \frac{1}{2}$, so higher realizations of θ correspond to more severe output losses. If the government

defaults, it retains only \bar{y}/θ , whereas full repayment requires transferring $\bar{y} - B$ to creditors. Default occurs whenever repayment is more costly than default:

$$\underbrace{\bar{y} - B}_{\text{Cost of repayment}} < \underbrace{\frac{\bar{y}}{\theta}}_{\text{Resources in default}} \iff \theta < \frac{1}{x}.$$

Intuitively, when debt obligations are large relative to output, the fiscal space x shrinks, so the government is willing to default even for relatively low realizations of θ .

Because lenders are risk neutral and the risk-free rate is normalized to one, the bond price equals the probability of repayment,

$$q(B) = \Pr(\theta > 1/x) = x^\alpha = \sqrt{x}. \quad (2)$$

Higher debt (lower slack x) reduces the repayment probability and hence the bond price. At $t = 3$, consumption c_3 represents the government's resources after deciding whether to repay or default. If it repays, it transfers $\bar{y} - c_3 = B$ to creditors; if it defaults, it avoids repayment but loses a fraction of output. Formally,

$$c_3 = \begin{cases} \bar{y} - B, & \text{if the government repays,} \\ \frac{\bar{y}}{\theta}, & \text{if the government defaults,} \end{cases} \quad (3)$$

where $\theta \geq 1$ governs the output loss in default. Consumption is therefore lower either because resources are used to service debt or because output is reduced by the default penalty. Given the pricing kernel (2), consumption (3), and the stochastic assumption on θ , the expected utility at $t = 3$ conditional on B is

$$\mathbb{E}[u(c_3) \mid B] = \frac{1}{\bar{y}} \left(1 - 2x^{-1/2} \right), \quad (4)$$

which captures the welfare cost of debt through two channels: a lower probability of repayment, embedded in $q(B)$, and reduced consumption in default states.

2.1 Sustaining Commitment through a Debt Ceiling

We now use a three-period framework to characterize how a debt ceiling can help sustain the commitment allocation. We begin by characterizing the commitment and no-commitment allocations. This benchmark allows us to isolate the fundamental tension between the

period-2 government's borrowing incentives and the welfare of the period-1 government. Under commitment, period-2 borrowing is predetermined and cannot respond myopically, while under no commitment the period-2 government issues excessive debt because it does not internalize how additional borrowing dilutes legacy bondholders and raises default risk. Having established these extremes, we then introduce an intermediate case in which the period-1 government can announce a debt ceiling, but the period-2 government may exceed it only by paying a fixed cost $\phi > 0$. This costly-deviation friction is irrelevant under commitment and non-binding when $\phi = 0$ (no commitment), but for $\phi > 0$ it partially disciplines period-2 borrowing and allows the incumbent to move the allocation toward the commitment benchmark.

Commitment. Under full commitment, the government can choose both b_1 and b_2 in period 1 to maximize lifetime welfare, fully internalizing how total debt affects future borrowing costs and repayment risk. Lifetime welfare is given by

$$V_1^C(b_1, b_2) = u(c_1) + \beta u(c_2) + \beta^2 \mathbb{E}[u(c_3) \mid B]$$

with $c_t = q(B)b_t$. Because the government internalizes the future price response $q(B)$, borrowing is disciplined across both bonds. In this three-period environment, commitment at $t = 1$ is equivalent to full commitment: choosing b_1 and b_2 jointly fixes the entire path of debt and precludes any subsequent reoptimization at $t = 2$. The first-order conditions characterizing the commitment allocation are

$$u'(c_1)(q(B) + b_1 q'(B)) + \beta \left(u'(c_2)b_2 q'(B) + \beta \frac{\partial}{\partial b_1} \mathbb{E}[u(c_3) \mid B] \right) = 0, \quad (5)$$

$$u'(c_1) b_1 q'(B) + \beta \left(u'(c_2)(q(B) + b_2 q'(B)) + \beta \frac{\partial}{\partial b_2} \mathbb{E}[u(c_3) \mid B] \right) = 0. \quad (6)$$

These conditions balance the marginal benefit of issuing an additional unit of debt at $t = 1$ or $t = 2$ with its marginal cost. The terms $q(B) + b_t q'(B)$ capture the direct effect of higher borrowing on time- t resources: issuing one more unit of b_t raises revenue through the bond price $q(B)$ but also lowers that price through $q'(B)$, reducing the value of outstanding debt.

In the condition for b_1 , the additional term $\beta u'(c_2)b_2 q'(B)$ reflects that increasing b_1 also changes the bond price at $t = 2$, thereby affecting period-2 consumption through its effect on total debt $B = b_1 + b_2$. In the condition for b_2 , the term $u'(c_1)b_1 q'(B)$ captures the symmetric effect on the previously chosen b_1 : when the planner selects b_2 , the induced change in $q(B)$ alters the valuation of existing obligations. Under commitment the planner internalizes this

effect, whereas a period-2 government under discretion would ignore it, which is the source of the familiar time-inconsistency problem.

Finally, the last terms, $\beta^2 \frac{\partial}{\partial b_t} \mathbb{E}[u(c_3) \mid B]$, capture the discounted marginal cost of increasing total obligations B , which lowers expected utility in period 3 through a higher probability of default or a greater repayment burden.

The solution to equations (5) and (6) is a pair (b_1^C, b_2^C) , which must satisfy the following equation obtained by dividing the two first-order conditions:

$$\beta \frac{u'(c_2)}{u'(c_1)} = 1 \quad (7)$$

Condition (7) is the standard Euler equation for intertemporal allocation under commitment. It requires the government to choose (b_1^C, b_2^C) so that the discounted marginal utilities of consumption in periods 1 and 2 are equalized. Equivalently, the implied discount factor between the two dates is one, reflecting that the government bond is the only asset and its price effects are fully internalized.

We can further simplify condition (7) to get the following simple proportionality:

$$b_2^C = \sqrt{\beta} b_1^C,$$

so the ratio between b_2^C and b_1^C is pinned down entirely by the discount parameter. Because $\sqrt{\beta} < 1$, the commitment allocation features more b_1^C than b_2^C , reflecting the government's ability to internalize future fiscal discipline. Substituting this relationship into the planner's problem yields a closed-form expression for b_1^C/\bar{y} , from which b_2^C follows immediately via the proportionality rule.⁴ The resulting debt levels are lower in both periods relative to the no-commitment case and borrowing is tilted toward period 1, where its marginal utility is highest, while the marginal cost of debt depends only on total exposure B . This choice contrasts with the no-commitment benchmark, where the period-2 government overissues debt at $t = 2$, generating higher overall borrowing.

No commitment. Without commitment, decisions are sequential. In period 2, the government takes the inherited debt stock b_1 as given and chooses b_2 to maximize current and future utility, since the government can't commit to any previously announce borrowing issuances.

⁴The explicit solution is provided in Appendix A.3.

The problem at $t = 2$ is:

$$V_2^{NC}(b_1) = \max_{b_2} \left\{ u(c_2) + \beta \mathbb{E}[u(c_3) \mid B] \right\},$$

with $c_2 = q(B) b_2$. The first-order condition with respect to b_2 is

$$u'(c_2)(q(B) + b_2 q'(B)) = -\beta \frac{\partial}{\partial b_2} \mathbb{E}[u(c_3) \mid B]. \quad (8)$$

Condition (8) is mathematically analogous to (6) under commitment, except that the period-2 government takes b_1 as predetermined. It equates the marginal benefit of issuing additional debt to its marginal cost. The left-hand side captures the immediate gain at $t = 2$: an extra unit of b_2 increases current resources by $q(B)$, but also lowers the bond price through $q'(B)$, which reduces the value of all outstanding short-term debt. The right-hand side reflects the discounted marginal cost of higher total obligations B at $t = 3$, which worsen expected utility through a higher likelihood or severity of default. Unlike under commitment, the period-2 government treats b_1 as predetermined and does not internalize how its borrowing decision interacts with the earlier choice of b_1 ; it optimally chooses b_2 taking b_1 as given, which will lead to excessive b_2 issuance relative to the commitment benchmark, as captured by Proposition 1.

Solving condition (8) yields the following closed-form policy function:

$$b_2^\dagger(b_1) = \frac{\bar{y}}{4\beta} \left[-3 + \sqrt{9 + 16\beta \left(1 - \frac{b_1}{\bar{y}} \right)} \right]. \quad (9)$$

The policy function $b_2^\dagger(b_1)$ is decreasing in the inherited long-term debt b_1 : a larger stock of outstanding obligations raises default risk and lowers the bond price, making additional borrowing less attractive. It is decreasing in β , since a more patient government discounts the future cost of repayment more heavily and therefore finds it optimal to issue less debt at $t = 2$. Equation (9) thus provides a closed-form characterization of the period-2 overborrowing motive.

At time 1, the government and the lenders anticipate that the time 2 government will reoptimize according to (9) and chooses b_1 to maximize

$$V_1^{NC}(b_1) = u(c_1) + \beta u(c_2(b_1)) + \beta^2 \mathbb{E}[u(c_3) \mid B],$$

with $c_1 = q(B) b_1$, $B = b_1 + b_2^\dagger(b_1)$, and $c_2(b_2) = q(B) b_2^\dagger(b_1)$. The first-order condition for b_1

under no commitment is

$$\begin{aligned}
& u'(c_1) \left(q(B) + b_1 q'(B) (1 + b_2^{\dagger'}(b_1)) \right) + \\
& + \beta \left[u'(c_2) \left(q(B) b_2^{\dagger'}(b_1) + b_2^{\dagger}(b_1) q'(B) (1 + b_2^{\dagger'}(b_1)) \right) + \beta \frac{\partial}{\partial B} \mathbb{E}[u(c_3) \mid B] (1 + b_2^{\dagger'}(b_1)) \right] = 0.
\end{aligned} \tag{10}$$

Condition (10) is a generalized Euler equation describing the rationale behind the optimal choice of b_1 when future governments reoptimize.⁵ As in the commitment case (5), the first term captures the marginal benefit of raising resources at $t = 1$: issuing additional debt expands current consumption by $q(B)$ but depresses the bond price through $q'(B) < 0$, lowering the value of all outstanding liabilities. The continuation value term inside the large brackets reflects the discounted effect of higher debt on future utility at $t = 2$ and $t = 3$. The key difference from commitment is the appearance of the blue terms involving $b_2^{\dagger'}(b_1)$, which measure how an increase in b_1 modifies the period-2 government's optimal issuance policy. Because the $t = 1$ government internalizes that its successor will adjust borrowing in response to changes in inherited debt, its Euler equation contains an additional channel: the effect of b_1 on the future price and quantity of debt issued by the period-2 government. Under commitment, this channel disappears—future choices are fixed—so the blue terms drop out and (5) reduces to the standard intertemporal condition balancing the marginal utility gain from extra resources at $t = 1$ with the discounted marginal cost of higher promised repayment. Thus, the no-commitment Euler equation differs from the commitment counterpart only through the policy-response terms $b_2^{\dagger'}(b_1)$, which encode the strategic anticipations absent under commitment.

Combining the period-2 condition (8) with the generalized Euler equation for b_1 under no commitment (9), we obtain the following generalized Euler equation:

$$\beta \frac{u'(c_2)}{u'(c_1)} = 1 + b_1 \frac{q'(B)}{q(B)} (1 + b_2^{\dagger'}(b_1)). \tag{11}$$

This condition has the same structure as the commitment Euler equation (7), except for the additional blue term. Under commitment, the discount factor between periods 1 and 2 satisfies $\beta u'(c_2)/u'(c_1) = 1$, so the planner equalizes discounted marginal utilities across the two pre-repayment dates. Under no commitment, the extra term in blue captures the *strategic bias*: it reflects how an increase in b_1 affects the successor's borrowing choice $b_2'(b_1)$ and, through $q'(B)$, the price of debt. The resulting condition (11) is a generalized Euler

⁵We refer to (10) as a generalized Euler equation because it incorporates the strategic interaction between the period-1 and period-2 governments through the derivative of the successor's policy function, $b_2^{\dagger'}(b_1)$.

equation with an effective discount factor distorted by the strategic interaction between the period-1 and period-2 governments. Specifically, relative to commitment, the strategic term in blue depresses the discount factor between periods 1 and 2: the government behaves as if it effectively discounts period-2 marginal utility more heavily (an “excess impatience” induced by the successor’s re-optimization). The size of the distortion depends on the bond-price elasticity $q'(B)/q(B)$ and on how aggressively the period-2 government adjusts its borrowing, $b_2^{\dagger'}(b_1)$. Since $q'(B)/q(B) < 0$ and $1 + b_2^{\dagger'}(b_1) > 0$, the entire blue term is negative, so the strategic bias always lowers the effective discount factor relative to the commitment benchmark.

The solution to (8)-(10) yields a pair (b_1^{NC}, b_2^{NC}) that differs systematically from the commitment benchmark. Because the period-2 government places too little weight on the future consequences of additional borrowing, it issues more debt than is dynamically efficient under commitment, and the period-1 government—anticipating this behavior—adjusts its own issuance accordingly. The resulting allocation features excessive total borrowing and an intertemporal composition tilted toward period 2.

Excessive Period-2 Borrowing Under No Commitment

Proposition 1. *Fix $\beta \in (0, 1)$ and $\bar{y} > 0$. Let (b_1^C, b_2^C) be the optimal borrowing choices under commitment at $t = 1$, and let (b_1^{NC}, b_2^{NC}) be the equilibrium choices in the no-commitment economy. Then borrowing undertaken at $t = 2$ – as a share of total borrowing – is strictly higher without commitment:*

$$\frac{b_2^{NC}}{b_1^{NC} + b_2^{NC}} \geq \frac{b_2^C}{b_1^C + b_2^C} = \frac{\sqrt{\beta}}{1 + \sqrt{\beta}}.$$

Proof. See Appendix A.2.

Proposition 1 formalizes this result by showing that the share of total borrowing undertaken at $t = 2$ is strictly higher without commitment than under commitment. Further details are provided in Appendix A.2.

