Dynamic Relationship Banking and the Equilibrium Cross-Section of Bank Markups *

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Abstract

Bank markups have risen substantially, dispersion across banks has increased, and large banks now charge higher markups than smaller institutions. We develop a general equilibrium model in which persistent borrower and depositor relationships endogenously generate heterogeneous bank market power. Banks are dynamic two-sided intermediaries that compete for customers while inheriting partially captive borrower and depositor bases. Relationship capital creates a trade-off between current margins and future franchise value, generating endogenous loan markups and deposit markdowns that vary across banks. Regulatory constraints and costly external equity make market power on one side of the balance sheet affect pricing on the other, linking deposit markdowns and loan markups. Quantitatively, two-sided bank market power has sizable macroeconomic implications, reducing financial intermediation and lowering aggregate output. Policy-rate changes alter franchise values, leading banks with different customer bases to adjust loan and deposit rates differently.

JEL Codes: D43, E44, G12, G21, L11.

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1 Introduction

Relationship banking has long been a defining feature of the U.S. financial system. Through repeated interactions across lending, deposit-taking, payments, and other financial services, banks develop durable relationships with customers that shape the allocation and pricing of financial products. These relationships are widely viewed as a source of bank market power. On the lending side, incumbent banks accumulate information and customer-specific capital that can reduce borrowers' willingness to switch to alternative sources of finance. On the deposit side, customer loyalty, switching costs, and the bundling of financial services can weaken competition for deposits. As a result, relationship banking generates franchise value by allowing banks to earn rents from both their lending relationships and their deposit franchises. As such, banks are dynamic two-sided platforms: they exercise oligopoly power over borrowers, oligopsony power over depositors, and manage both through constrained balance sheets.

A central feature of relationship banking is that its strength differs substantially across institutions. Banks with broader service offerings, stronger customer franchises, or more deeply entrenched relationships may be able to exercise greater pricing power than banks whose customers are more easily contested. Consistent with this view, we document a set of motivating facts using FDIC data from 1984 to 2022. While average loan markups have risen over time, markup dispersion has increased as well. Moreover, the cross-sectional profile of markups has become markedly steeper across the bank-size distribution. In particular, since 2010, larger banks have earned substantially higher markups than smaller banks. These patterns suggest that heterogeneity in relationship-based advantages may be an important source of differences in market power across banks.

In this paper, we ask how changes in the strength of banking relationships affect the cross-sectional distribution of bank markups and aggregate economic outcomes. The question is particularly relevant because the relationship-based business model of traditional banking is increasingly being challenged. On the liability side, technological innovations have lowered switching costs and expanded access to higher-yielding alternatives such as money market funds, reducing the value of established deposit franchises. On the asset side, private credit funds increasingly compete for borrowers that historically relied on bank financing. By making customer relationships more contestable, these developments may erode relationship-based rents and alter both the level and the distribution of market power across banks. Understanding these effects is important for assessing how ongoing changes in financial intermediation reshape competition in banking and, ultimately, the allocation of credit and the macroeconomy.

To study these questions, we develop a general equilibrium model in which bank market power arises from persistent relationships with borrowers and depositors. Banks compete for customers on both sides of the balance sheet by setting loan and deposit rates. Customer relationships are durable: only a fraction of borrowers and depositors reconsider their bank in any given period. As a result, banks inherit installed customer bases that are only partially contestable, giving rise to endogenous market power in both lending and deposit markets. Banks operate subject to financial frictions that link the two sides of the balance sheet, so that lending, funding, and pricing decisions are jointly determined.

The central mechanism of the model is a dynamic trade-off between current margins and future customer relationships. A bank can increase current profits by charging higher loan rates or paying lower deposit rates, but doing so reduces its future customer base. Conversely, more aggressive pricing expands future relationships at the expense of current margins. Because deposits relax funding constraints and loans use scarce balance-sheet capacity, the value of acquiring customers on one side of the balance sheet depends on conditions on the other side. Bank market power is therefore dynamic, two-sided, and state dependent.

The model yields two main theoretical results. First, optimal loan and deposit rates depend on the franchise value of customer relationships. Since banks differ in their inherited customer bases, relationship quality, and balance-sheet positions, franchise values vary across institutions, generating endogenous dispersion in markups. Relationship banking therefore provides a natural source of cross-sectional heterogeneity in market power. Second, relationship frictions and financial frictions play distinct roles. Relationship frictions generate market power by making customers partially captive, while financial frictions transmit that market power across the balance sheet by linking lending and funding decisions. Absent financial frictions, loan and deposit pricing become separable even though banks retain market power in both markets.

Quantitatively, we calibrate the model to match aggregate balance-sheet quantities, average lending and deposit spreads, and the observed dispersion in bank rates. We find that bank market power has large effects on both the scale and pricing of financial intermediation. Relative to a constrained-competitive benchmark, market power raises lending rates, lowers deposit rates, contracts bank balance sheets, and reduces aggregate output. These effects arise because market power distorts both the supply of credit and the supply of deposit funding, with the interaction between the two determining the equilibrium scale of intermediation.

The model also implies that observed spreads are not sufficient statistics for bank market power. Because current pricing affects future customer relationships, the relevant marginal costs of lending and deposit-taking are dynamic rather than static. Consequently, observed spreads reflect not only contemporaneous demand elasticities, but also franchise values associated with future customer relationships and the shadow value of bank balance-sheet capacity. Changes in spreads therefore need not map one-for-one into changes in market power.

Finally, the model generates novel implications for interest rate transmission. Changes in the risk-free rate are not passed through uniformly across banks or across customers. Banks with larger inherited customer bases adjust rates differently from banks facing more contestable demand, producing substantial heterogeneity in pass-through. In particular, in the wake of higher discount rates, banks with larger inherited customer bases raise rates relatively more as the present value of their future customer base declines. Changes in the interest rate thus affect the dynamic trade-off between current margins and future customer relationships. As a result, aggregate lending and deposit rates need not move in parallel with the rates faced by the average bank customer. Relationship banking therefore creates a wedge between customer-level and aggregate interest-rate pass-through.

Related literature. This paper contributes to four strands of work. First, it relates to the literature on bank market power and macroeconomic outcomes. A growing body of work studies how imperfect competition in financial intermediation affects credit allocation, investment, and the transmission of aggregate shocks. Corbae and D'Erasmo (2021) develop a quantitative model of banking industry dynamics to study the macroeconomic effects of bank market structure and regulation. Wang, Whited, Wu, and Xiao (2022) quantifies the role of banking-sector market power in monetary transmission and shows that it can be as important as other financial and regulatory frictions. Villa (2025) studies a real model with heterogeneous firms and a finite number of strategic banks, showing that oligopolistic intermediaries extract larger markups from firms with weaker outside options and that this distortion affects firm growth and aggregate dynamics. Our paper shares with this literature the view that bank market power is a first-order macroeconomic force. We differ by modeling banks as dynamic relationship intermediaries with market power on both sides of the balance sheet. Banks exercise oligopoly power in loan markets and oligopsony power in deposit markets, and these two sources of market power interact through bank financial frictions.

Second, the paper contributes to the literature on deposit-market power and interest-rate pass-through. Drechsler, Savov, and Schnabl (2017) emphasize the role of deposit market power in shaping the transmission of monetary policy to bank funding costs and deposit rates and argue that the deposit franchise has negative duration, helping banks hedge their interest rate exposure. DeMarzo, Krishnamurthy, and Nagel (2024) argue and providence supporting positive duration. Related empirical work, including Li, Loutskina, and Strahan (2019), shows that concentration and deposit-market power affect the pass-through of policy rates to deposit rates. Egan, Hortaçsu, and Matvos (2017) develop an empirical model of the U.S. banking sector in this framework, estimating deposit demand functions for both insured depositors and run-prone uninsured depositors who select among differentiated banks. Using a similar estimation approach, Egan, Stefan, and Sunderam (2022) provide a structural decomposition of the sources of bank value. Buchak, Matvos, Piskorski, and Seru (2024) evaluate the quantitative consequences of monetary and regulatory policies in this context. This literature typically focuses on the deposit side of banks' balance sheets and its implications for monetary transmission. We build on this insight but embed deposit-market power in a two-sided banking problem. In our model, deposits are not only a product over which banks have oligopsony power; they are also a funding input that relaxes lending constraints. As a result, deposit pricing affects lending capacity, and lending opportunities affect the value of deposit relationships. Empirically, Aguirregabiria, Clark, and Wang (2024) structurally estimate the economies of scope between bank deposits and loans.

Third, the paper relates to work on bank lending market power and the allocation of credit. Models with imperfectly competitive lenders show that banks' strategic pricing can distort firms' borrowing costs and real decisions. In Villa (2025), banks' loan-market power varies with firms' outside options and affects firm growth. Our paper instead focuses on the bank side of the relationship: banks inherit stocks of borrower and depositor relationships, and these installed customer bases determine the extent of market power. The relevant state variables are not only current prices or market shares, but the dynamic stocks of relationships that banks carry into the future. This allows loan markups and deposit markdowns to be endogenous, heterogeneous, and state dependent.

Fourth, the paper provides a quantitative perspective on the literature on relationship banking and dynamic customer markets in the spirit of Petersen and Rajan (1994), Petersen and Rajan (1995), or Boot and Thakor (2000), as surveyed in Boot (2000). Relationship lending and deposit relationships imply that customers do not continuously reoptimize their banking choices. We incorporate this idea into a tractable macroeconomic environment by assuming that only a fraction of borrowers and depositors reconsider their bank each period. Customers who reconsider compare banks based on posted rates, bank-specific relationship quality, and idiosyncratic tastes, which generate logit shares in equilibrium. Customers who do not reconsider remain attached to their incumbent bank. This structure gives banks installed customer bases and makes pricing dynamic: current loan and deposit rates affect both current quantities and future relationship stocks. Empirically, the persistence of banking relationships and the associated bank market power has recently linked to depositor inattention (Lu and Wu (2026)), or lack of depositor financial sophistication (Fleckenstein and Longstaff (2024)), for example.

Our main contribution is to integrate these forces in a dynamic general-equilibrium model in which banks simultaneously manage bilateral market power and financial frictions. Relationship frictions generate dynamic oligopoly power in loan markets and dynamic oligopsony power in deposit markets. Financial frictions determine how these two forms of market power interact. Without costly equity issuance and without a loan-to-deposit constraint, the bank's loan-pricing and deposit-pricing problems are separable: loan rates depend only on borrower relationships, and deposit rates depend only on depositor relationships. With financial frictions, the two sides are linked. Deposits relax lending constraints, loans use scarce balance-sheet capacity, and costly external finance makes internal funds valuable. To the best of our knowledge, this is the first dynamic macroeconomic general-equilibrium framework in which heterogeneous banks jointly manage persistent borrower and depositor relationships, endogenous loan markups and deposit markdowns, and balance-sheet frictions that transmit market power across the two sides of intermediation. More broadly, our paper is related to dynamic quantitative models with a rich banking sector, such as Begenau (2020), Begenau and Landvoigt (2022), Elenev, Landvoigt, and Van Nieuwerburgh (2021), Jermann and Xiang (2023), or Bolton, Li, Wang, and Yang (2025).

Outline. The rest of the paper is organized as follows. Section 2 presents stylized facts that motivate the analysis. Section 3 describes the model and discusses the main mechanism and its effects in detail. Section 4 explains the calibration and illustrates the results. Section 5 concludes.

2 Stylized Facts

This section documents the empirical background for the quantitative analysis. We use bank-level data for all U.S. banks registered in the Federal Financial Institutions Examination Council's Central Data Repository from 1984 to 2022. We aggregate banks at the holding-company level and use the resulting panel to construct asset-market shares, bank-level loan markups, and the cross-sectional distribution of markups across bank size.

Bank-level markups are computed in the spirit of Corbae and D'Erasmo (2021), following the implementation in Villa (2025). For each bank b and year t, the markup is defined as

$$\mu_{b,t} = \frac{\text{Interest Return on Loans}_{b,t}}{\text{Cost of Funds}_{b,t} + \text{Marginal Net Expenses}_{b,t}} - 1. \tag{1}$$

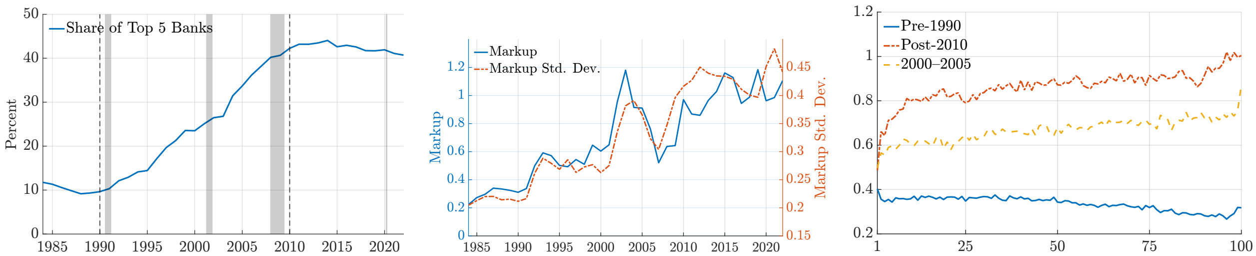
The interest return on loans is calculated as the ratio of interest income on loans to total loans. The cost of funds is calculated as the interest rate paid on deposits and Fed funds, relative to the total amount of deposits and Fed funds. Marginal net expenses are defined as marginal non-interest expenses net of marginal non-interest income. Marginal non-interest expenses and marginal non-interest income are obtained from trans-log cost and revenue functions, along the lines of Demirgüç-Kunt and Martinez Peria (2010). Thus, equation (1) measures the wedge between the return charged on loans and the marginal cost of funding and operating the lending technology.

Figure 1 summarizes the main patterns. Panel (a) reports the asset share of the five largest banks. This share increases substantially over the sample, reflecting the well-known consolidation of the U.S. banking sector. The vertical lines identify the periods used in panel (c) to compare the cross-sectional distribution of markups: a low-concentration period before the major consolidation wave and a high-concentration period after 2010.

Panel (b) reports the loan-weighted average markup and the loan-weighted cross-sectional standard deviation of bank-level markups. The increase in the average markup is consistent with the evidence in Corbae and D'Erasmo (2021) and Villa (2025) that bank market power has risen over time. The additional pattern emphasized here is that the dispersion of markups also increases. Hence, the rise in bank market power is not only reflected in a higher average wedge, but also in greater heterogeneity in pricing wedges across banks.

Panel (c) reports average markups by bank asset decile for the periods highlighted in panel (a). Asset percentile 1 contains the smallest banks, and asset percentile 100 contains the largest banks. The figure shows that the cross-sectional profile of markups changes across concentration regimes. In the more concentrated period, markups are higher among larger banks and the markup profile is steeper across the bank-size distribution.

Figure 1. Bank Concentration, Markups, and Markup Dispersion



(a) Concentration (b) Markups and dispersion (c) Markup distribution

Notes: Panel (a) reports the asset share of the five largest banks. The vertical markers indicate the periods used to compare the cross-sectional distribution of markups in Panel (c). Panel (b) reports the loan-weighted average markup, $\bar{\mu}_t = \sum_b \omega_{b,t}\mu_{b,t}$, and the loan-weighted markup standard deviation, $\sigma_{\mu,t}$, whose variance is $\sigma^2_{\mu,t} = \sum_b \omega_{b,t}(\mu_{b,t} - \bar{\mu}_t)^2$, where $\omega_{b,t} = L_{b,t}/\sum_{b'} L_{b',t}$. Panel (c) reports average bank markups by asset percentile. The horizontal axis orders banks by total assets, with percentile 1 containing the smallest banks and percentile 100 containing the largest banks. The data are from the Federal Financial Institutions Examination Council's Central Data Repository and are aggregated at the holding-company level.

The evidence in Figure 1 provides the empirical background for the analysis. Bank concentration has increased, average loan-side markups have risen, and markup dispersion across banks has become larger. The cross-sectional evidence also shows that the increase in markups is not uniform across the bank-size distribution. Motivated by these patterns, the model below studies dynamic banking platforms in general equilibrium: heterogeneous banks with market power in both loan and deposit markets. Banks differ in their installed loan and deposit relationships and in bank-specific relationship shifters; these differences generate heterogeneous equilibrium loan markups and deposit markdowns.

3 Model

Time is discrete, $t = 0, 1, 2, \ldots$. The economy is populated by a representative household, a representative nonfinancial firm, a government, and a unit mass of relationship banks indexed by $j \in [0, 1]$. Banks post loan and deposit prices and compete for borrowers and depositors. Market power arises from relationship frictions: in each period only a fraction of customers reconsider their bank, while the remaining customers stay with their existing relationship. Conditional on reconsidering, customers choose banks according to a logit demand system.

We work in the standard atomistic monopolistic-competition limit. Each bank takes aggregate quantities and the logit denominators as given when choosing its posted prices. A bank nevertheless internalizes that current prices affect its future relationship stocks.

3.1 Households

The representative household chooses $\{C_t, N_t, B_t, D_t\}_{t\geq 0}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(N_t) + \chi \frac{D_t^{1-\eta} - 1}{1-\eta} \right],$$

subject to

$$C_t + Q_t B_t + Q_t^D D_t = W_t N_t + B_{t-1} + D_{t-1} + \Pi_t^{\text{reb}} + T_t.$$

Here B_t denotes holdings of one-period government bonds, D_t denotes one-period bank deposits, Q_t^D is the aggregate deposit price, Π_t^{reb} are rebated firm and bank profits net of resource costs, and T_t are lump-sum transfers. Let $\Lambda_t \equiv u_C(C_t)$.

The household optimality conditions are

$$Q_t = \beta\, \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \right], \tag{2}$$

$$W_t = \frac{v_N(N_t)}{u_C(C_t)}, \tag{3}$$

$$\chi D_t^{-\eta} = \Lambda_t Q_t^D - \beta\, \mathbb{E}_t[\Lambda_{t+1}] = \Lambda_t (Q_t^D - Q_t). \tag{4}$$

Thus deposits provide liquidity services, and households require a wedge between the deposit price and the risk-free bond price. We define $R^M \equiv Q_t^{-1}$. In a stationary equilibrium,

$$D = \left(\frac{\chi}{u_C(C)(Q^D - \beta)} \right)^{1/\eta}, \qquad \frac{\partial D}{\partial Q^D} < 0. \tag{5}$$

Equivalently, deposit supply is increasing in the gross deposit rate $R^D = (Q^D)^{-1}$.

3.2 Firms

The representative firm produces

$$Y_t = AK_{t-1}^{\alpha} N_t^{1-\alpha}.$$

The firm issues one-period bank debt with face value L_t at price Q_t^L. Issuing L_t delivers proceeds $Q_t^L L_t$ at date t and requires repayment L_t at date $t+1$. The implied gross loan rate is $R_t^L = (Q_t^L)^{-1}$.

Corporate income is taxed at rate $\tau_c \in [0, 1)$. Interest payments are deductible. Since one-period debt is priced, the interest expense associated with debt issued at $t-1$ is $(1-Q_{t-1}^L)L_{t-1}$ and the corresponding tax shield is $\tau_c(1 - Q_{t-1}^L)L_{t-1}$.

We assume debt also entails a convex resource cost:

$$\Psi(L_t) = \frac{\phi_L}{2} L_t^2, \qquad \phi_L > 0.$$

We interpret $\Psi(L_t)$ as a reduced-form firm-side resource cost of debt finance. Although bank debt provides financing benefits, a larger debt position also exposes the firm to leverage-related distortions, including expected distress costs, renegotiation costs, agency costs between managers, creditors, and shareholders, and the organizational costs of maintaining a larger stock of debt obligations. Rather than modeling these frictions explicitly, we summarize them with a convex cost function. Convexity implies that the marginal cost of debt rises with the scale of borrowing, generating a well-defined interior demand for bank debt. The parameter ϕ_L governs the strength of this firm-side borrowing friction and helps discipline the steady-state loan-to-output ratio in the calibration.

Firm payout is

$$\begin{aligned}\pi_t^F &= (1 - \tau_c)(Y_t - W_t N_t) - I_t - L_{t-1} + Q_t^L L_t - \Psi(L_t) \\ &\quad + \tau_c(1 - Q_{t-1}^L)L_{t-1} + \tau_c \delta K_{t-1}.\end{aligned}$$

We allow for a reduced-form corporate-finance wedge in firms' intertemporal decisions. Firm managers discount future payouts using the effective discount factor $\beta^F \equiv \beta - \omega_F$, where $\omega_F \geq 0$. When $\omega_F = 0$, managers evaluate future payouts using the household discount factor. When $\omega_F > 0$, managers are more impatient than households and therefore put less weight on future payouts when choosing debt and capital. We interpret this wedge as a parsimonious way to capture corporate-finance frictions, such as managerial short-termism or agency distortions, without explicitly modeling the underlying agency problem, consistent with models in which agency conflicts distort the horizon of corporate policies (Stein, 1989; Gryglewicz, Mayer, and Morellec, 2020).

The Euler equation for debt is

$$Q_t^L - \phi_L L_t = \beta^F \,\mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(1 - \tau_c(1 - Q_t^L)\right) \right]. \tag{6}$$

In a non-stochastic steady state,

$$L = \frac{Q^L - \beta^F \left[1 - \tau_c(1 - Q^L)\right]}{\phi_L}, \qquad \frac{\partial L}{\partial Q^L} > 0. \tag{7}$$

Thus a higher loan price, equivalently a lower loan rate, raises loan demand. A higher corporate tax rate increases desired borrowing by strengthening the interest tax shield.

The Euler equation for capital is

$$1 = \beta^F \,\mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left((1 - \tau_c) MPK_{t+1} + \tau_c \delta + (1 - \delta)\right) \right], \tag{8}$$

where

$$MPK_{t+1} \equiv \alpha A K_t^{\alpha - 1} N_{t+1}^{1-\alpha}.$$

In steady state,

$$MPK = \frac{(\beta^F)^{-1} - (1 - \delta) - \tau_c \delta}{1 - \tau_c}. \tag{9}$$

3.3 Government

The government issues one-period bonds B_t^g at price Q_t and purchases an exogenous amount G_t of the final good. Market clearing for government bonds requires

$$B_t = B_t^g.$$

The government budget constraint is

$$Q_t B_t^g = B_{t-1}^g + G_t + T_t - \mathcal{T}_t,$$

where $\mathcal{T}_t$ denotes corporate tax revenue. We impose a constant debt target,

$$B_t^g = \bar{B}^g,$$

and transfers adjust residually to satisfy (3.3).

3.4 Relationship Banking

Each bank enters period t with predetermined loan and deposit relationships,

$$l_{j,t-1}, \qquad d_{j,t-1}.$$

It also has loan- and deposit-side relationship shifters,

$$s_{j,t}^L, \qquad s_{j,t}^D,$$

which summarize bank-specific relationship quality on the two sides of the balance sheet. The shifters are strictly positive and evolve exogenously according to

$$\log s_{j,t+1}^L = \rho_L \log s_{j,t}^L + \sigma_s^L \varepsilon_{j,t+1}^L, \tag{10}$$

$$\log s_{j,t+1}^D = \rho_D \log s_{j,t}^D + \sigma_s^D \varepsilon_{j,t+1}^D, \tag{11}$$

where $\rho_L, \rho_D \in [0, 1)$, $\sigma_s^L, \sigma_s^D > 0$, and

$$\varepsilon_{j,t+1}^L, \varepsilon_{j,t+1}^D \sim \mathcal{N}(0, 1).$$

The innovations are independent across banks, across time, and across the loan and deposit sides. The processes are stationary, so the cross-sectional distribution of relationship quality is time invariant in the stationary equilibrium.

Banks post loan and deposit prices $(Q_{j,t}^L, Q_{j,t}^D)$. A higher loan price corresponds to a lower loan rate, since $R_{j,t}^L = (Q_{j,t}^L)^{-1}$; a higher deposit price corresponds to a lower deposit rate, since $R_{j,t}^D = (Q_{j,t}^D)^{-1}$.

Customers do not continuously reoptimize their banking relationships. In each period, a fraction δ_L of borrowers and a fraction δ_D of depositors reconsider their bank. The remaining fractions, $1 - \delta_L$ and $1 - \delta_D$, stay with their existing relationships.

Conditional on reconsidering, customers choose among banks according to a random-utility problem. A borrower i who reoptimizes in period t chooses a bank

$$j_{i,t}^L \in \arg \max_{j \in [0,1]} \left\{ V_{j,t}^L + \varepsilon_{i,j,t}^L \right\},$$

where

$$V_{j,t}^L = \alpha_L \log Q_{j,t}^L + \log s_{j,t}^L, \qquad \alpha_L > 0.$$

A depositor i who reoptimizes chooses a bank

$$j_{i,t}^D \in \arg \max_{j \in [0,1]} \left\{ V_{j,t}^D + \varepsilon_{i,j,t}^D \right\},$$

where

$$V_{j,t}^D = -\alpha_D \log Q_{j,t}^D + \log s_{j,t}^D, \qquad \alpha_D > 0.$$

The signs of α_L and α_D reflect the fact that borrowers prefer lower loan rates, hence higher loan prices, while depositors prefer higher deposit rates, hence lower deposit prices.

The idiosyncratic taste shocks

$$\{\varepsilon^L_{i,j,t}\}_{j\in[0,1]}, \qquad \{\varepsilon^D_{i,j,t}\}_{j\in[0,1]},$$

are independent across banks, customers, and time, and are distributed type-I extreme value. Therefore, conditional on reoptimization, the probability that a borrower chooses bank j is

$$\mathcal{S}^L_{j,t} = \Pr(j^L_{i,t} = j) = \frac{\exp(V^L_{j,t})}{\int_0^1 \exp(V^L_{k,t})\,dk}, \tag{12}$$

and the probability that a depositor chooses bank j is

$$\mathcal{S}^D_{j,t} = \Pr(j^D_{i,t} = j) = \frac{\exp(V^D_{j,t})}{\int_0^1 \exp(V^D_{k,t})\,dk}. \tag{13}$$

Equivalently, defining the logit denominators

$$Z^L_t \equiv \int_0^1 \exp(V^L_{k,t})\,dk, \qquad Z^D_t \equiv \int_0^1 \exp(V^D_{k,t})\,dk,$$

we have $\mathcal{S}^L_{j,t} = \exp(V^L_{j,t})/Z^L_t$ and $\mathcal{S}^D_{j,t} = \exp(V^D_{j,t})/Z^D_t$.

Let $m^L_{j,t} \equiv l_{j,t}/L_t$ and $m^D_{j,t} \equiv d_{j,t}/D_t$ denote bank j's shares of loan and deposit relationships. These shares evolve as

$$m^L_{j,t} = (1-\delta_L)m^L_{j,t-1} + \delta_L \mathcal{S}^L_{j,t}, \tag{14}$$

$$m^D_{j,t} = (1-\delta_D)m^D_{j,t-1} + \delta_D \mathcal{S}^D_{j,t}. \tag{15}$$

Thus, continuing relationships remain with their incumbent bank, while reoptimizing relationships are allocated according to the logit shares. Equivalently, bank-level quantities satisfy

$$l_{j,t} = (1-\delta_L)\frac{l_{j,t-1}}{L_{t-1}}L_t + \delta_L \mathcal{S}^L_{j,t} L_t, \tag{16}$$

$$d_{j,t} = (1-\delta_D)\frac{d_{j,t-1}}{D_{t-1}}D_t + \delta_D \mathcal{S}^D_{j,t} D_t. \tag{17}$$

Integrating over banks gives consistency with aggregate loan and deposit quantities in every period:

$$\int_0^1 l_{j,t}\,dj = L_t, \qquad \int_0^1 d_{j,t}\,dj = D_t.$$

3.5 Bank Cash Flows and Financial Frictions

Bank j's pre-issuance dividend is

$$\widetilde{\text{Div}}^B_{j,t} = l_{j,t-1} - Q^L_{j,t} l_{j,t} - d_{j,t-1} + Q^D_{j,t} d_{j,t}. \tag{18}$$

The first two terms are the net cash flow from the loan side: the bank receives repayment on last period's loans and issues new loans at price $Q^L_{j,t}$. The last two terms are the net cash flow from the deposit side: the bank repays last period's deposits and raises new deposits at price $Q^D_{j,t}$.