Intermediate commitment. We retain the no-commitment structure described above but introduce a borrowing ceiling \bar{b} in period 2 and a fixed cost $\phi \geq 0$ that applies whenever the ceiling is exceeded. The period-2 government then solves

$$V_2(b_1, \bar{b}) = \max_{b_2} \left\{ u(c_2) - \phi \mathbf{1}_{\{b_2 > \bar{b}\}} + \beta \mathbb{E}[u(c_3) \mid B] \right\}.$$

Absent the penalty ($\phi = 0$), the interior first-order condition yields the unconstrained best response (9), identical to the $t = 2$ policy in the no-commitment case. Introducing the ceiling with cost ϕ leads to a simple comparison: the government either (i) chooses the interior point $b_2^\dagger(b_1)$ if it yields higher value than respecting the cap, or (ii) stops at the ceiling $b_2 = \bar{b}$ and avoid paying the cost.

To evaluate this decision transparently, define the value function excluding the penalty term:

$$f_0(b_2 \mid b_1) = u(c_2) + \beta \mathbb{E}[u(c_3) \mid B].$$

For any given (b_1, \bar{b}) , the period-2 government chooses b_2 according to

$$b_2(b_1, \bar{b}) = \begin{cases} b_2^\dagger(b_1), & \text{if } f_0(b_2^\dagger(b_1) \mid b_1) - \phi > f_0(\bar{b} \mid b_1), \\ \bar{b}, & \text{otherwise.} \end{cases} \quad (12)$$

That is, the government either borrows freely up to its unconstrained optimum $b_2^\dagger(b_1)$ or stops at the ceiling \bar{b} if the expected utility gain from additional borrowing does not outweigh the cost ϕ .

At time 1, the government anticipates that its successor will reoptimize according to (12) and chooses b_1 and \bar{b} to maximize

$$V_1(b_1, \bar{b}) = u(c_1) + \beta (u(c_2(b_1, \bar{b})) - \phi \mathbf{1}_{\{b_2(b_1, \bar{b}) > \bar{b}\}}) + \beta^2 \mathbb{E}[u(c_3) \mid B],$$

with $c_1 = q(B) b_1$, $B = b_1 + b_2(b_1, \bar{b})$, and $c_2(b_2, \bar{b}) = q(B) b_2(b_1, \bar{b})$.

The choice of the ceiling \bar{b} follows directly from the period-2 best response (12). For any fixed b_1 , the ceiling affects equilibrium only if it is *enforceable*, meaning the period-2 government weakly prefers respecting the cap to deviating to its unconstrained optimum. This requires the incentive constraint

$$f_0(b_2^\dagger(b_1) \mid b_1) - \phi \leq f_0(\bar{b} \mid b_1).$$

Among all ceilings satisfying this condition, the period-1 government chooses the *largest* enforceable value. The reason is that tighter ceilings reduce period-2 borrowing and therefore depress c_2 without providing any additional benefit once the incentive constraint is satisfied. Consequently, for any $\phi > 0$ that leads the planner to use the ceiling (i.e., values of ϕ that are too small to make the commitment allocation self-enforcing), the optimal \bar{b} lies exactly

on the indifference locus

$$f_0(b_2^\dagger(b_1) \mid b_1) - \phi = f_0(\bar{b} \mid b_1), \quad (13)$$

which pins down \bar{b} uniquely as a function of b_1 . At such a ceiling, the period-2 government is indifferent between deviating to $b_2^\dagger(b_1)$ and respecting \bar{b} , and the equilibrium debt at $t = 2$ satisfies $b_2(b_1, \bar{b}) = \bar{b}$. This ensures that period-2 borrowing is disciplined while imposing the minimal distortion on the allocation. Equation (13) implies the existence of a cutoff enforcement level ϕ_{\min} above which the commitment allocation $(b_1, \bar{b}) = (b_1^C, b_2^C)$ becomes sustainable. The smallest cost that makes the ceiling self-enforcing at $t = 2$ is the *value gap*:

$$\phi_{\min} = f_0(b_2^\dagger(b_1^C) \mid b_1^C) - f_0(b_2^C \mid b_1^C) > 0. \quad (14)$$

To characterize the optimal choice of b_1 under intermediate commitment, differentiate $V_1(b_1, \bar{b})$ with respect to b_1 . Using $c_1 = q(B)b_1$, $c_2 = q(B)\bar{b}(b_1)$, and $B = b_1 + \bar{b}(b_1)$, and letting $\bar{b}'(b_1) = \frac{d\bar{b}(b_1)}{db_1}$ denote the policy-response term implied by the indifference condition, the generalized Euler equation is

$$\begin{aligned} & u'(c_1)(q(B) + b_1 q'(B)(1 + \bar{b}'(b_1))) \\ & + \beta \left[u'(c_2)(q(B)\bar{b}'(b_1) + q'(B)\bar{b}(b_1)(1 + \bar{b}'(b_1))) + \beta \frac{\partial}{\partial B} \mathbb{E}[u(c_3) \mid B] (1 + \bar{b}'(b_1)) \right] = 0. \end{aligned} \quad (15)$$

Equation (15) has the same structure as the generalized Euler equation (10) under no commitment. The only difference is the policy-response term: under no commitment the relevant elasticity is $b_2^{\dagger'}(b_1)$, while under intermediate commitment the period-2 choice is determined by the ceiling, $b_2(b_1, \bar{b}) = \bar{b}(b_1)$, so the elasticity becomes $\bar{b}'(b_1)$. Substituting $\bar{b}(b_1)$ and $\bar{b}'(b_1)$ for $b_2^\dagger(b_1)$ and $b_2^{\dagger'}(b_1)$ transforms the no-commitment condition into (15).

The policy-response term $\bar{b}'(b_1)$ is naturally shaped by how the optimal ceiling adjusts when inherited debt changes. As seen in Figure 2, both b_1 and the induced ceiling \bar{b} decline with higher costs, and the slope of the intermediate commitment locus therefore governs the magnitude of $\bar{b}'(b_1)$. When the ceiling is tightly disciplined (high ϕ), small increases in b_1 require only small adjustments in \bar{b} , so $\bar{b}'(b_1)$ is close to zero; when the cost ϕ is low, the ceiling must respond more strongly to preserve incentive compatibility, generating a larger $\bar{b}'(b_1)$. Overall, the discipline imposed by the ceiling makes $\bar{b}'(b_1)$ generally smaller in magnitude than the unconstrained policy-response term that arises under no commitment. A detailed derivation is provided in Appendix A.4.

To summarize, when $\phi \geq \phi_{\min}$ the economy replicates the commitment allocation. When $\phi < \phi_{\min}$, the optimal pair (b_1, \bar{b}) lies between the commitment and no-commitment benchmarks, generating intermediate borrowing levels and maturities. Putting these two steps together yields a simple, threshold characterization summarized by Proposition 2.

Sustainable Allocations Under a Debt Ceiling

Proposition 2. *Fix a cost $\phi \geq 0$. The solution to the intermediate-commitment problem is a pair (b_1^*, b_2^*) such that:*

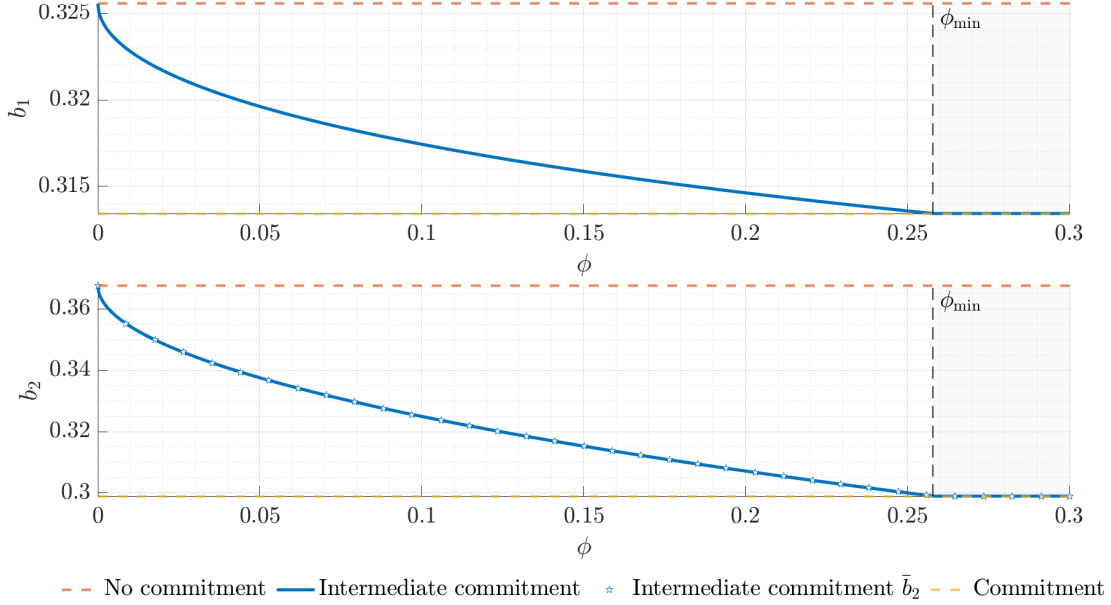
1. *If $\phi = 0$, the unique equilibrium is the no-commitment allocation $(b_1^*, b_2^*) = (b_1^{NC}, b_2^{NC})$, given by (9) and (8).*
2. *If $0 < \phi < \phi_{\min}$, the equilibrium is an intermediate-commitment allocation (b_1^*, b_2^*) characterized by (13) and (15), with $b_1^* \in (b_1^C, b_1^{NC})$ and $b_2^* \in (b_2^C, b_2^{NC})$. The planner sets $\bar{b} = b_2^*$.*
3. *If $\phi \geq \phi_{\min}$, the planner sets $\bar{b} = b_2^C$, and the economy sustains the commitment allocation $(b_1^*, b_2^*) = (b_1^C, b_2^C)$, given by (5) and (6), as a self-enforcing outcome at $t = 2$.*

Proof. See Appendix A.4.

Intuitively, the ceiling mitigates the period-2 overborrowing motive. For low costs it only partially binds, reducing—but not eliminating—the overborrowing bias. Once the cost reaches ϕ_{\min} , setting $\bar{b} = b_2^C$ fully restores the commitment plan; any additional enforcement beyond ϕ_{\min} is redundant.

Figure 2 illustrates how the sustainable debt levels (b_1^*, b_2^*) vary as a function of ϕ . The blue line shows the equilibrium with intermediate commitment, approaching the red and yellow benchmarks as enforcement weakens or strengthens. For $\phi = 0$, the economy exhibits the no-commitment outcome with excessive borrowing. As ϕ increases, the debt ceiling tightens and gradually aligns incentives across periods. Once ϕ reaches the threshold ϕ_{\min} (indicated by the vertical line and shaded region), the ceiling fully sustains the commitment plan and further increases in ϕ have no additional effect.

Figure 2: Optimal Debt Positions Under Varying Commitment Strength



Note. The figure plots optimal debt choices b_1 and b_2 , as functions of the cost ϕ . The blue solid line represents the case of intermediate commitment, while the red and yellow dashed lines correspond to the no-commitment and full-commitment benchmarks, respectively. The blue star markers plot the endogenous ceiling. The shaded gray region marks values of ϕ above the minimum threshold ϕ_{\min} required to sustain commitment.

Intermediate commitment with higher order costs. We allow the cost to follow a proportional parametric form, which nests the fixed cost as a special case. Suppose that whenever the period-2 government borrows above a ceiling \bar{b} , it incurs a period-2 utility cost given by

$$\Phi(b_2, \bar{b}) = \phi (b_2 - \bar{b})^\zeta \mathbf{1}_{\{b_2 > \bar{b}\}}, \quad \phi \geq 0, \quad \zeta \geq 0.$$

The parameter ζ governs the curvature of the cost function. Three benchmark cases are of interest:

- $\zeta = 0$: a fixed cost of violating the ceiling, identical to the specification in our baseline intermediate-commitment analysis;
- $\zeta = 1$: a piecewise-linear penalty with marginal costs that increase discretely once the ceiling is exceeded;
- $\zeta = 2$: a smooth quadratic penalty, differentiable at $b_2 = \bar{b}$ and thus suitable for characterizing the interior tradeoff between incentives and distortions.

In what follows, we focus on the case $\zeta = 2$, which yields a tractable first-order condition for the period-2 government and therefore provides a useful contrast with the environments

$\zeta = 0$ and $\zeta = 1$. Numerical results for both $\zeta = 1$ and $\zeta = 2$ are presented in Figure 3, which illustrate the different ways in which ceilings shape borrowing incentives.

Given (b_1, \bar{b}) , the period-2 government chooses b_2 to maximize

$$V_2(b_1, \bar{b}) = \max_{b_2} \left\{ u(c_2) - \phi(b_2 - \bar{b})^2 \mathbf{1}_{\{b_2 > \bar{b}\}} + \beta \mathbb{E}[u(c_3) \mid B] \right\},$$

with $c_2 = q(B)b_2$ and $B = b_1 + b_2$. Define

$$f_0(b_2 \mid b_1) = u(c_2) + \beta \mathbb{E}[u(c_3) \mid B].$$

For $b_2 \leq \bar{b}$, the penalty is inactive and the first-order condition is identical to the no-commitment case (8). For an interior optimum with $b_2 > \bar{b}$, differentiating the objective yields

$$\frac{\partial}{\partial b_2} f_0(b_2 \mid b_1) - 2\phi(b_2 - \bar{b}) = 0, \quad b_2 > \bar{b}, \quad (16)$$

where

$$\frac{\partial}{\partial b_2} f_0(b_2 \mid b_1) = u'(c_2)(q(B) + b_2 q'(B)) + \beta \frac{\partial}{\partial B} \mathbb{E}[u(c_3) \mid B].$$

Relative to the unconstrained best response $b_2^\dagger(b_1)$ in (9), the quadratic penalty introduces a smooth marginal cost $2\phi(b_2 - \bar{b})$ that grows linearly with the violation. Let $b_2^Q(b_1, \bar{b})$ denote the policy function solving the first-order condition (16). Because the quadratic penalty is differentiable at $b_2 = \bar{b}$, the problem is smooth and admits a single interior optimality condition.