Negative pre-issuance dividends require costly external equity. Let

$$E^B_{j,t} \equiv -\widetilde{\text{Div}}^B_{j,t}.$$

The equity issuance cost is

$$\Phi(\widetilde{\text{Div}}^B_{j,t}) = \begin{cases} 0, & \widetilde{\text{Div}}^B_{j,t} \geq 0, \\ \dfrac{\phi^B_E}{2}(E^B_{j,t})^2, & \widetilde{\text{Div}}^B_{j,t} < 0. \end{cases} \tag{19}$$

Net dividends are

$$\text{Div}^B_{j,t} = \widetilde{\text{Div}}^B_{j,t} - \Phi(\widetilde{\text{Div}}^B_{j,t}). \tag{20}$$

Banks also face a leverage constraint,

$$l_{j,t} \leq \bar{\lambda} d_{j,t}, \qquad \bar{\lambda} > 0, \tag{21}$$

with multiplier $\zeta_{j,t} \geq 0$.

The bank's state is

$$x_{j,t} = (l_{j,t-1}, d_{j,t-1}, s^L_{j,t}, s^D_{j,t}).$$

The bank solves

$$V(x_{j,t}) = \max_{Q^L_{j,t}, Q^D_{j,t}} \left\{ \mathrm{Div}^B_{j,t} + \beta\, \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V(x_{j,t+1}) \right] \right\}, \tag{22}$$

subject to (12)–(17), the shifter transition, and (21). Each bank takes aggregate quantities (L_t, D_t) and the logit denominators as given, but internalizes the effect of its posted prices on its future relationship stocks.

3.6 Bank Pricing

This subsection characterizes optimal bank prices. The key objects are the marginal values of installed relationships. Relationship persistence makes these values state dependent and gives rise to dynamic, state-dependent markups. Financial frictions then transmit these values across the two sides of the balance sheet.

Logit derivatives. In the atomistic monopolistic-competition limit, each bank takes the logit denominators as given. Hence

$$\frac{\partial \mathcal{S}^L_{j,t}}{\partial Q^L_{j,t}} = \frac{\alpha_L}{Q^L_{j,t}}\, \mathcal{S}^L_{j,t}, \qquad \frac{\partial \mathcal{S}^D_{j,t}}{\partial Q^D_{j,t}} = -\frac{\alpha_D}{Q^D_{j,t}}\, \mathcal{S}^D_{j,t}. \tag{23}$$

A higher loan price attracts borrowers because it lowers the loan rate, whereas a higher deposit price discourages depositors because it lowers the deposit rate.

Equity issuance wedge. The marginal issuance wedge is

$$\Phi_{E,j,t} \equiv \phi^B_E\, \mathbf{1}\{E^B_{j,t} > 0\} E^B_{j,t}.$$

For any bank-level choice variable z,

$$\frac{\partial \mathrm{Div}^B_{j,t}}{\partial z} = (1 + \Phi_{E,j,t})\, \frac{\partial \widetilde{\mathrm{Div}}^B_{j,t}}{\partial z}. \tag{24}$$

Thus costly external equity raises the marginal value of current internal funds.

Envelope conditions. Let V_l and V_d denote derivatives of the bank value function with respect to the predetermined loan and deposit stocks in the state,

$$x_{j,t} = (l_{j,t-1}, d_{j,t-1}, s^L_{j,t}, s^D_{j,t}).$$

Define

$$g^L_t \equiv \frac{L_t}{L_{t-1}}, \qquad g^D_t \equiv \frac{D_t}{D_{t-1}}.$$

The envelope conditions are

$$V_l(x_{j,t}) = (1 + \Phi_{E,j,t}) \left[1 - g^L_t(1-\delta_L)Q^L_{j,t}\right] - \zeta_{j,t} g^L_t (1-\delta_L) + \mathcal{M}^L_{j,t} g^L_t (1-\delta_L), \tag{25}$$

$$V_d(x_{j,t}) = (1 + \Phi_{E,j,t}) \left[-1 + g^D_t(1-\delta_D)Q^D_{j,t}\right] + \bar{\lambda}\zeta_{j,t} g^D_t (1-\delta_D) + \mathcal{M}^D_{j,t} g^D_t (1-\delta_D), \tag{26}$$

where the SDF-discounted marginal franchise values are

$$\mathcal{M}^L_{j,t} \equiv \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V_l(x_{j,t+1})\right], \tag{27}$$

$$\mathcal{M}^D_{j,t} \equiv \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} V_d(x_{j,t+1})\right]. \tag{28}$$

The envelope conditions (25)–(26) describe the value of inherited relationships. An additional inherited loan relationship has a direct benefit because it generates repayment today. It also affects current lending through the fraction $1-\delta_L$ of borrowers who remain with the bank. This continuation of the installed borrower base has three effects: it changes current cash flow, it uses balance-sheet capacity, and it changes the future relationship stock. These effects are reflected respectively in the current-dividend term, the leverage term $-\zeta_{j,t} g^L_t(1-\delta_L)$, and the continuation term $\mathcal{M}^L_{j,t} g^L_t (1-\delta_L)$.

An additional inherited deposit relationship has the opposite direct payoff implication because deposits must be repaid today. However, persistent depositor relationships also provide funding going forward. The term $\bar{\lambda}\zeta_{j,t} g^D_t(1-\delta_D)$ captures the fact that deposits relax the loan-to-deposit constraint and are therefore especially valuable when balance-sheet capacity is scarce. The continuation term $\mathcal{M}^D_{j,t} g^D_t (1-\delta_D)$ captures the value of carrying the depositor relationship into the future. The factors $g^L_t(1-\delta_L)$ and $g^D_t(1-\delta_D)$ measure how much of an inherited relationship survives into current bank-level quantities, adjusted for aggregate loan and deposit growth.

The objects $\mathcal{M}^L_{j,t}$ and $\mathcal{M}^D_{j,t}$ are the current values of marginally increasing next period's loan and deposit relationship stocks. The loan envelope is positive when an additional installed loan relationship raises future repayment income, whereas the deposit envelope is negative absent relationship benefits because an additional installed deposit is a future repayment obligation. Relationship persistence modifies both objects through their continuation values. The terms $g^L_t(1-\delta_L)$ and $g^D_t(1-\delta_D)$ capture the pass-through of inherited relationships into current bank-level quantities. In a stationary equilibrium, $g^L_t = g^D_t = 1$; outside steady state, aggregate loan and deposit growth scale the value of inherited relationships.

Pricing first-order conditions. Let $\zeta_{j,t} \geq 0$ be the multiplier on the leverage constraint

$$l_{j,t} \leq \bar{\lambda} d_{j,t}.$$

The Kuhn–Tucker conditions are

$$\zeta_{j,t} \geq 0, \qquad l_{j,t} - \bar{\lambda} d_{j,t} \leq 0, \qquad \zeta_{j,t}(l_{j,t} - \bar{\lambda} d_{j,t}) = 0.$$

Using the laws of motion for $l_{j,t}$ and $d_{j,t}$, the first-order condition for the deposit price is

$$0 = (1+\Phi_{E,j,t}) \left[d_{j,t} + Q^D_{j,t} \delta_D \frac{\partial \mathcal{S}^D_{j,t}}{\partial Q^D_{j,t}} D_t \right] + \mathcal{M}^D_{j,t} \delta_D \frac{\partial \mathcal{S}^D_{j,t}}{\partial Q^D_{j,t}} D_t - \zeta_{j,t} \frac{\partial}{\partial Q^D_{j,t}} \left(l_{j,t} - \bar{\lambda} d_{j,t} \right). \tag{29}$$

Similarly, the first-order condition for the loan price is

$$0 = (1+\Phi_{E,j,t}) \left[-l_{j,t} - Q^L_{j,t} \delta_L \frac{\partial \mathcal{S}^L_{j,t}}{\partial Q^L_{j,t}} L_t \right] + \mathcal{M}^L_{j,t} \delta_L \frac{\partial \mathcal{S}^L_{j,t}}{\partial Q^L_{j,t}} L_t - \zeta_{j,t} \frac{\partial}{\partial Q^L_{j,t}} \left(l_{j,t} - \bar{\lambda} d_{j,t} \right). \tag{30}$$

Equations (29)–(30) summarize the dynamic pricing trade-off faced by a relationship bank. Consider first the loan side. A higher loan price $Q^L_{j,t}$ corresponds to a lower loan rate and therefore attracts additional borrowers among the customers currently reconsidering their bank. This raises future loan relationships and creates a continuation benefit, captured by the marginal franchise value $\mathcal{M}^L_{j,t}$. At the same time, a higher loan price has two current-cash-flow effects. First, it lowers current cash flow mechanically on the bank's existing loan volume, captured by the term $-l_{j,t}$. Second, because a higher $Q^L_{j,t}$ attracts additional borrowers, it expands lending by

$$\frac{\partial l_{j,t}}{\partial Q^L_{j,t}} = \delta_L \frac{\partial \mathcal{S}^L_{j,t}}{\partial Q^L_{j,t}} L_t > 0,$$

which lowers current cash flow through the term

$$-Q^L_{j,t}\delta_L \frac{\partial \mathcal{S}^L_{j,t}}{\partial Q^L_{j,t}} L_t.$$

When the loan-to-deposit constraint binds, the same expansion in lending uses scarce balance-sheet capacity, as reflected by

$$-\zeta_{j,t} \frac{\partial}{\partial Q^L_{j,t}} \left(l_{j,t} - \bar{\lambda} d_{j,t} \right).$$

The loan-pricing FOC therefore equates the current marginal cost of offering more attractive credit terms to the discounted value of the additional borrower relationships, net of the shadow cost of the balance-sheet capacity used by those loans.

The deposit side features the mirror image of this trade-off. A higher deposit price $Q^D_{j,t}$ corresponds to a lower deposit rate. It raises current cash flow on the inframarginal deposit volume, captured by the term $d_{j,t}$, but it discourages depositors who are currently reconsidering their bank. The induced change in deposits is

$$\frac{\partial d_{j,t}}{\partial Q^D_{j,t}} = \delta_D \frac{\partial \mathcal{S}^D_{j,t}}{\partial Q^D_{j,t}} D_t < 0,$$

and this reduction in deposit funding affects current cash flow through the term

$$Q^D_{j,t}\delta_D \frac{\partial \mathcal{S}^D_{j,t}}{\partial Q^D_{j,t}} D_t.$$

A higher deposit price therefore improves current margins but weakens the bank's depositor base. The dynamic cost of losing these relationships is captured by

$$\mathcal{M}^D_{j,t}\delta_D \frac{\partial \mathcal{S}^D_{j,t}}{\partial Q^D_{j,t}} D_t.$$

Conversely, offering a more attractive deposit rate is costly today, but builds depositor relationships that are valuable because deposits relax the loan-to-deposit constraint. This balance-sheet benefit is captured in the first-order condition by

$$-\zeta_{j,t}\frac{\partial}{\partial Q^D_{j,t}}\left(l_{j,t} - \bar{\lambda} d_{j,t}\right),$$

or, equivalently, by the term $\bar{\lambda}\zeta_{j,t}$ in the markup equation.

The equity-issuance wedge $1+\Phi_{E,j,t}$ scales the current-cash-flow terms in both first-order conditions. When the bank is short of internal funds and external equity is costly, current cash flow becomes more valuable. The bank then places greater weight on current margins relative to the future value of expanding customer relationships. Thus, costly equity issuance tilts both loan and deposit pricing toward preserving internal funds, while the leverage constraint links the two sides of the balance sheet by making deposits valuable for lending capacity and loans costly in terms of scarce funding capacity.

Dynamic two-sided markup equations. Substituting the logit derivatives (23) into (29)–(30) and collecting terms gives

$$\underbrace{Q^D_{j,t}\left(\frac{d_{j,t}}{\alpha_D \delta_D \mathcal{S}^D_{j,t} D_t} - 1\right)}_{\text{deposit markdown } (\mu^D_{j,t})} = \frac{\mathcal{M}^D_{j,t} + \bar{\lambda}\zeta_{j,t}}{1+\Phi_{E,j,t}}, \tag{31}$$

$$\underbrace{Q^L_{j,t}\left(1 + \frac{l_{j,t}}{\alpha_L \delta_L \mathcal{S}^L_{j,t} L_t}\right)}_{\text{loan markup } (\mu^L_{j,t})} = \frac{\mathcal{M}^L_{j,t} - \zeta_{j,t}}{1+\Phi_{E,j,t}}. \tag{32}$$

Equations (31)–(32) are the dynamic two-sided markup equations. The left-hand sides are the static logit wedges. The right-hand sides are the dynamic and financial wedges.

The marginal franchise values $\mathcal{M}^D_{j,t}$ and $\mathcal{M}^L_{j,t}$ summarize the value of future relationships. When these values are high, the bank has an incentive to compress current margins in order to build or preserve relationships. The equity wedge $1+\Phi_{E,j,t}$ tilts pricing toward current cash flow when external finance is costly. The leverage multiplier has opposite effects across sides: deposits relax the leverage constraint and therefore become more valuable when the constraint binds, whereas loans tighten the constraint and therefore become more costly to expand.

It is useful to define installed-base-to-contestable-flow ratios

$$\kappa^D_{j,t} \equiv \frac{d_{j,t}}{\delta_D \mathcal{S}^D_{j,t} D_t}, \qquad \kappa^L_{j,t} \equiv \frac{l_{j,t}}{\delta_L \mathcal{S}^L_{j,t} L_t}. \tag{33}$$

Then the static wedges are

$$\mu^D_{j,t} = \frac{\kappa^D_{j,t}}{\alpha_D} - 1, \qquad \mu^L_{j,t} = 1 + \frac{\kappa^L_{j,t}}{\alpha_L}.$$

A larger installed base relative to the contestable flow raises market power. It increases the loan markup wedge and lowers deposit generosity. Higher α_L and α_D make demand more elastic and reduce these static wedges.