Anticipating $b_2^Q(b_1, \bar{b})$, the period-1 government chooses (b_1, \bar{b}) to maximize

$$V_1^Q(b_1, \bar{b}) = u(c_1) + \beta \left(u(c_2^Q) - \phi(b_2^Q - \bar{b})^2 \mathbf{1}_{\{b_2^Q > \bar{b}\}} \right) + \beta^2 \mathbb{E}[u(c_3) \mid B^Q],$$

with $c_1 = q(B^Q)b_1$, $c_2^Q = q(B^Q)b_2^Q(b_1, \bar{b})$, and $B^Q = b_1 + b_2^Q(b_1, \bar{b})$. Differentiating with respect to b_1 yields the generalized Euler equation

$$\begin{aligned} 0 = & u'(c_1) \left(q(B^Q) + b_1 q'(B^Q)(1 + b_2^{Q'}(b_1, \bar{b})) \right) \\ & + \beta \left[u'(c_2^Q) \left(q(B^Q) b_2^{Q'} + q'(B^Q) b_2^Q (1 + b_2^{Q'}) \right) + \beta \frac{\partial}{\partial B} \mathbb{E}[u(c_3) \mid B^Q] (1 + b_2^{Q'}) \right], \end{aligned} \quad (17)$$

where $b_2^{Q'}$ denotes $\partial b_2^Q / \partial b_1$. Equation (17) has the identical structure to the no-commitment Euler equation (10), except that the policy-response elasticity $b_2^{\dagger'}(b_1)$ is replaced by the

ceiling-induced response $b_2^{Q'}(b_1, \bar{b})$. As with the fixed-penalty case, the ceiling reduces the sensitivity of period-2 issuance to inherited debt, thereby partially restoring the discipline present under commitment.

To characterize the optimal ceiling, differentiate $V_1^Q(b_1, \bar{b})$ with respect to \bar{b} . Because b_2^Q solves the period-2 problem, the envelope theorem applies to V_2 , and the derivative decomposes into two terms:

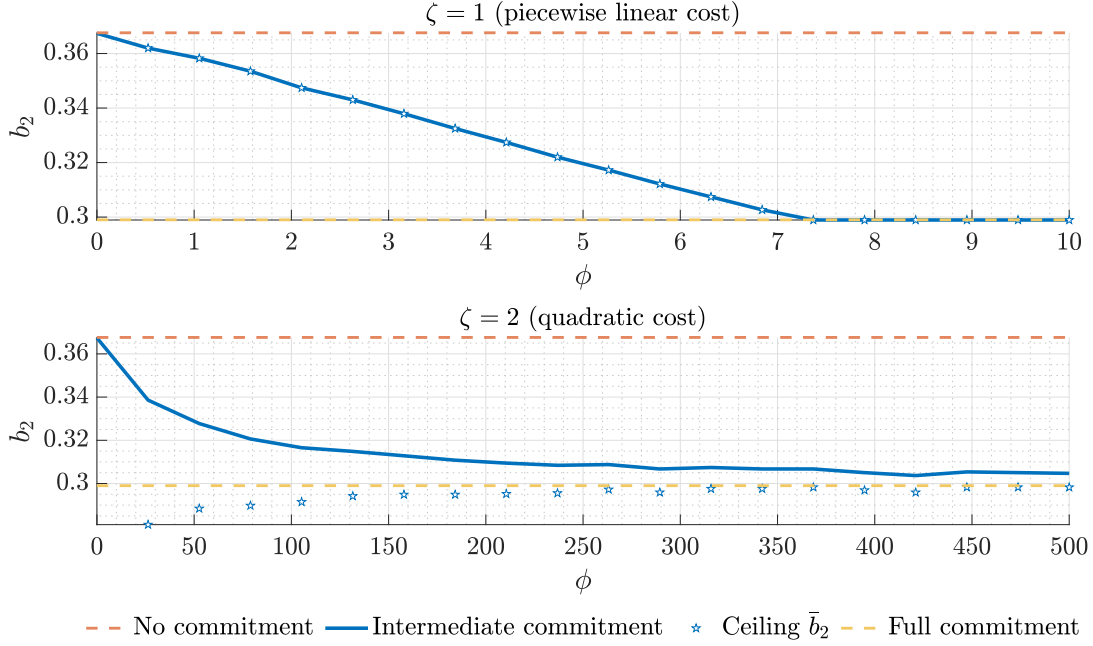
$$\frac{\partial V_1^Q}{\partial \bar{b}} = u'(c_1) q'(B^Q) b_1 \frac{\partial b_2^Q}{\partial \bar{b}} + 2\beta\phi (b_2^Q - \bar{b}) \mathbf{1}_{\{b_2^Q > \bar{b}\}}.$$

The second term captures the direct effect of relaxing the ceiling on the penalty paid at $t = 2$, while the first term reflects how the ceiling influences period-1 resources through its effect on the equilibrium borrowing b_2^Q and thus the bond price $q(B)$. In order to characterize the region in which the quadratic penalty is active, we focus on the region where $b_2^Q > \bar{b}$, so the indicator $\mathbf{1}_{\{b_2^Q > \bar{b}\}}$ equals one throughout and can be dropped. Setting the derivative equal to zero yields the optimality condition

$$u'(c_1) q'(B^Q) b_1 \frac{\partial b_2^Q}{\partial \bar{b}} + 2\beta\phi (b_2^Q - \bar{b}) = 0. \quad (18)$$

Equation (18) shows that, unlike the fixed-cost case $\zeta = 0$, a quadratic penalty does *not* imply $b_2^Q = \bar{b}$ in equilibrium. Relaxing the ceiling reduces the penalty but also increases b_2^Q , which lowers the bond price $q(B)$ and therefore depresses c_1 . The optimal ceiling balances these two forces, generating an interior gap $b_2^Q - \bar{b} > 0$. Figure 3 shows that under a quadratic penalty ($\zeta = 2$), the gap between b_2 and \bar{b}_2 shrinks as ϕ increases, but vanishes only asymptotically as $\phi \rightarrow \infty$, at which point the allocation converges to the commitment benchmark. By contrast, under a linear penalty ($\zeta = 1$), commitment is restored at a finite threshold ϕ_{\min} , similarly to the $\zeta = 0$'s case.

Figure 3: Optimal Period-2 Borrowing Under Linear and Quadratic Costs



Note. The figure plots optimal period-2 borrowing b_2 and the associated borrowing ceiling \bar{b}_2 as functions of the cost ϕ , under two functional forms: a piecewise-linear penalty ($\zeta = 1$) and a quadratic penalty ($\zeta = 2$). The blue solid line shows optimal constrained borrowing, the blue star markers plot the endogenous ceiling, and the red and yellow dashed lines denote the no-commitment and full-commitment benchmarks, respectively. Increasing enforcement strength progressively disciplines short-term borrowing, with the shape and rate of convergence depending on the curvature of the penalty function.

The behavior of breaches differs across the three cost specifications. Under the quadratic cost, breaches naturally arise in equilibrium: the marginal cost of exceeding the ceiling increases only gradually, so the period-2 government finds it optimal to violate the ceiling whenever the marginal value of additional borrowing exceeds the incremental political or institutional cost of doing so. By contrast, with a fixed cost ($\zeta = 0$) or a linear cost ($\zeta = 1$), breaches do not occur in the three-period benchmark. Because there is no uncertainty at period 2, the government never finds it worthwhile to incur a discrete political or institutional cost to obtain additional resources—the value of “purchasing” state contingency simply never materializes. In a stochastic environment, however, the same fixed or linear costs can generate breaches, as adverse states make the marginal value of extra borrowing high enough to justify paying the cost. Thus, the curvature of the cost governs how sharply ceilings deter violations, while uncertainty governs whether violations ever occur.

These insights motivate our transition to the stochastic infinite-horizon model, where uncertainty, long-term debt dynamics, and forward-announced ceilings interact to produce richer patterns of borrowing, occasional breaches, and endogenous discipline across time.

3 Model

We build on the canonical [Eaton and Gersovitz \(1981\)](#) framework of sovereign default, incorporating long-term debt and an endogenous debt ceiling. The government chooses both the next period's level of debt and a self-imposed ceiling on future debt, beyond which borrowing incurs a quadratic cost. These costs are only paid when the desired borrowing exceeds the previously promised debt ceiling, which captures frictions such as institutional constraints, political economy considerations, or additional market-imposed discipline from legacy bondholders.

3.1 Environment

Time is discrete and infinite. In each period, the government observes the realization of an exogenous endowment $y \in \mathcal{Y}$, which follows a Markov process with known transition probabilities. The government begins the period with outstanding debt $B \in \mathbb{R}_+$ and a debt ceiling $\bar{B} \in \mathbb{R}_+$. The debt ceiling is a choice variable in repayment and represents the upper bound above which new borrowing incurs additional costs.

3.2 Government and Lenders

The government chooses whether to repay or default. Let $d \in \{0, 1\}$ be the default indicator, where $d = 1$ denotes default. The government's value function is:

$$V(y, B, \bar{B}) = \max_{d \in \{0, 1\}} (1 - d)V^R(y, B, \bar{B}) + dV^D(y), \quad (19)$$

where V^R is the value of repayment and V^D is the value of default.

Repayment. If the government chooses to repay, it selects next period's debt $B' \in \mathcal{B}$ and a new ceiling $\bar{B}' \in \bar{\mathcal{B}}$. Consumption c satisfies the resource constraint:

$$c + (\delta + (1 - \delta)z)B = y + q(y, B', \bar{B}')[B' - (1 - \delta)B] - \Phi(B', \bar{B}'), \quad (20)$$

where $q(y, B', \bar{B}')$ is the price of long-term debt, δ is the fraction of debt maturing each period, and z is the coupon. The function $\Phi(B', \bar{B}')$ captures the additional cost of borrowing beyond the ceiling:

$$\Phi(B', \bar{B}) = \begin{cases} 0, & \text{if } B' \leq \bar{B}, \\ \phi \cdot (B' - \bar{B})^\zeta, & \text{if } B' > \bar{B}, \end{cases} \quad (21)$$

where $\phi \geq 0$ governs the severity of the penalty. The parameter $\zeta \geq 0$ governs how the political and institutional cost of exceeding the ceiling scales with the size of the breach. Consistently with Section 2, we consider two values for ζ . When $\zeta = 0$, any deviation from the ceiling triggers a single discrete cost ϕ , regardless of how large the breach is. This captures environments where the key friction is a fixed political or procedural hurdle—for example, reopening a fiscal law or bearing a reputational cost that arises once the ceiling is violated, but does not depend on the amount of additional borrowing. In contrast, when $\zeta = 2$, the penalty is convex in the size of the breach, so larger violations become disproportionately more costly. This specification embodies the idea that political resistance, legislative bargaining frictions, or reputational losses intensify with the magnitude of the deviation. Although these two forces may interact in practice, we consider them separately for clarity, allowing us to isolate how discrete versus convex costs affect the credibility and effectiveness of debt-ceiling announcements.

The government's recursive problem in repayment is thus:

$$V^R(y, B, \bar{B}) = \max_{c, B', \bar{B}'} u(c) + \beta \mathbb{E}_{y'} [V(y', B', \bar{B}')] , \quad (22)$$

subject to the implementability constraint above.

Default. In the event of default, the country is excluded from financial markets and receives autarky consumption $\varrho(y)$. Re-entry occurs with probability $\theta \in (0, 1)$, in which case the country returns with zero debt and the maximal allowable ceiling \bar{B}_{\max} . The default value function is:

$$V^D(y) = u(\varrho(y)) + \beta \mathbb{E}_{y'} [\theta V(y', 0, \bar{B}_{\max}) + (1 - \theta) V^D(y')] . \quad (23)$$

Lenders and bond pricing. There is a continuum of risk-neutral international lenders that discount at the constant gross rate $R = 1 + r$. They are competitive, so in equilibrium the bond price equals the expected discounted payoff per unit of debt. A bond issued at (y, B', \bar{B}') promises, if the government repays next period, a coupon z and the possibility of

reselling the remaining $(1 - \delta)$ fraction of the bond. Thus the equilibrium bond price satisfies

$$q(y, B', \bar{B}') = \frac{1}{1+r} \mathbb{E}_{y'|y} \left[\left((1-d(y', B', \bar{B}')) (\delta + (1-\delta) (z + q(y', B''(y', B', \bar{B}'), \bar{B}''(y', B', \bar{B}'))) \right) \right], \quad (24)$$

where $d(y', B', \bar{B}') \in \{0, 1\}$ is the default decision of the next-period government in state (y', B', \bar{B}') , and $B''(y', B', \bar{B}')$ and $\bar{B}''(y', B', \bar{B}')$ denote its optimal choices of next-period debt and ceiling. If default occurs ($d = 1$), lenders receive zero, so the payoff in (24) is zero in those states. The price schedule $q(\cdot)$ is therefore determined jointly with the government's policy functions in a recursive Markov-perfect equilibrium.

3.3 Calibration

We follow Chatterjee and Eyigungor (2012) in parameterizing preferences, the stochastic endowment process, and the structure of long-term debt. The model is calibrated at a quarterly frequency. The government discounts the future at rate β , households have CRRA utility with coefficient $\sigma = 2$, and international lenders discount at the risk-free rate $r = 0.01$.⁶ Long-term bonds mature geometrically at rate $\delta = 0.05$ per quarter and pay coupon $z = 0.03$. This implies an average maturity of $1/\delta = 20$ quarters, or five years. Upon default, the economy re-enters financial markets with probability $\theta = 0.0385$ each period.

Endowment process. Output follows an AR(1) process,

$$\log y_t = \rho \log y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma \varepsilon^2),$$

with persistence $\rho = 0.9485$ and innovation standard deviation $\sigma \varepsilon = 0.0271$, identical to the estimates in Chatterjee and Eyigungor (2012).

Default cost function. Following Chatterjee and Eyigungor (2012), the consumption loss in default is given by

$$\varrho(y) = y - \max\{0, d_0 y + d_1 y^2\},$$

where the coefficients (d_0, d_1) determine the magnitude and curvature of the output cost. As discussed below, these parameters are internally calibrated to match key external moments of the Argentine data.

⁶In line with the sovereign default literature, we allow for $r > 1/\beta$. This wedge captures political myopia, limited commitment, or unmodeled sovereign risks and helps generating realistic debt levels and spreads.