Equivalently, in gross-rate form,

$$R^D_{j,t} = \frac{1}{Q^D_{j,t}} = \mu^D_{j,t} \cdot \frac{1+\Phi_{E,j,t}}{\mathcal{M}^D_{j,t} + \bar{\lambda}\zeta_{j,t}}, \tag{34}$$

$$R^L_{j,t} = \frac{1}{Q^L_{j,t}} = \mu^L_{j,t} \cdot \frac{1+\Phi_{E,j,t}}{\mathcal{M}^L_{j,t} - \zeta_{j,t}}. \tag{35}$$

Equations (34)–(35) make clear that the relevant marginal costs in this economy are dynamic. Bank rates are markups (markdowns) over (under) dynamic marginal costs, which entail forward-looking franchise values and financial shadow costs. On the loan side, $\mathcal{M}^L_{j,t} - \zeta_{j,t}$ is the value of expanding borrower relationships net of the balance-sheet capacity used by additional lending. On the deposit side, $\mathcal{M}^D_{j,t} + \bar{\lambda}\zeta_{j,t}$ is the value of expanding depositor relationships including the benefit that deposits relax the loan-to-deposit constraint. The equity wedge $1+\Phi_{E,j,t}$ scales both terms by the marginal value of internal funds. Hence observed rates combine demand-side market power with a dynamic, endogenous marginal cost of intermediation. This mechanism parallels dynamic market-power problems in investment-goods and durable-goods markets, where current pricing affects future customer stocks and markups depend on the shadow value of future demand (Bertolotti, Lanteri, and Villa, 2025).

Equations (34)–(35) imply that the bank-level loan-deposit spread is

$$R^L_{j,t} - R^D_{j,t} = (1 + \Phi_{E,j,t}) \left[\frac{\mu^L_{j,t}}{\mathcal{M}^L_{j,t} - \zeta_{j,t}} - \frac{\mu^D_{j,t}}{\mathcal{M}^D_{j,t} + \bar{\lambda}\zeta_{j,t}} \right]. \tag{36}$$

This expression separates the intermediation spread into static demand-side wedges and dynamic shadow values. The terms $\mu^L_{j,t}$ and $\mu^D_{j,t}$ summarize the market-power components generated by logit demand and relationship lock-in: a bank with a larger installed base relative to contestable flow, $l_{j,t}/(\delta_L \mathcal{S}^L_{j,t} L_t)$ on the loan side and $d_{j,t}/(\delta_D \mathcal{S}^D_{j,t} D_t)$ on the deposit side, charges a higher loan rate and offers a less generous deposit rate. The term $\mathcal{M}^L_{j,t} - \zeta_{j,t}$ is the shadow value of expanding loan relationships, net of the balance-sheet cost of using scarce leverage capacity. The term $\mathcal{M}^D_{j,t} + \bar{\lambda}\zeta_{j,t}$ is the shadow value of expanding deposit relationships, including the benefit that deposits relax the leverage constraint. Thus the spread is not only a markdown (markup) below (over) funding costs: it also reflects forward-looking franchise values and the endogenous balance-sheet shadow cost of intermediation. Finally, the equity issuance wedge $1 + \Phi_{E,j,t}$ scales both sides of the spread by increasing the marginal value of current internal funds.

Proposition 1 (Relationship persistence and endogenous markups)**.** *Assume the leverage constraint is slack. Define the bank's wedge-adjusted intertemporal cost of funds as*

$$\mathcal{Q}^B_{j,t} \equiv \frac{\beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1 + \Phi_{E,j,t+1}) \right]}{1 + \Phi_{E,j,t}}. \tag{37}$$

If $\delta_L < 1$ or $\delta_D < 1$, current prices affect future relationship stocks. The optimal pricing rules then depend on the marginal franchise values $\mathcal{M}^L_{j,t}$ and $\mathcal{M}^D_{j,t}$, which vary with installed relationships and relationship shifters. Hence relationship persistence generates endogenous, state-dependent loan and deposit markups.

If instead $\delta_L = \delta_D = 1$, current prices affect only contemporaneous market shares.

The relationship-franchise channel disappears. The bank's prices reduce to static constant-elasticity rules around $\mathcal{Q}^B_{j,t}$:

$$Q^D_{j,t} = \frac{\alpha_D}{\alpha_D - 1}\,\mathcal{Q}^B_{j,t}, \tag{38}$$

$$Q^L_{j,t} = \frac{\alpha_L}{\alpha_L + 1}\,\mathcal{Q}^B_{j,t}. \tag{39}$$

Thus, absent relationship persistence, loan and deposit prices are constant logit markup/-markdowns over the same wedge-adjusted intertemporal cost of funds. If equity issuance costs are also absent, then $\mathcal{Q}^B_{j,t} = Q_t$, where

$$Q_t = \beta\mathbb{E}_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\right]$$

is the risk-free bond price.

Proof. When $\delta_L < 1$ or $\delta_D < 1$, the laws of motion

$$l_{j,t} = \left[(1-\delta_L)\frac{l_{j,t-1}}{L_{t-1}} + \delta_L\mathcal{S}^L_{j,t}\right]L_t, \qquad d_{j,t} = \left[(1-\delta_D)\frac{d_{j,t-1}}{D_{t-1}} + \delta_D\mathcal{S}^D_{j,t}\right]D_t$$

make current prices affect future relationship stocks. The pricing first-order conditions therefore contain the continuation values $\mathcal{M}^L_{j,t}$ and $\mathcal{M}^D_{j,t}$. Since these objects are derivatives of the value function with respect to installed relationship stocks, they vary with the bank's state and with the aggregate scaling of inherited relationships. Relationship persistence therefore generates endogenous, state-dependent markups.

When $\delta_L = \delta_D = 1$, relationship stocks satisfy

$$l_{j,t} = \mathcal{S}^L_{j,t}L_t, \qquad d_{j,t} = \mathcal{S}^D_{j,t}D_t.$$

Thus current prices affect only current allocations and do not affect future predetermined relationship stocks. The relationship-franchise continuation channel disappears. With a slack leverage constraint, the envelope conditions imply

$$V_l(x_{j,t+1}) = 1 + \Phi_{E,j,t+1}, \qquad V_d(x_{j,t+1}) = -(1 + \Phi_{E,j,t+1}).$$

Hence

$$\mathcal{M}^L_{j,t} = \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1 + \Phi_{E,j,t+1}) \right], \qquad \mathcal{M}^D_{j,t} = -\beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1 + \Phi_{E,j,t+1}) \right].$$

Substituting these expressions into the pricing first-order conditions and using

$$\frac{\partial \mathcal{S}^L_{j,t}}{\partial Q^L_{j,t}} = \frac{\alpha_L}{Q^L_{j,t}} \mathcal{S}^L_{j,t}, \qquad \frac{\partial \mathcal{S}^D_{j,t}}{\partial Q^D_{j,t}} = -\frac{\alpha_D}{Q^D_{j,t}} \mathcal{S}^D_{j,t},$$

gives (38) and (39). □

Proposition 1 clarifies the distinction between static and dynamic market power. Static logit demand gives banks market power even in a one-period pricing problem. Relationship persistence makes this market power dynamic: by changing current prices, a bank changes not only current quantities but also future relationship stocks. The shadow values of those stocks vary across banks and over time, generating endogenous state-dependent markups. In contrast, when all customers reoptimize every period, markups are constant functions of the logit elasticities. Equity issuance costs affect the bank's wedge-adjusted intertemporal cost of funds, $\mathcal{Q}^B_{j,t}$, around which banks set prices, but do not by themselves create relationship-based markup variation.

3.7 Market Power Separability without Financial Frictions

The model features two distinct sources of interaction between loan and deposit pricing. The first is technological: the bank chooses prices on both sides of its balance sheet. The second is financial: equity issuance costs and the leverage constraint make the marginal value of cash and balance-sheet capacity depend jointly on loan and deposit choices. The following proposition shows that the second source is essential. Without financial frictions linking the two sides, the bank's loan and deposit problems are separable.

Proposition 2 (Separability of loan and deposit pricing)**.** *Suppose that banks face no equity issuance costs, $\phi^B_E = 0$, and no leverage constraint, $\bar{\lambda} \to \infty$. Then the bank's dynamic problem separates into an independent loan-side problem and an independent deposit-side* *problem. In particular, the optimal loan price $Q^L_{j,t}$ depends only on loan-side states and primitives,*

$$(l_{j,t-1}, s^L_{j,t}; L_t, Z^L_t),$$

while the optimal deposit price $Q^D_{j,t}$ depends only on deposit-side states and primitives,

$$(d_{j,t-1}, s^D_{j,t}; D_t, Z^D_t),$$

where Z^L_t and Z^D_t denote the corresponding logit denominators. Consequently, loan and deposit market power operate independently.

Proof. With $\phi^B_E = 0$, net dividends equal pre-issuance dividends:

$$\text{Div}^B_{j,t} = l_{j,t-1} - Q^L_{j,t} l_{j,t} - d_{j,t-1} + Q^D_{j,t} d_{j,t}.$$

This payoff is additively separable in loan-side objects and deposit-side objects. The laws of motion are also separable:

$$l_{j,t} = \left[(1-\delta_L)\frac{l_{j,t-1}}{L_{t-1}} + \delta_L \mathcal{S}^L_{j,t}\right] L_t, \qquad d_{j,t} = \left[(1-\delta_D)\frac{d_{j,t-1}}{D_{t-1}} + \delta_D \mathcal{S}^D_{j,t}\right] D_t.$$

Absent the leverage constraint, there is no restriction linking $l_{j,t}$ and $d_{j,t}$. Therefore the Bellman equation is the sum of a loan-side Bellman equation and a deposit-side Bellman equation. The first-order condition for $Q^L_{j,t}$ contains only loan-side payoffs, loan-side aggregate scaling, loan-side transition terms, and the loan-side continuation value. The first-order condition for $Q^D_{j,t}$ contains only deposit-side payoffs, deposit-side aggregate scaling, deposit-side transition terms, and the deposit-side continuation value. Hence the two pricing problems are independent. □

The proposition clarifies the role of bank financial frictions. Relationship frictions generate market power on each side of the balance sheet. Equity issuance costs and leverage constraints determine whether that market power is transmitted across sides. When these financial frictions are absent, the bank behaves as two independent relationship intermediaries: one in the loan market and one in the deposit market. When they are present, the two sides interact because deposits relax the balance-sheet constraint and loans tighten it, while equity issuance costs make current internal funds valuable.

3.8 Stationary Equilibrium

3.8.1 Baseline Economy

A stationary equilibrium consists of constant aggregate allocations and prices

$$(C, N, K, I, Y, B^g, L, D, Q, Q^L, Q^D, W, T),$$

bank policy functions

$$Q_j^L = \mathcal{Q}^L(l_j, d_j, s_j^L, s_j^D), \qquad Q_j^D = \mathcal{Q}^D(l_j, d_j, s_j^L, s_j^D),$$

and an invariant cross-sectional distribution μ over bank states

$$x_j = (l_j, d_j, s_j^L, s_j^D),$$

such that the following conditions hold.

First, households optimize:

$$Q = \beta, \qquad W = \frac{v_N(N)}{u_C(C)}, \qquad D = \left(\frac{\chi}{u_C(C)(Q^D - \beta)}\right)^{1/\eta}.$$

Second, firms optimize:

$$Q^L - \phi_L L = \beta^F \left[1 - \tau_c(1 - Q^L)\right],$$

and

$$1 = \beta^F \left[(1 - \tau_c)MPK + \tau_c\delta + (1 - \delta)\right].$$

Third, each bank solves the stationary version of (22), taking aggregate quantities and logit denominators as given, subject to the relationship laws of motion and the leverage constraint.

Fourth, logit shares satisfy

$$\mathcal{S}_j^L = \frac{\exp(V_j^L)}{\int_0^1 \exp(V_k^L)\, d\mu(k)}, \qquad \mathcal{S}_j^D = \frac{\exp(V_j^D)}{\int_0^1 \exp(V_k^D)\, d\mu(k)}.$$

Fifth, since aggregate loan and deposit quantities are constant in a stationary equilibrium, bank-level relationship stocks evolve according to

$$l_j' = (1-\delta_L)l_j + \delta_L \mathcal{S}_j^L L, \qquad d_j' = (1-\delta_D)d_j + \delta_D \mathcal{S}_j^D D. \tag{40}$$

with

$$l_j' \leq \bar{\lambda} d_j', \qquad \zeta_j \geq 0, \qquad \zeta_j(l_j' - \bar{\lambda} d_j') = 0.$$

Sixth, aggregate loan and deposit stocks clear:

$$\int_0^1 l_j\, d\mu(j) = L, \qquad \int_0^1 d_j\, d\mu(j) = D.$$

Aggregate loan and deposit prices are quantity-weighted averages of posted prices:

$$Q^L L = \int_0^1 Q_j^L l_j'\, d\mu(j), \qquad Q^D D = \int_0^1 Q_j^D d_j'\, d\mu(j).$$

Seventh, government bonds clear, the government budget constraint holds with $B^g = \bar{B}^g$, and transfers adjust residually:

$$B = B^g, \qquad QB^g = B^g + T - \mathcal{T}.$$

Eighth, goods market clearing holds. Dividends are rebated to households and therefore do not directly enter the aggregate resource constraint. The resource costs in the economy are the firm debt cost and bank equity issuance costs, so

$$Y = C + I + \Psi(L) + \int_0^1 \Phi(\widetilde{\mathrm{Div}}_j^B)\, d\mu(j).$$

Finally, the distribution μ is invariant under the transition induced by the bank policy functions, the relationship laws of motion, and the exogenous shifter process.

3.8.2 First-best: Competitive Benchmark

The constrained-competitive benchmark is the stationary equilibrium of the economy in which bank market-power wedges are removed. Formally, the benchmark consists of constant aggregate allocations and prices

$$(C, N, K, I, Y, L, D, Q, Q^L, Q^D, W),$$

and a shadow value of balance-sheet capacity $\zeta \geq 0$, such that the following conditions hold.

First, households optimize:

$$Q = \beta, \qquad W = \frac{v_N(N)}{u_C(C)},$$

and the deposit Euler equation is

$$D = \left(\frac{\chi}{u_C(C)(Q^D - \beta)} \right)^{1/\eta}.$$

Second, firms optimize:

$$Q^L - \phi_L L = \beta^F \left[1 - \tau_c(1 - Q^L) \right],$$

and

$$1 = \beta^F \left[(1 - \tau_c) MPK + \tau_c \delta + (1 - \delta) \right].$$

Third, banks are competitive price takers on both sides of the balance sheet. Relationship turnover is one,

$$\delta_L = \delta_D = 1,$$

so current prices affect only current loan and deposit quantities. Bank market-power wedges vanish. This limit corresponds to taking the demand elasticities α_D and α_L to infinity. From

(33), this implies

$$\mu_j^D \to -1, \qquad \mu_j^L \to 1.$$

Equivalently, the deposit markdown component $1 + \mu_j^D$ and the loan markup component $\mu_j^L - 1$ both converge to zero. In addition, because $\delta_L = \delta_D = 1$, current bank prices affect only current quantities and not future installed relationship stocks. The relationship-franchise component of the envelope conditions in (26)–(25) therefore drops out. Using (31) and (32), the competitive bank pricing conditions become

$$Q^D = \beta + \bar{\lambda}\zeta, \qquad Q^L = \beta - \zeta.$$

The loan-to-deposit constraint is active:

$$L = \bar{\lambda}D, \qquad \zeta \geq 0.$$

Equivalently, combining the household deposit Euler equation with the binding constraint gives

$$Q^D = \beta + \frac{\chi}{u_C(C)D^\eta}, \qquad \zeta = \frac{Q^D - \beta}{\bar{\lambda}}.$$

Fourth, aggregate loan and deposit prices are common across banks:

$$Q_j^L = Q^L, \qquad Q_j^D = Q^D \quad \text{for all } j.$$

Since the competitive benchmark removes bank market-power wedges, the cross-sectional distribution of bank relationships is irrelevant for aggregate pricing. The aggregate balance sheet is summarized by L and D. Equivalently, the banking sector can be represented by a representative competitive bank. Its gross dividend is

$$\text{Div}^B = L - Q^L L - D + Q^D D.$$

In the stationary benchmark computed below this dividend is positive, so the representative bank does not issue equity and the equity issuance wedge is zero.

Fifth, goods market clearing holds:

$$Y = C + I + \Psi(L), \qquad \Psi(L) = \frac{\phi_L}{2} L^2,$$

with

$$Y = AK^{\alpha} N^{1-\alpha}, \qquad I = \delta K.$$

Note that when the loan-to-deposit constraint binds, the shadow value ζ creates a wedge between the competitive loan and deposit prices:

$$Q^D - Q^L = (1 + \bar{\lambda})\zeta.$$

This spread is therefore not a bank market-power spread. It reflects the shadow cost of scarce balance-sheet capacity in an economy where deposits provide liquidity services.

4 Quantitative Analysis

In this section, we solve and calibrate the stationary model. The calibration uses aggregate quantities, interest-rate spreads, and cross-sectional dispersion in bank rates to discipline the strength of relationship frictions, bank market power, and financial frictions. The calibrated economy is the benchmark for the quantitative analysis that follows.

4.1 Solution Method

We compute the stationary equilibrium with heterogeneous relationship banks. A bank's individual state is

$$x_j = (l_j, d_j, s_j^L, s_j^D),$$

where l_j and d_j are predetermined relationship stocks and s_j^L and s_j^D are bank-specific relationship shifters. We discretize the state space on a tensor-product grid. For a candidate aggregate environment, we solve the bank's dynamic pricing problem by value-function iteration, compute the invariant distribution induced by the resulting policy functions, and update the aggregate objects until the cross-sectional distribution is consistent with the aggregate loan and deposit markets. Appendix A provides the computational details.

In the stationary equilibrium, the bank takes as given the aggregate loan and deposit markets, L and D, and the logit denominators, Z^L and Z^D. These four objects enter the relationship laws of motion in (40) only through the contestable flows $\mathcal{S}_j^L L$ and $\mathcal{S}_j^D D$. Using

$$\mathcal{S}_j^L = \frac{\exp(V_j^L)}{Z^L}, \qquad \mathcal{S}_j^D = \frac{\exp(V_j^D)}{Z^D},$$

these flows can be written as

$$\mathcal{S}_j^L L = \exp(V_j^L)\frac{L}{Z^L}, \qquad \mathcal{S}_j^D D = \exp(V_j^D)\frac{D}{Z^D}.$$

Thus the bank problem depends on the aggregate environment only through two scaled market sizes,

$$\frac{L}{Z^L} \qquad \text{and} \qquad \frac{D}{Z^D}.$$

These objects have a simple interpretation: they are the sizes of the contestable loan and deposit markets per unit of aggregate competitor attractiveness. An increase in L or D expands the flow of customers available to the bank; an increase in the corresponding logit denominator dilutes the bank's share of that flow.

Given a candidate pair $(L/Z^L, D/Z^D)$, we solve the bank problem over posted prices (Q_j^L, Q_j^D), construct the transition matrix induced by the policy functions and the exogenous shifter process, and compute the invariant distribution μ. The stationary distribution implies aggregate relationship stocks and logit denominators. The fixed point requires

$$\frac{\int l_j'\, d\mu(j)}{\int \exp(V_j^L)\, d\mu(j)} = \frac{L}{Z^L}, \qquad \frac{\int d_j'\, d\mu(j)}{\int \exp(V_j^D)\, d\mu(j)} = \frac{D}{Z^D}.$$

The solution delivers bank pricing policies, the invariant distribution of banks, and all aggregate quantities and prices in the stationary equilibrium.

4.2 Calibration

We calibrate the stationary model at an annual frequency. Household preferences are

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}, \qquad v(N) = \psi_N \frac{N^{1+\nu_N}}{1+\nu_N}.$$

We set the coefficient of relative risk aversion to $\sigma = 2$, a standard curvature normalization in quantitative macroeconomic models. We set the inverse Frisch parameter to $\nu_N = 1$, corresponding to a Frisch elasticity of one, and choose ψ_N to normalize steady-state labor to one. Deposit liquidity services have curvature $\eta = 2$, which we set equal to the curvature of consumption utility. The level of deposit liquidity services is disciplined by the parameter χ, which is calibrated to match the deposit-to-output ratio.

The discount factor is $\beta = 0.9747$, which matches an annual risk-free rate of 2.6 percent. On the production side, we normalize aggregate productivity to $A = 1$. The capital share α and depreciation rate δ are chosen to discipline the capital-output and investment-output ratios, respectively. The corporate tax rate is set to $\tau_c = 0.21$, corresponding to the U.S. statutory federal corporate tax rate. We set the corporate-finance wedge to $\omega_F = 0.0247$, implying an effective firm discount factor $\beta^F = \beta - \omega_F = 0.95$. This wedge is set externally and captures, in reduced form, managerial impatience or agency distortions in firms' intertemporal borrowing and investment decisions. Quantitatively, the choice corresponds to a moderate annual wedge: firms discount future payouts at an effective annual rate of about 5.3 percent, compared with the household risk-free rate of 2.6 percent. Thus, managers are modestly more impatient than households, consistent with models in which managerial myopia or agency conflicts distort the horizon of corporate policies (Stein, 1989; Gryglewicz, Mayer, and Morellec, 2020).

The banking block is disciplined by quantities, prices, and dispersion. The debt-cost curvature ϕ_L controls firms' demand for bank debt and is primarily disciplined by the loan-to-output ratio. The loan- and deposit-side logit elasticities, α_L and α_D, discipline average pricing power on the two sides of the balance sheet. In particular, α_L is primarily disciplined by the average loan spread, $R^L - R^M$, while α_D is primarily disciplined by the average deposit markdown, $R^M - R^D$.

Cross-sectional dispersion in bank rates is disciplined by heterogeneity in bank-specific relationship quality. The shifters s_j^L and s_j^D enter the borrower and depositor logit choice problems and capture persistent differences in bank attractiveness among customers who are currently reconsidering their bank. In the quantitative implementation, the unconditional standard deviations of log relationship quality, σ_s^L and σ_s^D, are chosen to match the cross-sectional standard deviations of loan and deposit rates in the FDIC bank-level data. The persistence parameters are set externally to $\rho_L = \rho_D = 0.90$, so that bank-specific attractiveness is persistent but mean reverting.

We set the relationship turnover rates to

$$\delta_L = \delta_D = 0.25,$$

so that the expected duration of both loan and deposit relationships is four years. The loan-side value is disciplined by the evidence in Gopalan, Udell, and Yerramilli (2011): applying a simple duration calculation to their estimates implies an average lending relationship of about 3.7 years, which we round to four years. We set deposit turnover equal to loan turnover in the baseline. This choice is consistent with evidence that deposit markets feature meaningful switching costs and persistent bank–depositor relationships (Iyer and Puri, 2012). We study the sensitivity of the results to this assumption in the comparative statics of Subsection 4.3.

Finally, the equity issuance cost ϕ_E^B and the loan-to-deposit constraint $\bar{\lambda}$ govern the bank financial-friction block. The equity issuance cost makes the marginal value of internal funds state dependent when banks face negative pre-issuance dividends. The loan-to-deposit constraint is interpreted as a reduced-form funding or balance-sheet limit: deposits relax the constraint and therefore link loan and deposit pricing. These two parameters are fixed in the baseline calibration and varied in the quantitative exercises below. Table I reports the full set of parameter values.

Table I. Calibrated Parameters

Parameter	Symbol	Value	Calibration / discipline
Calibrated to targeted moments			
Discount factor	β	0.9747	Internally targeted; see Table II
Deposit liquidity taste	χ	0.003	Internally targeted; see Table II
Debt cost curvature	ϕ_L	0.008	Internally targeted; see Table II
Deposit-side logit elasticity	α_D	55.00	Internally targeted; see Table II
Loan-side logit elasticity	α_L	40.00	Internally targeted; see Table II
Std. dev. loan shifter	σ_s^L	0.10	Internally targeted; see Table II
Std. dev. deposit shifter	σ_s^D	0.10	Internally targeted; see Table II
Capital share	α	0.40	Internally targeted; see Table II
Depreciation rate	δ	0.07	Internally targeted; see Table II
Set externally or normalized			
Risk aversion	σ	2.00	Standard curvature normalization
Deposit liquidity curvature	η	2.00	Set equal to σ
Inverse Frisch parameter	ν_N	1.00	Frisch elasticity of one
Labor disutility scale	ψ_N	0.57	Normalizes steady-state labor to one
Productivity	A	1.00	Aggregate productivity normalization
Corporate tax rate	τ_c	0.21	U.S. statutory federal corporate tax rate
Corporate-finance wedge	ω_F	0.0247	Effective firm discount factor $\beta^F = 0.95$
Equity issuance cost	ϕ_E^B	0.40	Baseline financial friction; varied below
Loan-to-deposit constraint	$\bar{\lambda}$	1.10	Baseline funding limit; varied below
Loan relationship turnover	δ_L	0.25	Implies average duration of four years
Deposit relationship turnover	δ_D	0.25	Implies average duration of four years
Loan-side shifter persistence	ρ_L	0.90	Externally set persistence
Deposit-side shifter persistence	ρ_D	0.90	Externally set persistence

Notes: The table reports the baseline calibration. The first panel reports parameters calibrated to the targeted moments reported in Table II. The second panel reports parameters set externally or by normalization. The equity issuance cost and loan-to-deposit constraint are fixed in the baseline calibration and varied in the quantitative exercises. The persistence parameters are set externally because stationary rate-dispersion moments discipline the dispersion of relationship quality but not its persistence.

Table II compares the model-implied targeted moments with their empirical counterparts. The calibration matches the level of interest rates and the two main intermediation wedges closely. The annual risk-free rate is 2.60 percent in both the model and the data. The model loan spread is 2.85 percentage points, compared with 2.96 percentage points in the data, while the model deposit markdown is 0.67 percentage points, compared with 0.63 percentage points in the data. These moments discipline the average amount of market power on the loan and deposit sides.

The model also generates empirically plausible balance-sheet quantities, although it overpredicts the loan-to-output ratio. The deposit-to-output ratio is 54 percent in the model and 60 percent in the data. The loan-to-output ratio is 56 percent in the model and 46 percent in the data. The remaining targeted moments discipline the dispersion of bank rates and the real side of the economy. The model closely matches the dispersion of deposit rates and generates substantial, though somewhat lower than empirical, dispersion in loan rates. On the real side, the model reproduces the capital-output ratio closely and generates an investment-output ratio somewhat above its empirical counterpart.

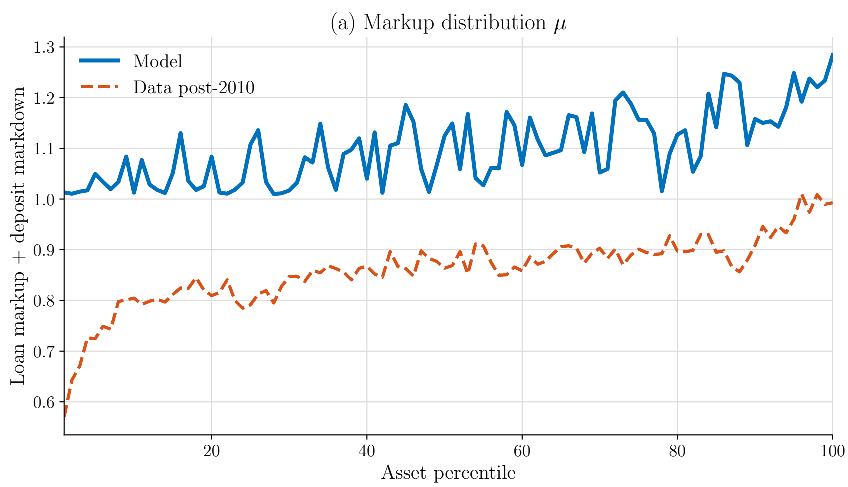
Table II. Calibration Moments: Model vs. Data

Target moment	Model	Data	Primary parameter
Risk-free rate, $R^M = \beta^{-1}$	2.596	2.601	β
Deposits to output, D/Y	53.369	59.843	χ
Loans to output, L/Y	55.979	46.446	ϕ_L
Deposit markdown, $R^M - R^D$	0.691	0.630	α_D
Loan spread, $R^L - R^M$	2.861	2.957	α_L
Std. dev. loan rates, $\sigma(R^L)$	1.334	1.741	σ_s^L
Std. dev. deposit rates, $\sigma(R^D)$	0.643	0.610	σ_s^D
Capital to output, K/Y	2.928	2.940	α
Investment to output, I/Y	20.494	17.456	δ

Notes: Balance-sheet ratios and investment ratios are reported in percent, except for K/Y. Interest rates, spreads, and rate standard deviations are annualized percentage points. The targeted moments are used jointly in the calibration. The column "Primary parameter" reports the parameter most directly disciplined by each moment, although all calibrated parameters are chosen jointly. The loan and deposit rates are reported as implied moments because, conditional on R^M, the targeted loan spread and deposit markdown determine their levels.