Solution method. Unlike Chatterjee and Eyigungor (2012), we solve the model using the extreme-value shock framework of Dvorkin et al. (2021), which yields smooth policy functions and improve convergence. The correlation and variance of the extreme value shocks are set to $p = 0.37$ and $v = 5.7 \times 10^{-3}$. Because this solution method alters the shape of policy rules relative to the original discrete-choice formulation, we recalibrate the three internally chosen parameters. A detailed description of the computational strategy is reported in Appendix B.

Internally calibrated parameters. The parameters β , d_0 , and d_1 are selected to match three targeted moments from the Argentine data: the mean debt-to-output ratio, the mean sovereign spread, and the volatility of spreads. Table 1 reports the targets, model moments, and corresponding parameter values.

Table 1: Targeted Moments in Percent and Corresponding Parameters

Moment	Target	Model	Parameter	Value
Mean Level of Debt	70	71	β	0.943
Mean Spread	7.3	8.0	d_0	0.017
Volatility of the Spread	4.0	4.1	d_1	0.446

Note: The table reports the internally calibrated parameters and their corresponding targeted moments.

In what follows, we introduce an endogenous ceiling and study its implications under two alternative cost structures for breaching it: a fixed penalty ($\zeta = 0$) and a proportional quadratic penalty ($\zeta = 2$). For each cost function, we illustrate the core intuition in Subsection 3.4 by setting $\phi = 0.40$, a sufficiently large value that highlights the role played by the ceiling. In the quantitative welfare analysis of Subsection 3.5, we report results for the entire interval $\phi \in [0, 0.4]$ for both the fixed-cost and quadratic-cost cases, and translate these penalties into their implied GDP magnitudes and associated consumption-equivalent welfare gains.

3.4 The Role of the Debt Ceiling: Commitment vs Flexibility

In our benchmark with $\phi = 0$ (blue line in Figure 4), bond prices are determined only by borrowing choices and the exogenous income shock.⁷ We then introduce a debt ceiling by setting $\phi = 0.4$, which activates the endogenous penalty $\Phi(B', \bar{B})$ when borrowing exceeds

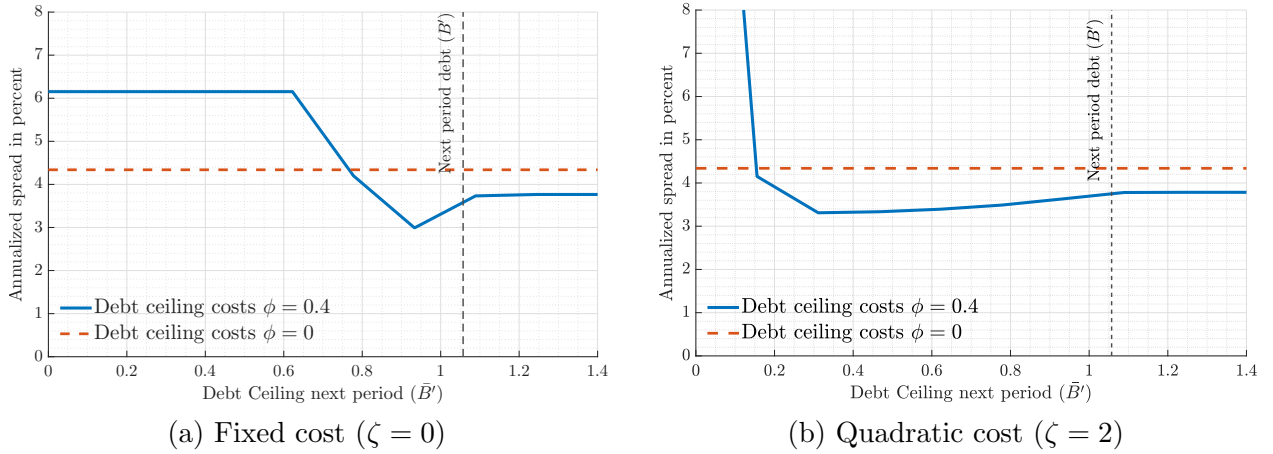
⁷If borrowing above the debt ceiling is costless, any debt ceiling announcement is a non informative signal to future bondholders and as a result the model collapses exactly into the set up of Chatterjee and Eyigungor (2012).

the self-imposed ceiling. The debt ceiling modifies equilibrium outcomes by altering the government’s intertemporal incentives.

The underlying mechanism hinges on a trade-off between commitment and fiscal flexibility. On the one hand, by choosing and maintaining a ceiling on future borrowing, the government partially commits not to excessively dilute outstanding debt. This mitigates the classic “debt dilution” problem in [Eaton and Gersovitz \(1981\)](#)-type models with long-term debt: when the government issues additional debt today, it lowers the value of legacy bonds tomorrow by raising the probability of default. Anticipating this, investors demand higher yields, which depresses bond prices ([Hatchondo and Martinez, 2009](#); [Chatterjee and Eyigungor, 2012](#)). Promising a credible ceiling can reduce this incentive, raise bond prices, and ultimately lower borrowing costs. On the other hand, the same ceiling constrains the government’s ability to smooth consumption and respond to adverse shocks by issuing additional debt. In tight fiscal states, this restriction can make default more attractive, increasing the risk of repayment failure.

The net effect of promising a “reasonable” value of the tomorrow’s debt ceiling thus reflects the balance of these two forces: lower interest rates in the current period at the cost of more limited borrowing capacity in the next period. [Figure 4](#) illustrates this mechanism.

Figure 4: Debt Ceilings, Bond Prices, and Cost Specifications



Note: Each panel plots the equilibrium annualized spread as a function of the next-period debt ceiling announcement. In both cases, moderate ceilings raise bond prices and lower credit spreads by mitigating debt dilution, whereas excessively tight ceilings increase default risk, thereby reducing prices and widening spreads. Panel [a](#) assumes a fixed cost of breaching the ceiling ($\zeta = 0$), while Panel [b](#) assumes a convex quadratic cost ($\zeta = 2$), which makes larger violations disproportionately more costly. Solid blue lines correspond to the model with a high debt-ceiling penalty ($\phi = 0.40$), while the dashed red lines refer to the costless-ceiling case ($\phi = 0$).

The x-axis in [Figure 4](#) reports the level of the promised next-period debt ceiling \bar{B}' , while

the y-axis shows the annualized credit spreads which are computed as

$$\left(1 + \frac{\delta + (1 - \delta)z}{q(y, B', \bar{B}')} - \delta\right)^4 - (1 + r)^4,$$

corresponding to a fixed level of next-period debt (B' , dashed vertical line). The red dashed line depicts the bond price in the benchmark model without ceiling frictions ($\phi = 0$). The blue continuous line shows the price in our model where the debt ceiling cost is $\phi = 0.4$. Panel **a** uses a fixed breach cost ($\zeta = 0$) and Panel **b** uses a proportional quadratic cost ($\zeta = 2$).

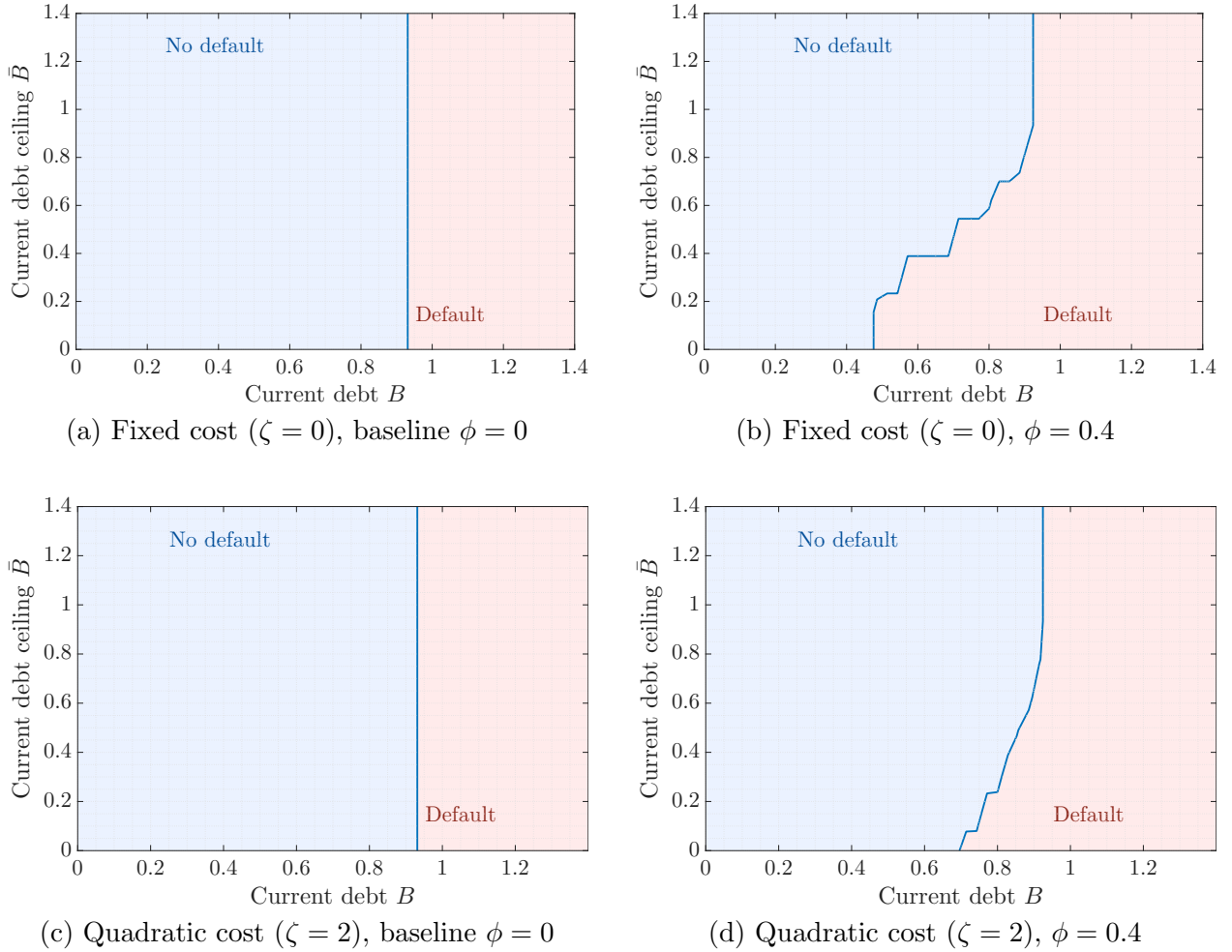
For a wide range of values of \bar{B}' around and moderately below the fixed next-period debt, the announced ceiling raise bond prices and lower credit spreads relative to the benchmark without ceiling costs. In this region, the ceiling mitigates debt dilution and lowers borrowing costs. When the ceiling becomes extremely tight, however—well below any level consistent with rolling over existing debt—bond prices fall sharply and credit spreads increase. Very small ceilings imply that the government would almost surely need to violate the ceiling in adverse states, which raises default risk and depresses prices. Under the fixed-cost specification (Panel **a**), this shows up as a low, flat segment: once the ceiling is tight enough to make a breach nearly certain, the discrete penalty leads lenders to price in a high probability of default. Under the quadratic specification (Panel **b**), the effect is even more pronounced. Because large violations become disproportionately costly, very tight ceilings generate sharply higher default risk and a steep collapse in bond prices. In both cases, excessively low ceilings are counterproductive: instead of reinforcing commitment, they undermine it by making future policy too inflexible to absorb shocks.

In this latter case, the prospect of being unable to respond to future shocks raises the likelihood of default, as shown in Figure 5. The associated increase in default risk leads to higher yields and lower bond prices, consistent with Figure 4. Figure 5 also shows that the default region is larger under fixed costs, since any ceiling breach triggers a discrete penalty and makes repayment unattractive over a broad set of states. By contrast, bond prices fall more sharply under the quadratic specification when ceilings are extremely tight: even if the government continues to repay in many states, lenders price in the possibility of large violations, which become disproportionately costly and raise the ex-ante default probability. Hence Figures 4 and 5 speak to different margins—ex-ante pricing versus the government’s realized default decisions—and together highlight how overly tight ceilings can undermine, rather than reinforce, policy credibility.

Finally, the non-monotonicity reported in Figure 5 highlights a central result: promising reasonable austerity can improve credit conditions, while excessive austerity can backfire by

undermining repayment incentives. These considerations suggest that the adoption of a debt ceiling can generate welfare gains. By alleviating debt dilution without excessively restricting fiscal flexibility, a moderate debt ceiling can improve the trade-off faced by the government, enabling it to borrow at lower spreads while preserving enough space to respond to adverse shocks. As we show in Subsection 3.5, this mechanism can raise ex-ante expected utility relative to both the no-ceiling benchmark and overly rigid fiscal rules.

Figure 5: Tight Ceilings Can Trigger Default Under Alternative Cost Structures

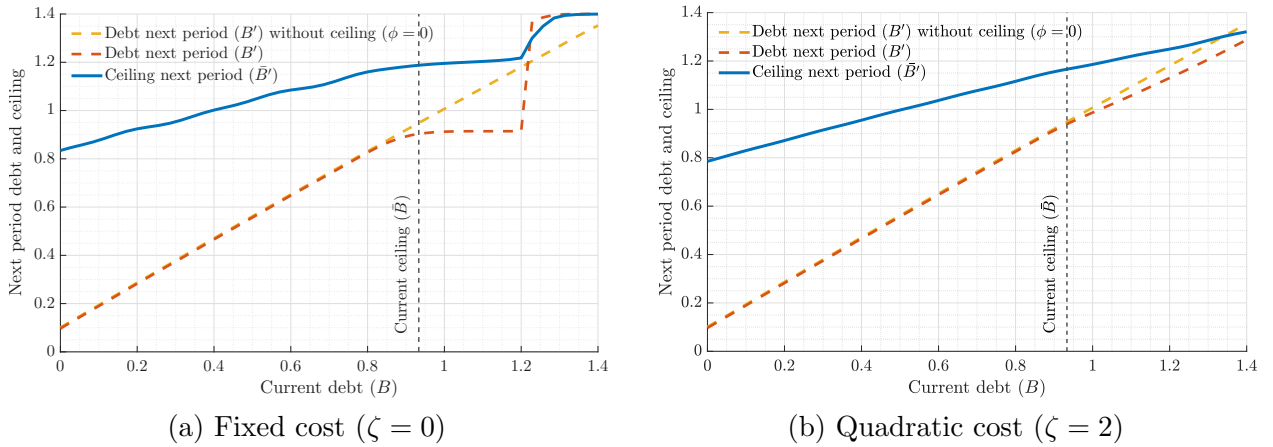


Note: Each panel shows the default region in the space of current debt B and current debt ceiling \bar{B} . Tight ceilings can trigger default by weakening repayment incentives. Increasing ϕ from 0 to 0.4 shifts the economy toward higher default risk when the ceiling binds at high debt levels. Fixed costs ($\zeta = 0$) create sharper threshold effects, whereas quadratic costs ($\zeta = 2$) generate smoother distortions in the default boundary.

Finally, we analyze the policy functions for the debt ceiling itself. Figure 6 shows how borrowing and debt-ceiling choices vary with the level of current debt (x-axis), holding the endowment and the inherited ceiling—marked by the vertical dashed line—fixed. In

Panel **b**, which assumes a quadratic breaching cost, the government's adjustment behavior is considerably smoother than under the fixed-cost specification. For current debt levels below the ceiling, borrowing is only mildly disciplined and both next-period debt and the newly announced ceiling move gradually with the state. As current debt approaches and exceeds the ceiling, the marginal cost of violating it rises, keeping next-period debt (the red line) below the unconstrained benchmark (the yellow line) and inducing the government to revise the ceiling (the blue line) cautiously. Unlike the discrete jump observed under the fixed-cost case in Panel **a**, the quadratic specification produces a continuous response: the ceiling and borrowing choices adjust smoothly, with larger breaches becoming progressively more expensive. Panel **a** indicates that the government finds it optimal to pay the breaching cost only when current debt lies well above the inherited ceiling, at which point it adjusts the announced ceiling upward more decisively.

Figure 6: Debt Ceilings and Borrowing choices



Note: Each panel plots equilibrium borrowing and debt-ceiling choices as a function of current debt, holding the endowment and the current period ceiling fixed. In both cases, borrowing is partially constrained by the existing ceiling, while future ceilings are chosen slightly above next-period debt. The plots are generated for a high-endowment state in which default risk is nil. Panel **a** assumes a fixed cost of breaching the ceiling ($\zeta = 0$), whereas Panel **b** assumes a convex quadratic cost ($\zeta = 2$). Dashed red and solid blue lines correspond to the model with a high debt-ceiling penalty ($\phi = 0.40$), while the dashed yellow lines refer to the costless-ceiling case ($\phi = 0$).

3.5 Welfare Gains from an Endogenous Debt Ceilings

We now quantify the welfare implications of introducing a debt ceiling by computing the lifetime utility of a patient social planner under different values of the debt ceiling penalty parameter ϕ . Specifically, we evaluate the consumption-equivalent welfare gain relative to the benchmark economy without debt ceiling frictions ($\phi = 0$), holding the initial state fixed

at (y_0, B_0) .

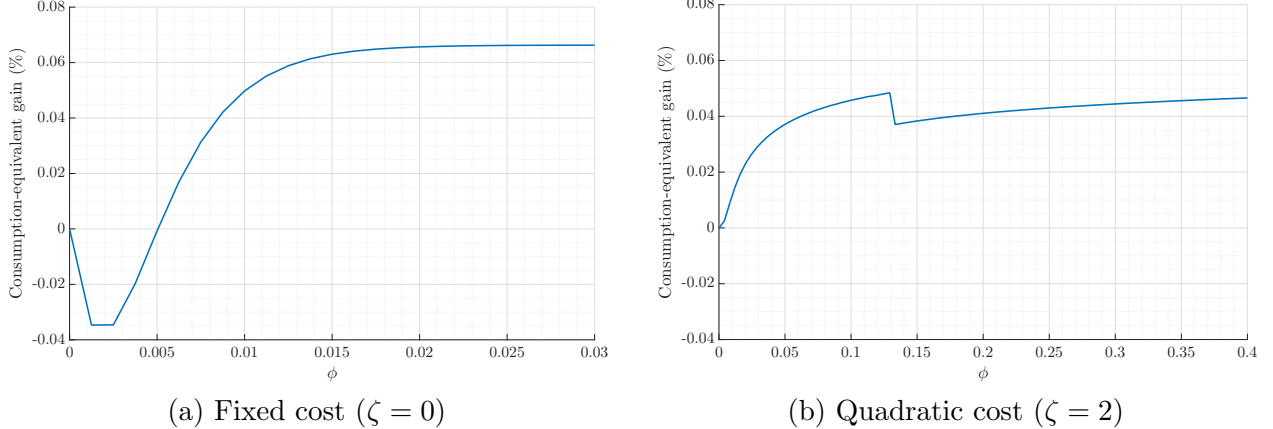
The planner's welfare is computed by simulating the economy under the actual policy functions of the impatient government—that is, taking as given the decision rules \mathcal{D}_G^ϕ , $\mathcal{B}_G^{\prime\phi}$, and $\bar{\mathcal{B}}_G^{\prime\phi}$ induced by a given value of ϕ —and integrating discounted utility forward:

$$V^{SP}(y_0, B_0 \mid \phi) = \sum_{t=0}^{\infty} \frac{u \left[\mathcal{C}(\mathcal{D}_G^\phi(t), \mathcal{B}_G^{\prime\phi}(t), \bar{\mathcal{B}}_G^{\prime\phi}(t)) \right]}{(1+r)^t}. \quad (25)$$

This approach reflects the planner's evaluation of welfare under the actual behavior of the government, capturing both the benefits of enhanced credibility and the costs of reduced fiscal flexibility.

Figure 7 reports the consumption-equivalent welfare effects of introducing a debt-ceiling commitment for different values of the penalty parameter ϕ , which governs the marginal cost of exceeding the self-imposed ceiling. Higher values of ϕ do not mechanically tighten the chosen ceiling, but they increase the incentive to keep future debt issuance within its announced bound, thereby enhancing the credibility of the commitment.

Figure 7: Welfare Gains from Debt Ceilings Under Alternative Cost Structures



Note: Each panel reports ex-ante welfare gains from introducing a debt ceiling as a function of the penalty parameter ϕ . Moderate values of ϕ raise welfare by inducing credible, but not excessively restrictive, ceilings. Panel **a** assumes a fixed cost of breaching the ceiling ($\zeta = 0$), while panel **b** assumes a convex quadratic cost ($\zeta = 2$), which penalizes larger violations more heavily.

The welfare effects of increasing ϕ differ across cost structures but share the feature that moderate penalties improve outcomes by making endogenous ceilings more credible. Under a fixed cost of violation ($\zeta = 0$), welfare initially declines for very small values of ϕ . When the discrete penalty is tiny, it generates almost no commitment value—ceilings remain effectively non-binding—yet the government still incurs the cost in the rare states in which the ceiling

is exceeded. As a result, the economy pays the cost without obtaining meaningful reductions in debt dilution. Once ϕ reaches roughly 0.5 percent of GDP, the ceiling becomes sufficiently credible that borrowing costs fall and welfare turns positive. As ϕ increases further, welfare rises sharply and stabilizes around 2–3 percent of GDP, a range in which violations are too costly to be frequent but fiscal flexibility remains largely intact. Because the penalty is lump-sum, tighter enforcement does not progressively restrict fiscal flexibility: once violations are costly, further increases in ϕ no longer sharpen incentives at the margin, and the welfare effects plateau.

By contrast, with a quadratic cost ($\zeta = 2$), welfare rises immediately as ϕ increases from zero. Because the marginal cost of violation grows smoothly with the size of the breach, even small penalties make promised ceilings credible enough to reduce dilution and improve market access. Unlike the fixed-cost case—where commitment benefits only emerge once the penalty is sufficiently large—the quadratic case yields welfare gains right away. As ϕ continues to increase, welfare improves smoothly until a maximum around $\phi \approx 0.1293$ (corresponding to roughly 3.66 percent of GDP). Beyond this threshold, the economy moves into a regime in which default is triggered more often and where the equilibrium switches to a tighter ceiling that is occasionally—but not frequently—revised. For higher values of ϕ , up to $\phi = 0.40$ (corresponding to roughly 12 percent of GDP), welfare remains positive but lower, reflecting the fact that very steep marginal penalties limit the government’s ability to respond to adverse shocks. For these higher values, the rising marginal cost of violating the ceiling limits the government’s ability to absorb adverse shocks, eventually raising default risk and producing a non-monotonic welfare profile.

This non-monotonicity illustrates a broader insight: welfare can be improved when governments are able to credibly commit to sustainable borrowing paths without fully surrendering their fiscal flexibility. In our model, the parameter ϕ captures the strength of that commitment—higher values induce the government to honor self-imposed debt ceilings more strictly. But when this commitment becomes too strong, the loss of flexibility could increase default risk and reduce welfare. In Figure 7, the presence of an interior optimum for ϕ thus reflects the importance of designing fiscal frameworks that strike a balance between credibility and adaptability. In practice, this speaks to the value of intermediate forms of commitment—such as institutional rules, fiscal compacts, or market-based reputational mechanisms—that discourage over-borrowing but still allow governments to respond to shocks. These “soft” rules can outperform both hard constraints that rigidly bind fiscal policy and

unconstrained regimes that allow full discretion.⁸

4 Conclusion

This paper has shown that debt-ceiling announcements can function as endogenous commitment devices through which governments discipline their future selves. Embedding this mechanism in a sovereign-default framework with long-term debt reveals that self-imposed debt ceilings—when costly to revise—provide a tractable form of intermediate commitment: they curb debt dilution by partially constraining future borrowing, yet remain flexible enough to accommodate adverse shocks.

A three-period benchmark isolates the logic of the mechanism. Without commitment, the government overborrows at the short end because it fails to internalize how additional issuance dilutes legacy bondholders and raises default risk. Announcing a ceiling backed by a deviation cost counteracts this bias. For low costs, the ceiling binds only partially and yields an allocation strictly between the no-commitment and full-commitment benchmarks. Once the cost reaches a simple threshold, the ceiling fully sustains the commitment allocation. These results formally characterize the conditions under which an incumbent can induce its preferred allocation through an appropriately chosen ceiling that its successor optimally finds it costly to violate.

Extending the mechanism to a quantitative infinite-horizon model shows that the same logic operates in a realistic stochastic environment with long-term debt. Calibrated to Argentina, the model predicts that governments voluntarily adopt “reasonable” debt ceilings: ones that are sufficiently tight to mitigate dilution and lower spreads, but not so tight as to unduly restrict fiscal flexibility. The resulting ceilings reduce borrowing costs and raise welfare relative to an economy without such rules. However, excessive violation costs produce ceilings that are too rigid, increasing default risk and reducing welfare. The welfare effects of strengthening the ceiling are thus non-monotonic, highlighting the importance of intermediate—rather than extreme—levels of commitment.

Overall, our results reinforce the view that fiscal frameworks function best when they combine discipline with flexibility. Self-imposed debt ceilings that are costly but can be

⁸In our framework, a “hard” constraint corresponds to a very high value of ϕ that makes the ceiling effectively binding for the successor. This is a one-period-ahead form of commitment, distinct from the fixed time-0 rules imposed under full commitment in [Hatchondo et al. \(2022\)](#) and [Roch and Roldán \(2023\)](#). Our analysis complements this literature by showing how governments endogenously generate state-dependent degrees of intermediate commitment through period-by-period ceiling choices that may be revised at a political or institutional cost.

revised can deliver meaningful commitment gains even without external enforcement. This suggests that fiscal institutions that shape how governments announce, update, and justify borrowing limits may play a larger role in debt sustainability than previously recognized.

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A Mathematical Details of the Three-Period Model

This appendix provides derivations for the three-period benchmark in Section 2. We first derive expected utility at $t = 3$ and the pricing function, then characterize the no-commitment and commitment allocations, and finally analyze the intermediate-commitment case with a debt ceiling.

A.1 Environment and Default

Time is $t \in \{1, 2, 3\}$. The government receives endowments $y_1 = y_2 = 0$ and $y_3 = \bar{y} > 0$. Preferences are CRRA with risk aversion $\sigma = 2$:

$$u(c) = -\frac{1}{c}, \quad \beta \in (0, 1].$$

All debt matures at $t = 3$. Let b_1 denote face value issued at $t = 1$ and b_2 face value issued at $t = 2$, so total obligations at $t = 3$ are

$$B = b_1 + b_2.$$

Following the main text, define the repayment slack

$$x \equiv \frac{\bar{y} - B}{\bar{y}} \in (0, 1], \quad \text{so that} \quad B = \bar{y}(1 - x).$$

Default cost at $t = 3$ is governed by a Pareto random variable $\theta \sim \text{Pareto}(\alpha)$ on $[1, \infty)$ with shape parameter $\alpha = \frac{1}{2}$. If the government repays, it consumes $c_3 = \bar{y} - B = \bar{y}x$. If it defaults, it consumes $c_3 = \bar{y}/\theta$. Default occurs whenever

$$\frac{\bar{y}}{\theta} \geq \bar{y} - B \iff \theta \leq \frac{\bar{y}}{\bar{y} - B} = \frac{1}{x}.$$

With risk-neutral lenders and a unit gross risk-free rate, the bond price equals the probability of repayment. Using the CDF of the Pareto distribution,

$$q(B) = \Pr(\theta > 1/x) = x^\alpha = \sqrt{x}.$$

Expected utility at $t = 3$ conditional on B is

$$\begin{aligned}
\mathbb{E}[u(c_3) \mid B] &= \Pr(\theta > 1/x) u(\bar{y} - B) + \int_1^{1/x} u\left(\frac{\bar{y}}{\theta}\right) \alpha \theta^{-(\alpha+1)} d\theta \\
&= x^{1/2} \left(-\frac{1}{\bar{y}x} \right) - \int_1^{1/x} \frac{1}{\bar{y}} \frac{1}{2} \theta^{-1/2} d\theta \\
&= -\frac{x^{1/2}}{x\bar{y}} - \frac{1}{\bar{y}} \left[\theta^{1/2} \right]_1^{1/x} \\
&= -\frac{x^{1/2}}{x\bar{y}} - \frac{1}{\bar{y}} \left(x^{-1/2} - 1 \right) \\
&= \frac{1}{\bar{y}} \left(1 - 2x^{-1/2} \right),
\end{aligned}$$

which is equation (4) in the main text.