As an additional validation exercise, Figure 2 compares the cross-sectional profile of model-implied banking markups with the empirical markup profile documented in Panel (c) of Figure 1.

Figure 2. Cross-Sectional Distribution of Banking Markups



Notes: The solid line reports the model-implied total banking markup by bank asset percentile, computed as $\mu_j^B = \mu_j^L + \mu_j^D$, where μ_j^L is the loan-side markup factor and μ_j^D is the deposit-side markdown term defined in equations (32) and (31). Equivalently, $\mu_j^B = (\mu_j^L - 1) + (1 + \mu_j^D)$, where $\mu_j^L - 1$ is the loan-side markup component and $1 + \mu_j^D$ is the deposit-side markdown component. The dashed line reports the empirical markup $\mu_{b,t}$ by bank asset percentile for the post-2010 period, constructed from bank balance-sheet data as in equation (1) and reported in Panel (c) of Figure 1.

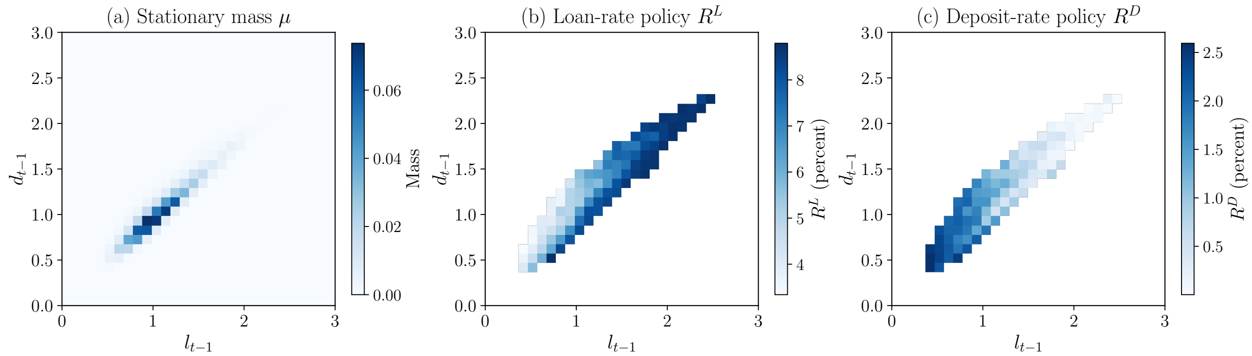
To compare the model with the empirical markup profile, we construct a model-implied total banking markup that aggregates the two static market-power components in the bank pricing equations. From equations (31)–(32), the loan-side markup factor is μ_j^L, so the loan markup net of its unit component is $\mu_j^L - 1$. On the deposit side, the corresponding static markdown component is $1 + \mu_j^D$. We therefore define

$$\mu_j \equiv (\mu_j^L - 1) + (1 + \mu_j^D) = \mu_j^L + \mu_j^D = \frac{\kappa_j^L}{\alpha_L} + \frac{\kappa_j^D}{\alpha_D}.$$

The terms κ_j^L and κ_j^D are the installed-base-to-contestable-flow ratios defined in equation (33). They measure the size of a bank's inherited loan and deposit relationships relative to the flow of customers that is currently contestable. The elasticities α_L and α_D govern how responsive borrowers and depositors are to posted prices. Thus, total banking markups are high when banks have large installed relationship bases relative to contestable flows, or when customers are less price elastic. Figure 2 compares μ_j, averaged by bank asset percentile in the stationary model, with the empirical markup profile $\mu_{b,t}$ constructed from balance-sheet data in equation (1). The comparison is not used as a calibration target; it is intended to assess whether the model reproduces the cross-sectional slope of banking markups across bank size. The model reproduces the qualitative cross-sectional pattern in the data: banking markups increase with bank size.

Distribution, Markups, and Policy Functions. Figure 3 summarizes the stationary distribution and the associated bank pricing policies. Panel (a) reports the invariant distribution of banks over inherited loan and deposit relationships. The distribution is concentrated along an upward-sloping diagonal: banks with larger loan relationships also tend to have larger deposit relationships, reflecting the balance-sheet link imposed by the loan-to-deposit constraint. Panels (b) and (c) report the stationary loan-rate and deposit-rate policies over the same state space. Banks with larger inherited loan positions set higher loan rates, while banks with larger deposit positions tend to offer lower deposit rates. These patterns reflect the endogenous pricing effects of installed relationships and the role of deposits in relaxing the bank balance-sheet constraint.

Figure 3. Stationary Distribution and Bank Pricing Policies



Notes: Panel (a) reports the invariant distribution of banks over inherited loan relationships, l_{t-1}, and inherited deposit relationships, d_{t-1}. Panels (b) and (c) report the stationary loan-rate and deposit-rate policy functions, R^L and R^D, over the same state space. The color scale in Panel (a) denotes stationary mass. The color scales in Panels (b) and (c) denote annualized loan and deposit rates.

Results. Table III compares the baseline economy with the constrained-competitive benchmark. Removing bank market-power wedges substantially compresses intermediation spreads. The average lending rate falls from 6.4 percent to 2.7 percent, while the average deposit rate rises from 1.3 percent to 2.5 percent. The resulting decline in the loan–deposit spread is accompanied by a large expansion in bank balance sheets: deposits increase by a factor of 2.3 and loans by a factor of 2.4 relative to the baseline.

These changes in intermediation have significant real effects. Output, labor, and capital are all about 2.1 percent higher in the competitive benchmark. The increase in capital implies a proportional increase in steady-state investment, since $I = \delta K$. Thus, bank market power depresses aggregate activity by raising the cost of external finance and limiting the scale of intermediation. At the same time, consumption is 1.0 percent lower in the competitive benchmark. This decline does not reflect lower productive capacity. Instead, the higher-output economy allocates more resources to investment and supports a substantially larger stock of private debt. Because borrowing entails the convex resource cost $\Psi(L)$, the expansion in leverage absorbs part of the additional output. This term captures, in reduced form, the real costs of a more leveraged private sector, including monitoring, enforcement, agency costs, and the higher expected distress or default-related costs associated with a larger debt burden relative to firm capital. Thus, the larger loan stock supports higher capital accumulation and output, but also uses resources that would otherwise be available for consumption.

Table III. Stationary Equilibrium

Variable	Baseline	Competitive benchmark	Difference
Output Y	1.915	1.956	+2.1%
Consumption C	1.517	1.501	-1.0%
Labor N	0.936	0.956	+2.1%
Capital K	5.607	5.726	+2.1%
Avg. bank lending rate R^L	6.389	2.654	-3.7 pp
Avg. bank deposit rate R^D	1.278	2.532	+1.3 pp
Deposits D	1.022	3.330	+2.3x
Loans L	1.072	3.663	+2.4x

Notes: Entries are stationary-equilibrium values. Quantities are reported in model levels. Interest rates are annualized net rates in percent, computed as quantity-weighted averages of bank-level lending and deposit rates. The competitive benchmark is defined in Section 3.8.2. The Difference column reports the percentage change from Baseline to the competitive benchmark for real quantities, the percentage-point difference for lending and deposit rates, and the change relative to Baseline in multiples for deposits and loans.

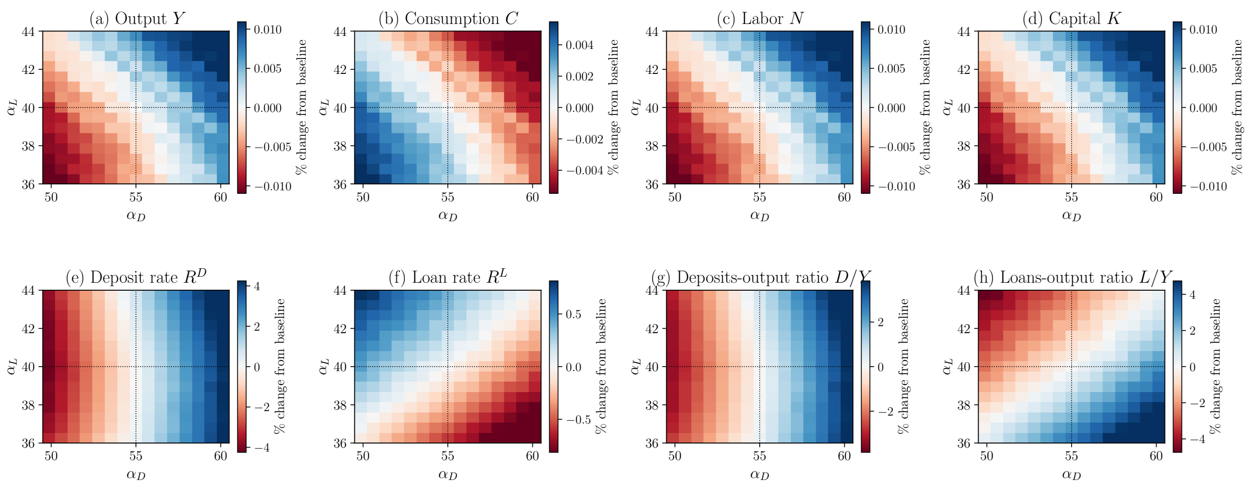
4.3 Comparative Statics

This section studies how the stationary equilibrium changes with the main banking frictions. We vary each set of parameters around the baseline calibration and recompute the equilibrium. The first exercises focus on market power margins: demand elasticities, which govern static pricing power, and relationship turnover, which governs dynamic relationship rents. The second exercises focus on balance-sheet frictions: costly equity issuance and the loan-to-deposit constraint. All outcomes are reported as percent deviations from the baseline equilibrium.

4.3.1 Market Power Margins

Figure 4 varies the loan- and deposit-side logit elasticities. The horizontal axis varies α_D, while the vertical axis varies α_L. Higher values of these parameters make contestable customers more price elastic and therefore weaken static bank market power. Increasing α_D weakens deposit-side market power: banks offer more attractive deposit terms, the deposit rate rises, and deposits expand. Because deposits relax the bank balance-sheet constraint, the expansion in deposit funding supports lending and raises aggregate output, labor, and capital. Increasing α_L weakens loan-side market power: borrowers become more responsive to loan prices, lending conditions improve, and loans and capital expand. The real responses therefore reflect the joint determination of bank prices and balance-sheet quantities on the two sides of the banking platform. Consumption need not move with output. In regions where intermediation and capital accumulation expand, a larger share of output is absorbed by steady-state investment and by the resource cost of sustaining a larger loan stock.

Figure 4. Comparative Statics: Bank Demand Elasticities



Notes: The figure varies the loan-side and deposit-side logit elasticities, α_L and α_D, around their baseline values and recomputes the stationary equilibrium. The horizontal axis reports α_D, and the vertical axis reports α_L. Each panel reports the percent change in the corresponding outcome relative to the baseline equilibrium. The dashed vertical and horizontal lines mark the baseline calibration, where all outcomes are normalized to zero.

Figure 5 reports the corresponding responses of spreads and markup components. The figure shows that changes in demand elasticities affect bank pricing margins, but also that spreads and markup components are not identical objects in a dynamic relationship model. The static components satisfy

$$1 + \mu_j^D = \frac{\kappa_j^D}{\alpha_D}, \qquad \mu_j^L - 1 = \frac{\kappa_j^L}{\alpha_L},$$

where

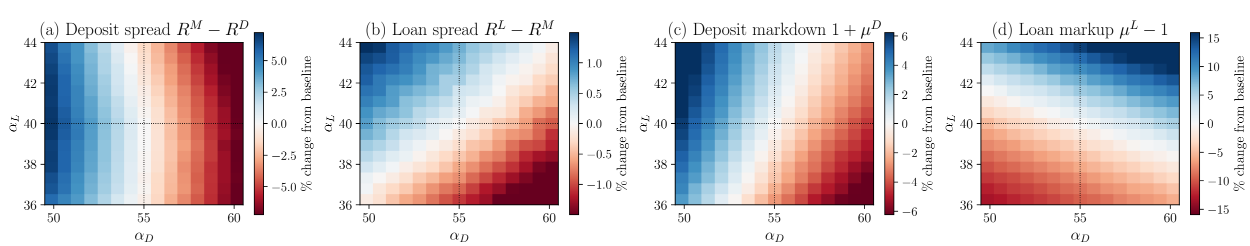
$$\kappa_j^D = \frac{d_j}{\delta_D \mathcal{S}_j^D D}, \qquad \kappa_j^L = \frac{l_j}{\delta_L \mathcal{S}_j^L L}.$$

Thus, holding the installed-base-to-contestable-flow ratios κ_j^D and κ_j^L fixed, higher elasticities reduce the corresponding markdown and markup components. In equilibrium, however, the κ's also respond because prices change market shares, relationship stocks, and the shadow value of balance-sheet capacity.

Along the deposit-side elasticity dimension, a higher α_D reduces the deposit spread, $R^M - R^D$, and lowers the deposit markdown component, $1 + \mu^D$. This is the direct pricing implication of weaker deposit-side market power: when depositors are more price elastic, banks must offer more attractive deposit rates. The same change also affects the loan side because deposits are a source of balance-sheet capacity. By expanding deposit funding and relaxing the loan-to-deposit constraint, stronger deposit competition changes the marginal value of lending and therefore affects loan spreads and loan markup components.

Along the loan-side elasticity dimension, a higher α_L generally lowers the loan spread, $R^L - R^M$, so firms face cheaper credit. The loan markup component, $\mu^L - 1$, however, increases in the figure. This reflects the endogenous response of κ^L. Higher loan-side elasticity weakens market power holding relationship stocks fixed, but it also changes lending quantities, logit shares, and installed borrower relationships. In the calibrated equilibrium, the induced increase in the installed-base-to-contestable-flow ratio is large enough to offset the direct $1/\alpha_L$ effect. Hence the loan spread and the loan markup component can move in opposite directions. This distinction is central to the model: in a dynamic banking platform, observed spreads reflect both static pricing power and shadow values, while markup components also depend on endogenous relationship stocks and contestable customer flows.

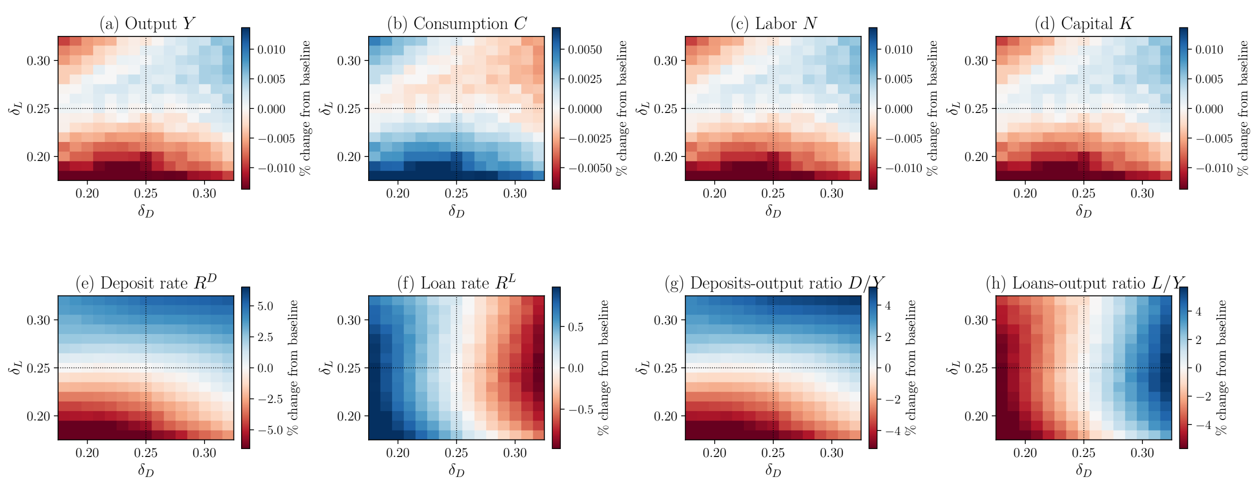
Figure 5. Markup Responses: Bank Demand Elasticities



Notes: *Notes*: The figure reports the response of model-implied spreads and markup components to changes in the loan-side and deposit-side logit elasticities, α_L and α_D. The horizontal axis reports α_D, and the vertical axis reports α_L. Panels (a) and (b) report the deposit spread, $R^M - R^D$, and the loan spread, $R^L - R^M$. Panels (c) and (d) report the deposit markdown component, $1 + \mu^D$, and the loan markup component, $\mu^L - 1$, where μ^D and μ^L are defined in equations (31) and (32). Each panel reports the percent change in the corresponding object relative to the baseline equilibrium. The dashed vertical and horizontal lines mark the baseline calibration.

Figure 6 varies relationship turnover on the loan and deposit sides. The horizontal axis varies δ_D, while the vertical axis varies δ_L. Higher turnover weakens relationship lock-in because a larger fraction of customers reoptimizes each period. Increasing δ_D reduces the value of installed depositor relationships and changes banks' incentives to attract deposit funding. Increasing δ_L reduces the value of installed borrower relationships and changes loan pricing and credit quantities. The heatmaps show that loan-side turnover is especially important for lending and loan rates because it directly affects firms' financing conditions. Deposit-side turnover primarily affects deposit rates and deposits, but it also feeds back into lending through the balance-sheet role of deposits. As in the elasticity exercise, the input ratios N/Y and K/Y remain essentially unchanged, so the main real effects operate through bank pricing, credit quantities, and the allocation of output across uses rather than through changes in factor intensities.

Figure 6. Comparative Statics: Relationship Turnover



Notes: The figure varies the loan-side and deposit-side relationship turnover rates, δ_L and δ_D, around their baseline values and recomputes the stationary equilibrium. A higher value of δ_L or δ_D means that a larger fraction of borrowers or depositors reoptimizes its bank relationship each period. The horizontal axis reports δ_D, and the vertical axis reports δ_L. Each panel reports the percent change in the corresponding outcome relative to the baseline equilibrium. The dashed vertical and horizontal lines mark the baseline calibration, where all outcomes are normalized to zero.

Figure 7 shows the associated responses of spreads and markup components. Relationship turnover affects pricing power by changing the importance of installed relationships relative to contestable customer flows. Using the definitions in equations (33),

$$1 + \mu_j^D = \frac{\kappa_j^D}{\alpha_D}, \qquad \mu_j^L - 1 = \frac{\kappa_j^L}{\alpha_L},$$

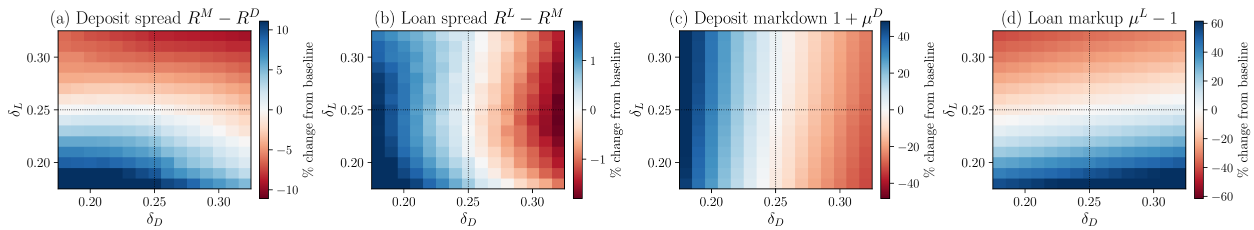
where

$$\kappa_j^D = \frac{d_j}{\delta_D \mathcal{S}_j^D D}, \qquad \kappa_j^L = \frac{l_j}{\delta_L \mathcal{S}_j^L L}.$$

Thus, holding equilibrium stocks and logit shares fixed, higher turnover reduces the corresponding markup component by increasing the contestable flow in the denominator. The figure shows that this force is quantitatively important. A higher δ_D lowers the deposit markdown component, $1+\mu^D$, while a higher δ_L lowers the loan markup component, $\mu^L - 1$. Conversely, more persistent relationships strengthen relationship-based market power because installed customer bases become large relative to the flow of customers currently up for reoptimization.

The spread panels show how these pricing wedges are transmitted through the two-sided bank balance sheet. A higher δ_D lowers the deposit spread, $R^M - R^D$, because depositors become easier to contest and banks offer more attractive deposit rates. The same change also lowers the loan spread, reflecting the balance-sheet role of deposits: stronger deposit competition expands funding capacity and changes the shadow value of lending. On the loan side, a higher δ_L lowers the loan markup component and affects the loan spread directly by making borrower relationships less durable. It also affects the deposit spread because changes in lending incentives alter the value of deposit funding. These cross-side responses are the central implication of the banking-platform structure: relationship turnover on one side changes pricing on the other side through endogenous relationship stocks and the loan-to-deposit constraint.

Figure 7. Markup Responses: Relationship Turnover



Notes: The figure reports the response of model-implied spreads and markup components to changes in the loan-side and deposit-side relationship turnover rates, δ_L and δ_D. The horizontal axis reports δ_D, and the vertical axis reports δ_L. Panels (a) and (b) report the deposit spread, $R^M - R^D$, and the loan spread, $R^L - R^M$. Panels (c) and (d) report the deposit markdown component, $1 + \mu^D$, and the loan markup component, $\mu^L - 1$, where μ^D and μ^L are defined in equations (31) and (32). Each panel reports the percent change in the corresponding object relative to the baseline equilibrium. The dashed vertical and horizontal lines mark the baseline calibration.

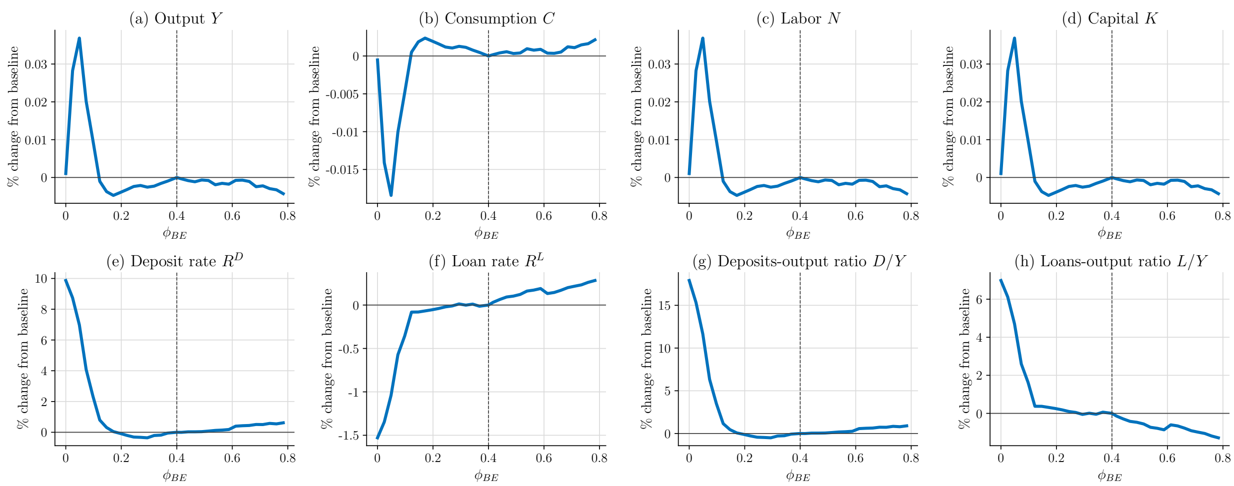
4.3.2 Bank Balance-Sheet Frictions

This subsection studies the role of the two bank financial frictions: the equity issuance cost, ϕ_E^B, and the loan-to-deposit constraint, $\bar{\lambda}$. The exercises vary one parameter at a time around its baseline value and recompute the stationary equilibrium. All outcomes are reported as percent deviations from the baseline equilibrium, so the vertical dashed line corresponds to the calibrated economy.

Figure 8 shows the effects of changing the equity issuance cost. A higher equity issuance cost increases the marginal value of internal funds for banks. As a result, banks require higher compensation for expanding lending and raise loan rates. Deposit rates fall, reflecting the higher value of deposit funding when external finance is more costly. Quantitatively, the main effects occur on bank prices and balance-sheet quantities: as ϕ_E^B rises above the baseline, loans decline and the loan-to-output ratio falls, while deposit funding becomes more valuable. The figure also shows a pronounced nonlinearity near $\phi_E^B = 0$. When external equity is essentially costless, banks can expand their balance sheets at low marginal cost; lending, deposits, and aggregate activity are therefore higher, and loan rates are lower. Introducing even moderate equity issuance costs sharply raises the marginal value of internal funds and compresses this expansion.

The real effects operate mainly through output and the allocation of output between consumption and other uses. The non-monotonic response of output reflects two opposing resource-cost effects. A higher equity issuance cost raises the resource cost of external bank finance, which tends to reduce output. At the same time, it contracts lending. Since borrowing entails the deadweight cost $\Psi(L)$, lower lending reduces real resource losses on the firm side. For moderate increases in ϕ_E^B, the reduction in lending-related resource costs partially offsets the higher cost of external bank finance; for sufficiently high values of ϕ_E^B, the contraction in lending and the higher marginal cost of external finance dominate.

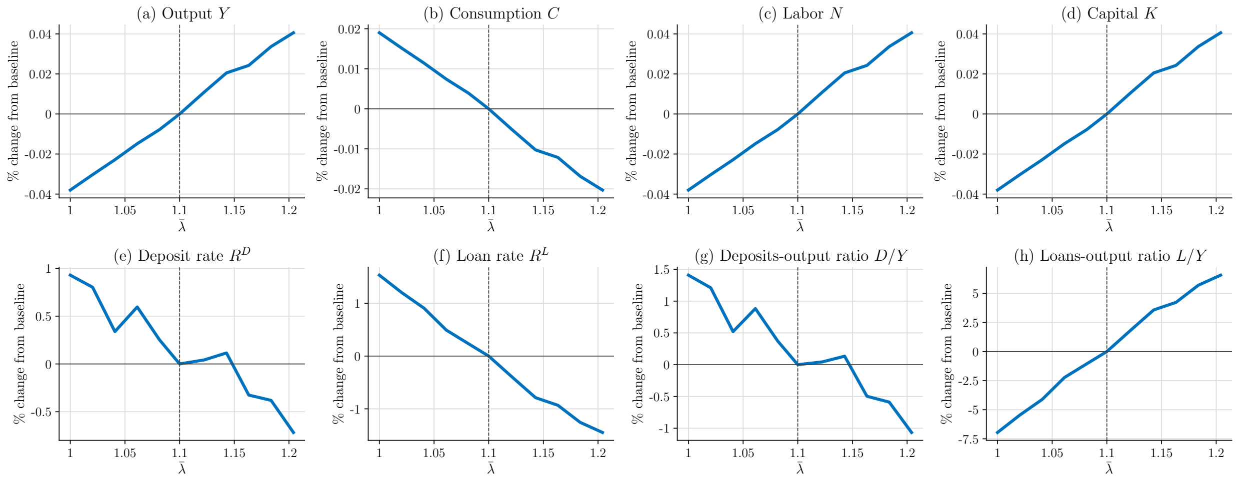
Figure 8. Comparative Statics: Equity Issuance Cost



Notes: The figure varies the bank equity issuance cost, ϕ_E^B, and recomputes the stationary equilibrium. The horizontal axis reports ϕ_E^B. Each panel reports the percent change in the corresponding outcome relative to the baseline equilibrium. The vertical dashed line denotes the baseline calibration.

Figure 9 varies the loan-to-deposit constraint. A higher value of $\bar{\lambda}$ relaxes the balance-sheet limit by allowing banks to support more lending per unit of deposits. Relaxing the constraint lowers loan rates and expands credit, so firms obtain cheaper external finance and produce more. Deposits decline because deposit funding becomes less valuable at the margin when the balance-sheet constraint is looser. On the real side, output rises with the expansion in credit. Consumption declines, despite higher output, because part of the additional output is absorbed by higher steady-state investment and by the deadweight resource cost associated with the larger stock of bank loans, $\Psi(L)$.

Figure 9. Comparative Statics: Loan-to-Deposit Constraint



Notes: The figure varies the loan-to-deposit constraint, $\bar{\lambda}$, and recomputes the stationary equilibrium. A higher value of $\bar{\lambda}$ relaxes the balance-sheet constraint by allowing banks to hold more loans per unit of deposits. The horizontal axis reports $\bar{\lambda}$. Each panel reports the percent change in the corresponding outcome relative to the baseline equilibrium. The vertical dashed line denotes the baseline calibration.