A.2 No Commitment

We first solve for the period-2 best response $b_2^\dagger(b_1)$ taking b_1 as given, and then characterize the period-1 choice b_1^{NC} .

Period-2 Problem

Given b_1 , total debt is $B = b_1 + b_2$, the slack is $x = 1 - (b_1 + b_2)/\bar{y}$, and the bond price is $q(B) = \sqrt{x}$. Period-2 consumption is

$$c_2 = q(B) b_2 = \sqrt{x} b_2.$$

The period-2 government chooses b_2 to maximize

$$\begin{aligned}
V_2^{NC}(b_1) &= \max_{b_2} \left\{ u(c_2) + \beta \mathbb{E}[u(c_3) \mid B] \right\} \\
&= \max_{b_2} \left\{ -\frac{1}{\sqrt{x} b_2} + \frac{\beta}{\bar{y}} \left(1 - 2x^{-1/2} \right) \right\},
\end{aligned}$$

where $x = 1 - (b_1 + b_2)/\bar{y}$ and $dx/db_2 = -1/\bar{y}$.

The first-order condition (FOC) is

$$\frac{\partial V_2^{NC}}{\partial b_2} = \frac{x^{-1/2}}{b_2^2} - \frac{1}{2\bar{y}} \frac{x^{-3/2}}{b_2} - \frac{\beta}{\bar{y}^2} x^{-3/2} = 0.$$

Multiplying by $x^{3/2}\bar{y}^2b_2^2$ and using $x = 1 - (b_1 + b_2)/\bar{y}$, this simplifies to

$$-\frac{b_2}{2\bar{y}} + x - \frac{\beta b_2^2}{\bar{y}^2} = 0.$$

Substituting $x = 1 - (b_1 + b_2)/\bar{y}$ and rearranging yields the quadratic in b_2/\bar{y} :

$$\beta\left(\frac{b_2}{\bar{y}}\right)^2 + \frac{3}{2}\left(\frac{b_2}{\bar{y}}\right) + \left(\frac{b_1}{\bar{y}} - 1\right) = 0. \quad (26)$$

We take the positive root and define

$$b_2^\dagger(b_1) = \frac{\bar{y}}{4\beta} \left[-3 + \sqrt{9 + 16\beta\left(1 - \frac{b_1}{\bar{y}}\right)} \right], \quad (27)$$

which coincides with equation (9) in the main text.

Using (26), the associated slack can be written as

$$x^\dagger(b_1) = 1 - \frac{b_1 + b_2^\dagger(b_1)}{\bar{y}} = \beta\left(\frac{b_2^\dagger(b_1)}{\bar{y}}\right)^2 + \frac{1}{2}\left(\frac{b_2^\dagger(b_1)}{\bar{y}}\right). \quad (28)$$

Differentiating (27) with respect to b_1 gives

$$\frac{db_2^\dagger}{db_1}(b_1) = -\frac{2}{\sqrt{9 + 16\beta\left(1 - \frac{b_1}{\bar{y}}\right)}} = -\frac{2}{4\beta\frac{b_2^\dagger(b_1)}{\bar{y}} + 3}. \quad (29)$$

From (28) we also obtain

$$\frac{dx^\dagger}{db_1}(b_1) = -\frac{1 + \frac{db_2^\dagger}{db_1}(b_1)}{\bar{y}}. \quad (30)$$

The value of following the optimal no-commitment policy at $t = 2$ is

$$V_2^{NC}(b_1) = -\frac{(x^\dagger(b_1))^{-1/2}}{b_2^\dagger(b_1)} + \frac{\beta}{\bar{y}}\left(1 - 2(x^\dagger(b_1))^{-1/2}\right). \quad (31)$$

Differentiating (31) and using (29)–(30) yields

$$\frac{dV_2^{NC}}{db_1}(b_1) = -\left(1 + \frac{db_2^\dagger}{db_1}\right)\frac{(x^\dagger)^{-3/2}}{\bar{y}}\left[\frac{1}{2b_2^\dagger} + \frac{\beta}{\bar{y}}\right] + (x^\dagger)^{-1/2}\frac{\frac{db_2^\dagger}{db_1}}{(b_2^\dagger)^2}, \quad (32)$$

where $x^\dagger = x^\dagger(b_1)$ and $b_2^\dagger = b_2^\dagger(b_1)$.

Period-1 Problem

Anticipating the period-2 policy $b_2^\dagger(b_1)$, the period-1 government solves

$$V_1^{NC}(b_1) = u(c_1) + \beta V_2^{NC}(b_1) = -\frac{1}{\sqrt{x^\dagger(b_1)} b_1} + \beta V_2^{NC}(b_1),$$

where $x^\dagger(b_1)$ and $V_2^{NC}(b_1)$ are given in (28) and (31). The FOC is

$$\begin{aligned} 0 &= \frac{dV_1^{NC}}{db_1}(b_1) \\ &= -\left[\frac{d}{db_1}((x^\dagger)^{-1/2}) \cdot \frac{1}{b_1} + x^{\dagger-1/2} \cdot \frac{d}{db_1}\left(\frac{1}{b_1}\right) \right] + \beta \frac{dV_2^{NC}}{db_1}(b_1). \end{aligned}$$

Using

$$\frac{d}{db_1}((x^\dagger)^{-1/2}) = \frac{1 + \frac{db_2^\dagger}{db_1}}{2\bar{y}} (x^\dagger)^{-3/2}, \quad \frac{d}{db_1}\left(\frac{1}{b_1}\right) = -\frac{1}{b_1^2},$$

and substituting (32), after simplification we obtain

$$0 = (x^\dagger) \left[1 - \beta \frac{\left(\frac{b_1}{\bar{y}}\right)^2}{\left(\frac{b_2^\dagger(b_1)}{\bar{y}}\right)^2} \right] - \frac{1}{2} \left(\frac{b_1}{\bar{y}}\right) \frac{4\beta \left(\frac{b_2^\dagger(b_1)}{\bar{y}}\right) + 1}{4\beta \left(\frac{b_2^\dagger(b_1)}{\bar{y}}\right) + 3}. \quad (33)$$

(33) together with (27) characterizes the no-commitment allocation $(b_1^{NC}, b_2^{NC}) = (b_1^*, b_2^*)$ in the main text.

Proof of Proposition 1

We now prove the maturity comparison in Proposition 1, reproduced here for convenience:

Fix $\beta \in (0, 1)$ and $\bar{y} > 0$. Let (b_1^C, b_2^C) be the commitment allocation and (b_1^{NC}, b_2^{NC}) the no-commitment allocation. Then

$$\frac{b_2^{NC}}{b_1^{NC} + b_2^{NC}} > \frac{b_2^C}{b_1^C + b_2^C} = \frac{\sqrt{\beta}}{1 + \sqrt{\beta}}.$$

Under commitment, the FOCs (derived below) imply $b_2^C/b_1^C = \sqrt{\beta}$. Suppose, for a contradiction, that the no-commitment allocation were relatively more front-loaded:

$$\frac{b_2^C}{b_1^C} = \sqrt{\beta} > \frac{b_2^{NC}}{b_1^{NC}}.$$

Equivalently,

$$\beta > \left(\frac{b_2^{NC}}{b_1^{NC}} \right)^2 = \left(\frac{\frac{b_2^{NC}}{\bar{y}}}{\frac{b_1^{NC}}{\bar{y}}} \right)^2 \implies 1 - \beta \frac{\left(\frac{b_1^{NC}}{\bar{y}} \right)^2}{\left(\frac{b_2^{NC}}{\bar{y}} \right)^2} < 0.$$

But at (b_1^{NC}, b_2^{NC}) , the no-commitment FOC (33) requires

$$x^\dagger(b_1^{NC}) \left[1 - \beta \frac{\left(\frac{b_1^{NC}}{\bar{y}} \right)^2}{\left(\frac{b_2^{NC}}{\bar{y}} \right)^2} \right] - \frac{1}{2} \left(\frac{b_1^{NC}}{\bar{y}} \right) \frac{4\beta \left(\frac{b_2^{NC}}{\bar{y}} \right) + 1}{4\beta \left(\frac{b_2^{NC}}{\bar{y}} \right) + 3} = 0.$$

Since $x^\dagger(b_1^{NC}) > 0$ and the second term is nonnegative, the bracketed expression must be nonnegative, contradicting the inequality above. Hence

$$\frac{b_2^{NC}}{b_1^{NC}} \geq \sqrt{\beta},$$

which in turn implies

$$\frac{b_2^{NC}}{b_1^{NC} + b_2^{NC}} \geq \frac{\sqrt{\beta}}{1 + \sqrt{\beta}} = \frac{b_2^C}{b_1^C + b_2^C}.$$

■

A.3 Commitment Problem

Under commitment, the period-1 government chooses both b_1 and b_2 , taking into account their joint effect on prices and default risk.

Using $B = b_1 + b_2$, $x = 1 - B/\bar{y}$, and $q(B) = \sqrt{x}$, period-1 and period-2 consumptions are $c_1 = q(B)b_1$ and $c_2 = q(B)b_2$, and the lifetime objective is

$$V_1^C(b_1, b_2) = -\frac{1}{q(B)b_1} - \frac{\beta}{q(B)b_2} + \beta^2 \mathbb{E}[u(c_3) \mid B], \quad (34)$$

with $\mathbb{E}[u(c_3) \mid B]$ given by (4).

Since $x = (\bar{y} - B)/\bar{y}$ and $q = \sqrt{x}$,

$$q'(B) = \frac{dq}{dB} = \frac{dq}{dx} \frac{dx}{dB} = \frac{1}{2\sqrt{x}} \left(-\frac{1}{\bar{y}} \right) = -\frac{1}{2\bar{y}\sqrt{x}},$$

and

$$\frac{d}{dB} \mathbb{E}[u(c_3) \mid B] = \frac{d}{dB} \left[\frac{1}{\bar{y}} \left(1 - 2x^{-1/2} \right) \right] = -\frac{1}{\bar{y}^2 x^{3/2}}.$$

Differentiating (34) with respect to b_1 and b_2 , and using $\partial B/\partial b_1 = \partial B/\partial b_2 = 1$, we obtain:

FOC with respect to b_1 .

$$\frac{\partial V_1^C}{\partial b_1} = \frac{q + b_1 q'}{q^2 b_1^2} + \beta \frac{q'}{q^2 b_2} + \beta^2 \frac{d}{dB} \mathbb{E}[u(c_3) \mid B] = 0. \quad (35)$$

FOC with respect to b_2 .

$$\frac{\partial V_1^C}{\partial b_2} = \frac{q'}{q^2 b_1} + \beta \frac{q + b_2 q'}{q^2 b_2^2} + \beta^2 \frac{d}{dB} \mathbb{E}[u(c_3) \mid B] = 0. \quad (36)$$

Subtracting (36) from (35) eliminates the common term $\beta^2 \frac{d}{dB} \mathbb{E}[u(c_3) \mid B]$:

$$\begin{aligned} 0 &= \frac{q + b_1 q'}{q^2 b_1^2} + \beta \frac{q'}{q^2 b_2} - \frac{q'}{q^2 b_1} - \beta \frac{q + b_2 q'}{q^2 b_2^2} \\ &= \frac{qb_2^2 - \beta qb_1^2}{q^2 b_1^2 b_2^2}, \end{aligned}$$

which implies

$$\frac{b_2^C}{b_1^C} = \sqrt{\beta}. \quad (37)$$

This is the maturity ratio reported in the main text.

Using $b_2^C = \sqrt{\beta} b_1^C$ and $B^C = b_1^C + b_2^C$, we have

$$x^C = 1 - \frac{B^C}{\bar{y}} = 1 - \frac{b_1^C}{\bar{y}}(1 + \sqrt{\beta}), \quad q^C = \sqrt{x^C}.$$

Substituting these expressions into (35) (or equivalently (36)) and simplifying yields a quadratic in b_1^C/\bar{y} :

$$\beta^2 \left(\frac{b_1^C}{\bar{y}} \right)^2 + \frac{3(1 + \sqrt{\beta})}{2} \left(\frac{b_1^C}{\bar{y}} \right) - 1 = 0.$$

The positive root is

$$\frac{b_1^C}{\bar{y}} = \frac{-\frac{3(1+\sqrt{\beta})}{2} + \sqrt{4\beta^2 + \frac{9}{4}(1 + \beta + 2\sqrt{\beta})}}{2\beta^2}, \quad (38)$$

and, by (37),

$$\frac{b_2^C}{\bar{y}} = \frac{-\frac{3(1+\sqrt{\beta})}{2} + \sqrt{4\beta^2 + \frac{9}{4}(1 + \beta + 2\sqrt{\beta})}}{2\beta^{3/2}}. \quad (39)$$

These are the closed-form expressions reported in the main text and used to construct the commitment benchmark.

A.4 Intermediate Commitment with a Debt Ceiling

We now derive the key objects for the intermediate-commitment case with a debt ceiling \bar{b} and enforcement cost ϕ .

Time-2 Problem and the Value Gap

At $t = 2$, given inherited b_1 and a ceiling \bar{b} , the government chooses b_2 to maximize

$$V_2(b_1, \bar{b}) = \max_{b_2} \left\{ u(c_2) - \phi \mathbf{1}_{\{b_2 > \bar{b}\}} + \beta \mathbb{E}[u(c_3) \mid B] \right\},$$

with $c_2 = q(B)b_2$, $B = b_1 + b_2$, $x = 1 - B/\bar{y}$ and $q(B) = \sqrt{x}$. Define the continuation value excluding the penalty:

$$f_0(b_2 \mid b_1) = u(c_2) + \beta \mathbb{E}[u(c_3) \mid B] = -\frac{1}{b_2 \sqrt{x}} + \frac{\beta}{\bar{y}} \left(1 - 2x^{-1/2} \right), \quad x = 1 - \frac{b_1 + b_2}{\bar{y}}. \quad (40)$$

When $\phi = 0$, the FOC coincides with the no-commitment case and yields the interior best response $b_2^\dagger(b_1)$ in (27).

With the ceiling and enforcement cost, the period-2 policy is

$$b_2(b_1, \bar{b}) = \begin{cases} b_2^\dagger(b_1), & \text{if } f_0(b_2^\dagger(b_1) \mid b_1) - \phi > f_0(\bar{b} \mid b_1), \\ \bar{b}, & \text{otherwise.} \end{cases}$$

For given b_1 , a ceiling \bar{b} is *enforceable* if the period-2 incentive constraint

$$f_0(b_2^\dagger(b_1) \mid b_1) - \phi \leq f_0(\bar{b} \mid b_1)$$

holds. Among enforceable ceilings, the period-1 government chooses the largest one. This implies that, whenever the ceiling is used (i.e., $b_2(b_1, \bar{b}) = \bar{b}$ in equilibrium), it lies on the indifference locus

$$f_0(b_2^\dagger(b_1) \mid b_1) - \phi = f_0(\bar{b} \mid b_1), \quad (41)$$

which determines \bar{b} as a function of b_1 .

To sustain the commitment allocation (b_1^C, b_2^C) , the planner sets $\bar{b} = b_2^C$ and chooses ϕ so

that the period-2 government is indifferent between deviating to $b_2^\dagger(b_1^C)$ and respecting the ceiling:

$$f_0(b_2^\dagger(b_1^C) \mid b_1^C) - \phi = f_0(b_2^C \mid b_1^C).$$

The smallest enforcement level that satisfies this is the *value gap*

$$\phi_{\min} = f_0\left(b_2^\dagger(b_1^C) \mid b_1^C\right) - f_0(b_2^C \mid b_1^C) > 0, \quad (42)$$

which coincides with equation (14) in the main text.

Derivatives of the Continuation Value

For notational convenience, define

$$x(b_1, b_2) = 1 - \frac{b_1 + b_2}{\bar{y}}, \quad s(b_1, b_2) = \sqrt{x(b_1, b_2)}.$$

Then the continuation value (40) can be written as

$$V(b_1, b_2) = -\frac{1}{b_2 s(b_1, b_2)} + \frac{\beta}{\bar{y}} \left(1 - \frac{2}{s(b_1, b_2)}\right). \quad (43)$$

We will use this representation to derive the generalized Euler equation under intermediate commitment.

Using $x = 1 - (b_1 + b_2)/\bar{y}$ and $\partial x / \partial b_j = -1/\bar{y}$ for $j = 1, 2$, we obtain

$$\frac{\partial s}{\partial b_j} = \frac{1}{2} x^{-1/2} \frac{\partial x}{\partial b_j} = -\frac{1}{2\bar{y}} x^{-1/2}, \quad \frac{\partial}{\partial b_j} \left(\frac{1}{s}\right) = \frac{1}{2\bar{y}} x^{-3/2}, \quad j = 1, 2.$$

Differentiating (43) gives:

Derivative with respect to b_1 .

$$\begin{aligned} \frac{\partial V}{\partial b_1}(b_1, b_2) &= -\frac{1}{b_2} \frac{\partial}{\partial b_1} \left(\frac{1}{s}\right) - \frac{2\beta}{\bar{y}} \frac{\partial}{\partial b_1} \left(\frac{1}{s}\right) \\ &= -\left(\frac{1}{2\bar{y} b_2} + \frac{\beta}{\bar{y}^2}\right) x(b_1, b_2)^{-3/2}. \end{aligned} \quad (44)$$

Derivative with respect to b_2 .

$$\begin{aligned}\frac{\partial V}{\partial b_2}(b_1, b_2) &= \frac{1}{b_2^2 s} - \frac{1}{b_2} \frac{\partial}{\partial b_2} \left(\frac{1}{s} \right) - \frac{2\beta}{\bar{y}} \frac{\partial}{\partial b_2} \left(\frac{1}{s} \right) \\ &= \frac{1}{b_2^2 \sqrt{x(b_1, b_2)}} - \left(\frac{1}{2\bar{y} b_2} + \frac{\beta}{\bar{y}^2} \right) x(b_1, b_2)^{-3/2}.\end{aligned}\tag{45}$$

Envelope Result for the Disciplined Continuation Value

For each b_1 , denote by $b_2^*(b_1)$ the disciplined policy that respects the ceiling and solves the indifference condition (41), and define the associated continuation value

$$V^*(b_1) = V(b_1, b_2^*(b_1)).$$

For comparison, denote by $b_2^\dagger(b_1)$ the unconstrained best response from (27). A standard implicit-function argument shows that

$$\frac{dV^*}{db_1}(b_1) = \frac{\partial V}{\partial b_1}(b_1, b_2^\dagger(b_1)) + \frac{\partial V}{\partial b_2}(b_1, b_2^\dagger(b_1)) \frac{db_2^\dagger}{db_1}(b_1),\tag{46}$$

so dV^*/db_1 depends only on the unconstrained threat policy $b_2^\dagger(b_1)$ and its derivative. Substituting (44), (45), and (29) into (46) yields an explicit expression for dV^*/db_1 in terms of b_2^\dagger , $x(b_1, b_2^\dagger)$, and the parameters (β, \bar{y}) .

Time-1 Problem and Proof of Proposition 2

At time 1, with a ceiling policy that induces $b_2^*(b_1)$, the government chooses b_1 to maximize

$$V_1(b_1) = u(c_1) + \beta V^*(b_1) = -\frac{1}{\sqrt{x^*(b_1)} b_1} + \beta V^*(b_1),$$

where

$$x^*(b_1) = 1 - \frac{b_1 + b_2^*(b_1)}{\bar{y}}.$$

The FOC is

$$0 = -\frac{1 + b_2'(b_1)}{2\bar{y}} \frac{(x^*(b_1))^{-3/2}}{b_1} + \frac{(x^*(b_1))^{-1/2}}{b_1^2} + \beta \frac{dV^*}{db_1}(b_1),\tag{47}$$

where dV^*/db_1 is given by (46). Rewriting (47) in terms of $B = b_1 + b_2^*(b_1)$ and $x^* = 1 - B/\bar{y}$ yields the generalized Euler equation (15) in the main text.

We are now in a position to prove Proposition 2.

Proof of Proposition 2. Part (1) follows immediately from the fact that when $\phi = 0$ the penalty term in $V_2(b_1, \bar{b})$ is inactive, so the period-2 best response is $b_2^\dagger(b_1)$ from (27). The period-1 FOC then coincides with the no-commitment condition (10) in the main text (equivalently (33) here), yielding (b_1^{NC}, b_2^{NC}) .

For part (2), fix $0 < \phi < \phi_{\min}$. For each b_1 , the ceiling is enforceable only if the period-2 incentive constraint

$$f_0(b_2^\dagger(b_1) \mid b_1) - \phi \leq f_0(\bar{b} \mid b_1)$$

is satisfied. Since tighter ceilings reduce period-2 consumption without relaxing future constraints, the period-1 government chooses the largest enforceable ceiling, which lies on the indifference condition (41). At this ceiling, $b_2(b_1, \bar{b}) = \bar{b} = b_2^\star(b_1)$, and substituting $B = b_1 + b_2^\star(b_1)$ into the period-1 problem yields the generalized Euler equation (15). Comparing with the commitment and no-commitment benchmarks, and using the fact that the ceiling partially disciplines the period-2 overborrowing motive but does not eliminate it, implies $b_1^\star \in (b_1^C, b_1^{NC})$ and $b_2^\star \in (b_2^C, b_2^{NC})$.

Part (3) follows from the definition of ϕ_{\min} in (42). For $\phi = \phi_{\min}$, the ceiling $\bar{b} = b_2^C$ exactly satisfies the incentive constraint at $b_1 = b_1^C$, making the commitment allocation (b_1^C, b_2^C) self-enforcing at $t = 2$. For any $\phi > \phi_{\min}$, deviating to $b_2^\dagger(b_1^C)$ is strictly worse for the period-2 government, so the same ceiling remains enforceable and (b_1^C, b_2^C) solves the period-1 problem. Thus, for all $\phi \geq \phi_{\min}$ the unique sustainable allocation coincides with the commitment benchmark. ■

B Computational Strategy

This appendix describes how we compute the Markov-perfect equilibrium of the infinite-horizon model with long-term debt and endogenous debt-ceiling announcements.

B.1 State Space, Grids, and Interpolation

The government's state at the beginning of a period is

$$(y, B, \bar{B}),$$

where y is the current endowment realization, B is the stock of outstanding long-term debt carried into the period, and \bar{B} is the inherited debt ceiling announced in the previous period. The exogenous process for y follows the AR(1) specification described in Section 3, discretized

using a finite Markov chain with N_y nodes and transition matrix P_y .

We approximate the endogenous state variables on finite grids:

- A grid $\mathcal{B} = \{B_1, \dots, B_{N_B}\}$ for outstanding long-term debt.
- A grid $\bar{\mathcal{B}} = \{\bar{B}_1, \dots, \bar{B}_{N_{\bar{B}}}\}$ for the announced debt ceiling.

Bounds $[B_{\min}, B_{\max}]$ and $[\bar{B}_{\min}, \bar{B}_{\max}]$ are chosen to be wide enough that the equilibrium support of debt and ceilings lies in the interior of each grid. We use evenly spaced points for both B and \bar{B} in the baseline implementation.

For expectations and bond pricing, we repeatedly evaluate objects such as value functions, default probabilities, and policy probabilities at off-grid points. To do so we construct multidimensional interpolants (Matlab's `griddedInterpolant`) on the tensor product grid

$$\mathcal{S} = \mathcal{B} \times \bar{\mathcal{B}} \times \mathcal{Y},$$

and use multilinear interpolation in (B, \bar{B}, y) whenever necessary.

B.2 Extreme-Value Formulation and Policy Probabilities

We follow [Dvorkin et al. \(2021\)](#) and solve the government's problem in a stochastic choice environment with additive Type I extreme-value shocks. For each state (y, B, \bar{B}) the government has two types of actions:

1. Default: exit financial markets, consume $\varrho(y)$, and transition into autarky according to (23).
2. Repay and choose next period's debt and ceiling (B', \bar{B}') , subject to the implementability constraint (20) and the ceiling cost $\Phi(B', \bar{B})$.

Let $v^D(y, B, \bar{B})$ denote the continuation value from default, and let

$$v^R(y, B, \bar{B}; B', \bar{B}')$$

denote the value from repaying and choosing (B', \bar{B}') optimally in the future.

We assume that:

- The default option is subject to an idiosyncratic taste shock ε^D .

- Each repayment alternative (B', \bar{B}') is subject to a shock $\varepsilon^R(B', \bar{B}')$.

Shocks are Type I extreme value, with scale parameter $v > 0$, and we allow for correlation among repayment alternatives through a correlation parameter $p \in (0, 1)$, which nests the standard logit when $p = 1$. Under this specification, the probability of choosing a particular repayment action (B', \bar{B}') conditional on repaying is

$$G(B', \bar{B}' \mid y, B, \bar{B}) = \frac{\exp\left(\frac{v^R(y, B, \bar{B}; B', \bar{B}')}{pv}\right)}{\sum_{(B'', \bar{B}'')} \exp\left(\frac{v^R(y, B, \bar{B}; B'', \bar{B}'')}{pv}\right)},$$

and the probability of default is

$$D(y, B, \bar{B}) = \frac{\exp\left(\frac{v^D(y, B, \bar{B})}{v}\right)}{\exp\left(\frac{v^D(y, B, \bar{B})}{v}\right) + \left[\sum_{(B'', \bar{B}'')} \exp\left(\frac{v^R(y, B, \bar{B}; B'', \bar{B}'')}{pv}\right)\right]^p}.$$

In the implementation we work with “adjusted” value objects that factor out common components of v^R and v^D and compute G and D using vectorized expressions based on these formulas.

B.3 Joint Iteration on Values and Bond Prices

Equilibrium is characterized by a fixed point in the space of (i) government value functions, (ii) default and policy probabilities, and (iii) the bond-price function $q(y, B', \bar{B}')$. We solve for this fixed point by iterating on the joint mapping induced by the government’s Bellman equation and the lenders’ zero-profit condition.

Initialization. We start from an initial guess for:

- The repayment value kernel $v_0^R(y, B, \bar{B}; B', \bar{B}')$ (equivalently, the object denoted v in the code).
- The default value $v_0^D(y, B, \bar{B})$.
- The bond-price function $q_0(y, B', \bar{B}')$.

The initial conditions are either taken from the no-ceiling benchmark or from a previously solved nearby parameterization to accelerate convergence.

Step 1: Probabilities G and D . Given current value objects (v_t^R, v_t^D) , we compute:

- The conditional repayment choice probabilities $G_t(B', \bar{B}' \mid y, B, \bar{B})$ as in the logit formula above.
- The default probability $D_t(y, B, \bar{B})$.

Both objects are stored on the tensor-product grid and represented by `griddedInterpolant` objects for efficient evaluation.

Step 2: Expected continuation values. Using G_t and D_t , we compute expected continuation values. For each state (y, B, \bar{B}) we form

$$\mathbb{E}_y[V(y', B', \bar{B}')] = \sum_{y'} P_y(y' \mid y) [D_t(y', B', \bar{B}') v_t^D(y', B', \bar{B}') + (1 - D_t(y', B', \bar{B}')) \tilde{v}_t^R(y', B', \bar{B}')],$$

where \tilde{v}_t^R is the expected value under the repayment-choice distribution G_t in the following period.

Step 3: Updating the value kernels. We then update the repayment and default values using the Bellman equations:

$$\begin{aligned} v_{t+1}^R(y, B, \bar{B}; B', \bar{B}') &= u(c(y, B, \bar{B}; B', \bar{B}')) + \beta \mathbb{E}_y[V_t(y', B', \bar{B}')], \\ v_{t+1}^D(y, B, \bar{B}) &= u(\varrho(y)) + \beta \mathbb{E}_y[\theta V_t(y', 0, \bar{B}_{\max}) + (1 - \theta) v_t^D(y')], \end{aligned}$$

where consumption $c(\cdot)$ satisfies the implementability constraint with the chosen ceiling cost,

$$c + (\delta + (1 - \delta)z)B = y + q_t(y, B', \bar{B}')(B' - (1 - \delta)B) - \Phi(B', \bar{B}).$$

To ensure numerical stability we apply a relaxation step:

$$v_{t+1}^R \leftarrow \lambda v_{t+1}^R + (1 - \lambda) v_t^R, \quad v_{t+1}^D \leftarrow \lambda v_{t+1}^D + (1 - \lambda) v_t^D,$$

with $\lambda \in (0, 1)$.

Step 4: Updating the bond-price function. Given default probabilities $D_t(y', B', \bar{B}')$ and the repayment-policy distribution G_t , we impose the lenders' zero-profit condition to

obtain a new price function. For each (y, B', \bar{B}') ,

$$q_{t+1}(y, B', \bar{B}') = \frac{1}{1+r} \mathbb{E}_{y'} \left[\left(1 - D_t(y', B', \bar{B}') \right) \left(\delta + (1-\delta)z + (1-\delta) \mathbb{E}_{(B'', \bar{B}'')|G_t} [q_t(y', B'', \bar{B}'')] \right) \right],$$

where the inner expectation integrates over next-period debt and ceiling choices using G_t .⁹

Step 5: Convergence. Steps 1–4 are iterated until the sup-norm of value-function differences is below a small tolerance,

$$\max \left\{ \|v_{t+1}^R - v_t^R\|_\infty, \|v_{t+1}^D - v_t^D\|_\infty \right\} < 10^{-5},$$

at which point we treat (v^R, v^D, G, D, q) as an equilibrium fixed point for the given value of the debt-ceiling penalty parameter ϕ and cost curvature ζ .

B.4 Debt-Ceiling Costs and Variants

The cost of exceeding the debt ceiling is given by

$$\Phi(B', \bar{B}) = \begin{cases} 0, & B' \leq \bar{B}, \\ \phi(B' - \bar{B})^\zeta, & B' > \bar{B}, \end{cases}$$

and we consider two cases in the quantitative analysis:

- **Fixed-cost case** ($\zeta = 0$): any breach of the announced ceiling triggers a discrete penalty ϕ , independent of the size of the violation.
- **Quadratic-cost case** ($\zeta = 2$): larger violations of the ceiling are disproportionately more costly, $\Phi \propto (B' - \bar{B})^2$.

For each specification we solve the equilibrium using the joint iteration described above. The numerical algorithm is identical across cases; only the evaluation of $\Phi(B', \bar{B})$ changes.

B.5 Simulation and Welfare Computation

Once the equilibrium objects (G, D, q) have been computed for a given ϕ , we simulate the model to obtain model-implied moments and welfare.

⁹In the code this is implemented by looping over future grid points (B'', \bar{B}'') , accumulating the continuation price, and then averaging with respect to both the income transition matrix and the policy probabilities.

Simulation. We simulate long histories of the Markov state (y_t, B_t, \bar{B}_t) using:

1. Draw a sequence of shocks $\{y_t\}$ from the Markov chain P_y .
2. At each t , compute default $D(y_t, B_t, \bar{B}_t)$ and repayment probabilities $G(B_{t+1}, \bar{B}_{t+1} \mid y_t, B_t, \bar{B}_t)$.
3. Draw default vs repayment using D , and conditional on repayment, draw (B_{t+1}, \bar{B}_{t+1}) using G .

We discard an initial burn-in and use the remainder to compute the distribution of debt, default frequencies, average and volatility of spreads, and the behavior of announced ceilings.

Welfare. To evaluate the welfare effects of introducing debt-ceiling frictions we compute expected lifetime utility under the equilibrium policies for each value of ϕ . Let $V^{SP}(y_0, B_0 \mid \phi)$ denote the planner's evaluation of welfare under the Markov-perfect government policies associated with ϕ , computed as

$$V^{SP}(y_0, B_0 \mid \phi) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where the expectation is taken over simulated histories consistent with the equilibrium policy functions. We report consumption-equivalent welfare gains relative to the benchmark without debt-ceiling costs ($\phi = 0$), defined as the constant percentage adjustment in consumption that makes the representative household indifferent between the two environments.

In all cases the algorithm converges rapidly and produces smooth policy functions and bond-price schedules, thanks to the extreme-value formulation and the use of interpolation on a moderate-sized multidimensional grid.

C Additional Figures and Tables

Table 2: Regression Results for Monthly EMBIG

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Lagged dummies	Full model dummies	Lagged haircuts	Full haircuts	Dummies + haircuts	+ rating	Fundamentals	Full model
Fiscal rule	-42.168* (23.690)	-59.280*** (20.931)	-44.871* (23.436)	-60.552*** (20.296)	-34.343 (23.241)	-34.800* (21.016)	-42.239* (22.292)	-52.712** (20.919)
Restructuring dummy, 1 year lag	193.362*** (62.135)	116.593* (65.816)			217.225** (107.542)	222.122** (113.010)	97.124 (109.145)	91.950 (118.450)
Restructuring dummy, 2 year lag	89.724* (50.654)	100.703** (49.986)			79.590 (96.852)	79.192 (99.556)	34.454 (95.717)	87.405 (96.803)
Restructuring dummy, 3 year lag	29.323 (43.674)	50.934 (38.757)			-95.417 (68.717)	-96.716 (59.653)	-87.042 (64.259)	-20.725 (62.423)
Restructuring dummy, 4 and 5 year lag	13.717 (33.621)	50.128* (28.860)			-149.403** (73.299)	-150.527** (60.360)	-116.785* (66.517)	-82.993 (58.449)
Restructuring dummy, 6 and 7 year lag	-3.562 (31.936)	17.732 (26.701)			-169.371*** (62.548)	-169.795*** (50.134)	-172.215*** (57.650)	-148.988*** (47.820)
US low-grade corporate yield	56.243*** (4.205)	55.155*** (4.122)	55.996*** (4.200)	54.983*** (4.120)	56.090*** (4.210)	56.099*** (4.151)	54.961*** (4.167)	55.100*** (4.128)
Residuals		-42.801*** (5.256)		-42.323*** (5.183)		-58.536*** (6.215)		-41.427*** (5.266)
Public debt to GNI		3.984*** (1.232)		4.060*** (1.213)			4.823*** (1.089)	3.857*** (1.229)
GDP real growth		-9.945*** (2.558)		-10.659*** (2.566)			-13.559*** (3.143)	-10.367*** (2.634)
Reserves to imports		-1.573*** (0.452)		-1.613*** (0.452)				-1.585*** (0.453)
Inflation		0.265** (0.123)		0.214* (0.124)				0.198 (0.121)
Budget balance to GDP		-12.366*** (3.867)		-12.494*** (3.832)				-12.219*** (3.933)
Current account to GDP		-6.228*** (1.637)		-5.789*** (1.591)				-5.826*** (1.647)
Political Risk Index (ICRG)		-12.593*** (1.665)		-12.765*** (1.683)			-14.084*** (1.922)	-12.645*** (1.674)
Haircut (SZ), 1 year lag			3.414*** (1.315)	2.377** (0.981)	-0.914 (1.994)	-0.902 (1.863)	-0.117 (2.009)	0.578 (1.986)
Haircut (SZ), 2 year lag			1.853* (1.006)	2.055** (0.799)	0.027 (1.885)	0.020 (1.823)	0.740 (1.843)	0.154 (1.683)
Haircut (SZ), 3 year lag			1.594 (1.015)	1.545* (0.797)	3.323** (1.679)	3.331** (1.405)	2.845** (1.427)	1.828 (1.266)
Haircut (SZ), 4 and 5 year lag			1.233* (0.688)	1.787*** (0.619)	4.058*** (1.522)	4.076*** (1.276)	3.899*** (1.274)	3.343*** (1.277)
Haircut (SZ), 6 and 7 year lag			0.875 (0.608)	1.289** (0.526)	4.159*** (1.179)	4.167*** (1.005)	4.327*** (1.075)	4.179*** (0.923)
Constant	-65.316* (36.288)	772.939*** (119.196)	-64.273* (36.640)	789.493*** (121.066)	-66.876* (36.385)	-66.584* (35.365)	836.802*** (132.951)	780.092*** (119.708)
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	9194	9186	9194	9186	9194	9186	9194	9186
R-squared	0.596	0.690	0.595	0.691	0.603	0.661	0.648	0.693
Adjusted. R-squared	0.593	0.687	0.591	0.687	0.598	0.657	0.644	0.689

Notes: This table reports coefficients from unbalanced panel regressions of monthly EMBIG sovereign yield spreads—measured as the average country spread over U.S. Treasury bonds (EMBIG stripped spread, in basis points)—on fiscal rules, restructuring history, haircut measures, global risk factors, and macroeconomic fundamentals. The sample covers 57 emerging and developing economies over 1993–2019, augmenting the *Cruces and Trebesch (2013)* dataset with a fiscal-rule indicator constructed from the IMF Fiscal Rules Database (the same source underlying Figure 1) and using monthly restructuring and haircut variables following *Cruces and Trebesch (2013)* and *Arce and Fourakis (2025)*. The country sample includes Angola, Argentina, Azerbaijan, Bulgaria, Belarus, Bolivia, Brazil, Chile, China, Cameroon, Colombia, Costa Rica, Dominican Republic, Ecuador, Egypt, Ethiopia, Gabon, Ghana, Guatemala, Honduras, Croatia, Hungary, Indonesia, India, Iraq, Jamaica, Jordan, Kazakhstan, Kenya, Lebanon, Sri Lanka, Lithuania, Morocco, Mexico, Mongolia, Mozambique, Malaysia, Nigeria, Pakistan, Panama, Peru, Philippines, Papua New Guinea, Poland, Paraguay, Russia, El Salvador, Serbia, Thailand, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, Vietnam, South Africa, and Zambia. Country-specific fundamentals are lagged 12 months, while the U.S. low-grade corporate yield and the ICRG Political Risk Index are lagged 1 month. All regressions include country and time fixed effects, and robust standard errors are clustered at the country level.

*** Significant at the 1 percent level.

** Significant at the 5 percent level.

* Significant at the 10 percent level.

Table 3: Regression Results for Monthly Standard Deviation of Daily EMBIG

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Lagged dummies	Full model dummies	Lagged haircuts	Full haircuts	Dummies + haircuts	Dummies + haircuts + rating	Fundamentals	Full model
Fiscal rule	-4.755*** (1.688)	-4.888*** (1.592)	-4.859*** (1.656)	-4.903*** (1.571)	-4.301** (1.670)	-4.236*** (1.616)	-4.602*** (1.652)	-4.524*** (1.601)
Restructuring dummy, 1 year lag	-3.704 (4.195)	-7.229 (4.506)			1.237 (6.395)	1.191 (6.601)	-5.056 (6.942)	-4.897 (7.171)
Restructuring dummy, 2 year lag	-2.645 (4.814)	-1.876 (5.026)			2.596 (8.843)	2.576 (9.050)	0.594 (8.901)	3.377 (9.173)
Restructuring dummy, 3 year lag	-7.135* (3.888)	-5.920 (3.888)			-6.216 (6.606)	-6.298 (6.495)	-5.523 (6.375)	-2.023 (6.487)
Restructuring dummy, 4 and 5 year lag	-3.488 (3.065)	-2.115 (2.859)			-12.187** (5.755)	-12.256** (5.330)	-10.586* (5.396)	-8.195 (5.245)
Restructuring dummy, 6 and 7 year lag	-4.078* (2.412)	-3.142 (2.342)			-14.147*** (4.502)	-14.184*** (4.240)	-14.488*** (4.274)	-13.128*** (4.236)
US low-grade corporate yield	3.885*** (0.453)	3.836*** (0.452)	3.876*** (0.453)	3.832*** (0.453)	3.877*** (0.453)	3.877*** (0.454)	3.827*** (0.451)	3.834*** (0.452)
Residuals		-1.746*** (0.369)		-1.741*** (0.365)		-2.549*** (0.433)		-1.692*** (0.371)
Public debt to GNI		0.139 (0.096)		0.142 (0.096)			0.211** (0.089)	0.130 (0.096)
GDP real growth		-0.583*** (0.191)		-0.564*** (0.192)			-0.802*** (0.217)	-0.587*** (0.195)
Reserves to imports		-0.040 (0.034)		-0.038 (0.034)				-0.040 (0.034)
Inflation		0.032*** (0.008)		0.032*** (0.008)				0.030*** (0.008)
Budget balance to GDP		-0.749*** (0.265)		-0.764*** (0.264)				-0.749*** (0.269)
Current account to GDP		-0.258** (0.116)		-0.278** (0.113)				-0.257** (0.120)
Political Risk Index (ICRG)		-0.596*** (0.143)		-0.601*** (0.142)			-0.679*** (0.153)	-0.605*** (0.144)
Haircut (SZ), 1 year lag			-0.109 (0.072)	-0.150** (0.071)	-0.156 (0.108)	-0.155 (0.105)	-0.101 (0.115)	-0.074 (0.118)
Haircut (SZ), 2 year lag			-0.084 (0.073)	-0.071 (0.074)	-0.164 (0.145)	-0.167 (0.149)	-0.127 (0.147)	-0.161 (0.149)
Haircut (SZ), 3 year lag			-0.141* (0.081)	-0.143* (0.073)	-0.041 (0.139)	-0.042 (0.135)	-0.066 (0.128)	-0.123 (0.126)
Haircut (SZ), 4 and 5 year lag			-0.014 (0.062)	-0.004 (0.056)	0.209* (0.118)	0.208* (0.109)	0.204* (0.105)	0.146 (0.104)
Haircut (SZ), 6 and 7 year lag			-0.018 (0.046)	-0.003 (0.045)	0.251*** (0.086)	0.247*** (0.085)	0.266*** (0.084)	0.249*** (0.082)
Constant	-9.527*** (3.608)	28.370*** (10.371)	-9.754*** (3.696)	28.145*** (10.386)	-9.613*** (3.626)	-9.608*** (3.637)	34.665*** (10.385)	29.052*** (10.431)
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N	8996	8989	8996	8989	8996	8989	8996	8989
R-squared	0.245	0.269	0.244	0.269	0.248	0.260	0.261	0.271
Adj. R-squared	0.238	0.262	0.237	0.261	0.240	0.252	0.253	0.263

Notes: This table reports coefficients from unbalanced panel regressions with robust standard errors clustered at the country level. The dependent variable is the monthly volatility of daily EMBIG returns, measured as their standard deviation. The dataset, country coverage, and variable construction follow Table 2. The analysis covers 1994–2019. Country-specific fundamentals are lagged 12 months, while the U.S. low-grade corporate yield and the ICRG Political Risk Index are lagged 1 month.

*** Significant at the 1 percent level.

** Significant at the 5 percent level.

* Significant at the 10 percent level.