4.3.3 Changes in Interest Rates

Table IV reports the stationary effects of a 50 basis point decline in β, holding the corporate-finance wedge $\omega_F = \beta - \beta^F$ fixed. The shock raises the frictionless safe rate and contracts productive capacity: output falls by 3.0 percent, consumption by 1.8 percent, and capital by 7.7 percent, while labor rises slightly. The increase in loans may appear surprising because lending rates rise. The firm debt Euler equation clarifies this result. In steady state,

$$L = \frac{Q^L - \beta^F \left[1 - \tau_c(1 - Q^L)\right]}{\phi_L},$$

so

$$dL = \frac{1 - \beta^F \tau_c}{\phi_L}\, dQ^L - \frac{1 - \tau_c(1 - Q^L)}{\phi_L}\, d\beta^F.$$

A higher lending rate lowers Q^L and reduces desired borrowing, but holding ω_F fixed implies that the decline in β also lowers β^F. This second force raises desired borrowing by reducing the present value of future repayment obligations. In the table, the firm-discounting effect dominates the loan-price effect, so loans rise even as capital and output fall. The shock therefore increases leverage rather than productive capacity.

Table IV. Stationary Equilibrium at $\bar{\lambda} = 1.1$

Variable	Baseline change	Competitive benchmark change
Output Y	-3.0%	-2.8%
Consumption C	-1.8%	-1.9%
Labor N	$+0.3\%$	$+0.5\%$
Capital K	-7.7%	-7.6%
Macro lending rate $R^{L,macro}$	$+0.6$ pp	$+0.5$ pp
Macro deposit rate $R^{D,macro}$	$+0.7$ pp	$+0.5$ pp
Avg. bank lending rate R^L	$+0.3$ pp	$+0.5$ pp
Avg. bank deposit rate R^D	$+0.2$ pp	$+0.5$ pp
Deposits D	$+9.9\%$	$+3.4\%$
Loans L	$+7.6\%$	$+3.4\%$

Notes: Entries report the change in the stationary equilibrium induced by lowering β by 50 basis points, holding $\omega_F = \beta - \beta^F$ fixed. For quantities, entries are percentage changes relative to the corresponding pre-shock stationary equilibrium within each regime. Macro-rate rows report changes in rates computed from aggregate prices Q^L and Q^D, in percentage points. Average bank-rate rows report changes in quantity-weighted average rates across banks, also in percentage points. This table fixes $\bar{\lambda} = 1.1$. The competitive benchmark is defined in Section 3.8.2.

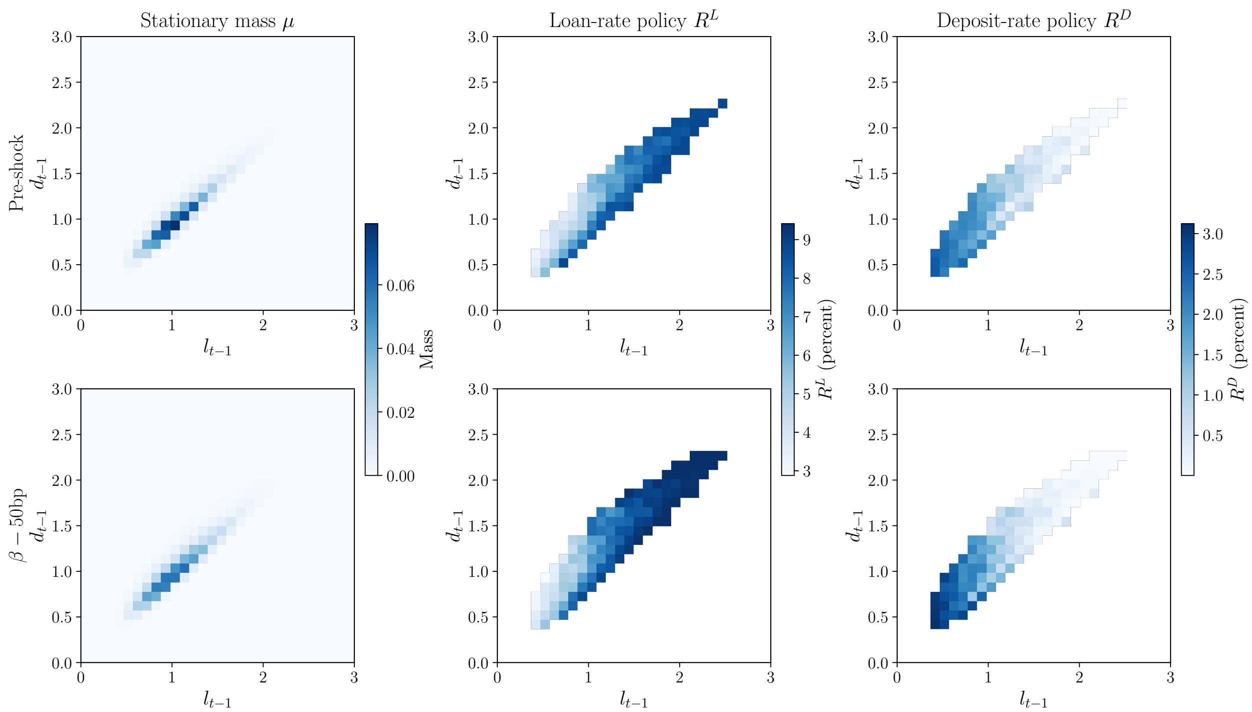
The table also shows that interest-rate pass-through differs sharply between the competitive benchmark and the baseline economy with relationship banks. In the competitive benchmark, pass-through is essentially one-for-one: loan and deposit rates rise by about 50 basis points. In the baseline economy, pass-through to the average bank-customer relationship is muted. Average bank lending rates rise by about 30 basis points, and average bank deposit rates rise by about 20 basis points. Hence relationship banking dampens customer-level pass-through, especially on the deposit side. Macro rates, however, move more than average bank rates. This difference arises because aggregate loan and deposit rates are computed from quantity-weighted prices,

$$R^{L,\text{macro}} = \frac{1}{\int Q_j^L (l_j/L)\, d\mu(j)}, \qquad R^{D,\text{macro}} = \frac{1}{\int Q_j^D (d_j/D)\, d\mu(j)},$$

whereas average bank rates average the bank-level rates $1/Q_j^L$ and $1/Q_j^D$ directly. Thus, aggregate pass-through depends not only on how much individual banks change rates, but also on where loans and deposits are located in the bank-state distribution.

Figure 10 clarifies the economic mechanism. The shock does not primarily operate by moving a large amount of mass across bank types. The invariant distribution remains concentrated along the diagonal in inherited loan and deposit relationships, although mass becomes somewhat less concentrated around the center. The main adjustment is instead through state-dependent pricing policies. Loan-rate policies shift upward strongly, especially for banks with larger inherited loan and deposit relationships. These banks have larger installed customer bases relative to the flow of customers currently reconsidering their bank, and therefore face less contestable borrower demand. They can pass the higher safe rate through to borrowers more aggressively.

Figure 10. Bank Distribution and Pricing after an Interest-Rate Change



Notes: The figure compares the stationary distribution and bank pricing policies before and after the 50 basis point decline in β, holding $\omega_F = \beta - \beta^F$ fixed. The top row reports the pre-shock stationary equilibrium. The bottom row reports the stationary equilibrium after the decline in β. The first column reports the invariant mass of banks over inherited loan and deposit relationships, l_{t-1} and d_{t-1}. The second and third columns report the corresponding loan-rate and deposit-rate policy functions, R^L and R^D, in annualized percent. The color scale in the first column denotes stationary mass. The color scales in the second and third columns denote annualized bank-level rates.

Deposit-rate policies respond very differently. Deposit rates increase less, and the visible increases are concentrated among banks with smaller inherited relationship stocks. Large relationship banks raise deposit rates only weakly. This asymmetry reflects the oligopsony side of relationship banking: banks with large installed depositor bases do not need to compete as aggressively for marginal deposits, while smaller banks must offer more attractive deposit rates to attract or preserve funding. The result is an uneven widening of intermediation margins across the bank distribution. Large relationship banks raise loan rates substantially while increasing deposit rates only modestly; smaller banks display stronger deposit-rate pass-through but have less scope to exploit borrower relationships.

Taken together, the table and figure show that relationship banking changes the nature of interest-rate transmission. A higher safe rate is not passed through uniformly across banks or across the two sides of the balance sheet. Instead, pass-through depends on inherited customer relationships and balance-sheet positions. Customer-level pass-through is incomplete, especially for depositors, while aggregate rates also reflect the quantity weights attached to different regions of the bank distribution. The key mechanism is therefore not a large reallocation of banks across states, but the state-dependent response of loan and deposit rates among heterogeneous relationship banks.

5 Conclusion

This paper develops a theory of relationship banking in which banks exercise market power on both sides of the balance sheet through persistent relationships with borrowers and depositors. We model banks as dynamic two-sided intermediaries that compete for customers while operating subject to financial frictions that jointly determine lending, funding, and pricing decisions. The key mechanism is a dynamic trade-off between current margins and future franchise value: banks can exploit existing customer relationships to earn rents today, but doing so affects the future evolution of their borrower and depositor bases. The model implies that market power depends not only on contemporaneous demand elasticities but also on the value of future customer relationships and balance-sheet capacity. As a result, differences in relationship strength generate endogenous heterogeneity in bank markups, providing a natural explanation for the substantial cross-sectional dispersion in market power that we document in the data.

Quantitatively, we find that relationship-based two-sided market power has significant consequences for financial intermediation and aggregate activity. Relative to a constrained-competitive benchmark, market power raises lending rates, lowers deposit rates, contracts bank balance sheets, and reduces output. Moreover, relationship banking generates heterogeneity in interest rate transmission, with an interest-rate pass-through varying systematically across banks and customers depending on the strength of existing relationships. Changes in policy rates alter the value of future customer relationships through the discounting of franchise value, leading banks with different inherited customer bases to adjust loan and deposit rates differently. These findings suggest that ongoing changes in financial intermediation—including greater competition from money market funds and private credit—may affect not only the level of bank market power but also its distribution across institutions, with potentially important implications for credit allocation, interest rate transmission, and aggregate economic performance.

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APPENDIX

A Computational Appendix

This appendix describes the algorithm used to compute the stationary bank-side equilibrium in the atomistic limit using a brute-force value function iteration (VFI) and a stationary-distribution computation.

A.1 State space and grids

We discretize the bank state

$$x = (l_{t-1}, d_{t-1}, s_t^L, s_t^D)$$

on tensor-product grids:

$$l_{t-1} \in \{\ell^1, \dots, \ell^{n_L}\}, \quad d_{t-1} \in \{d^1, \dots, d^{n_D}\}, \quad s_t^L \in \{s_1^L, \dots, s_{n_{SL}}^L\}, \quad s_t^D \in \{s_1^D, \dots, s_{n_{SD}}^D\}.$$

Shifters (s^L, s^D) follow finite-state Markov chains with transition matrices (P^{sL}, P^{sD}).

Banks choose posted prices (Q^L, Q^D) on discrete grids:

$$Q^L \in \{Q_1^L, \dots, Q_{n_{QL}}^L\}, \qquad Q^D \in \{Q_1^D, \dots, Q_{n_{QD}}^D\}.$$

A.2 Atomistic logit objects and scaling

Given (Q^L, Q^D) and shifters, the deterministic utilities are

$$V^L = \nu_L + \alpha_L \log Q^L + \gamma_L \log s^L, \qquad V^D = \nu_D - \alpha_D \log Q^D + \gamma_D \log s^D.$$

Let

$$Z^L \equiv \int_0^1 e^{V_j^L} dj, \qquad Z^D \equiv \int_0^1 e^{V_j^D} dj, \qquad \mathcal{S}_j^L = \frac{e^{V_j^L}}{Z^L}, \quad \mathcal{S}_j^D = \frac{e^{V_j^D}}{Z^D}.$$

In computation we solve for scaled aggregates

$$\frac{L}{Z^L} \quad \text{and} \quad \frac{D}{Z^D}$$

directly. In the stationary equilibrium, aggregate loan and deposit quantities are constant, so the dynamic scaling terms L_t/L_{t-1} and D_t/D_{t-1} equal one. Given $(L/Z^L, D/Z^D)$, the law of motion for bank-level *levels* is therefore evaluated as

$$l_t = (1-\delta_L)\, l_{t-1} + \delta_L\, e^{V^L} \frac{L}{Z^L}, \qquad d_t = (1-\delta_D)\, d_{t-1} + \delta_D\, e^{V^D} \frac{D}{Z^D},$$

which is algebraically equivalent to $l_t = (1-\delta_L)l_{t-1} + \delta_L \mathcal{S}^L L$ and similarly for deposits.

A.3 Bank Bellman equation (discrete choice of prices)

Given current state x and a candidate choice (Q^L, Q^D), the bank computes implied next-period levels (l_t, d_t), then pre-issuance dividends

$$\widetilde{\mathrm{Div}}^B = l_{t-1} - Q^L l_t - d_{t-1} + Q^D d_t - \Omega,$$

where Ω is the operating/size cost (**??**) evaluated at (l_t, d_t). Net dividends are $\mathrm{Div}^B = \widetilde{\mathrm{Div}}^B - \Phi(\widetilde{\mathrm{Div}}^B)$.

The Bellman operator is

$$V(x) = \max_{Q^L, Q^D} \left\{ \mathrm{Div}^B(x, Q^L, Q^D) + \beta\, \mathbb{E}\left[V(x') \mid x, Q^L, Q^D \right] \right\},$$

where $x' = (l_t, d_t, s^{L\prime}, s^{D\prime})$ and the expectation is taken over the shifter transitions, subject to the leverage constraint $l_t \leq \bar{\lambda} d_t$.

A.4 Interpolation and stationary distribution

Because (l_t, d_t) generally do not lie exactly on the (l, d) grid, continuation values are computed by bilinear interpolation on (l, d) for each shifter pair (s^L, s^D).

Given the optimal policy functions, we build a sparse transition matrix P over the discretized state space by: (i) evolving (l, d) deterministically to (l_t, d_t) and splitting mass across the four neighboring grid points (bilinear mass-splitting), and (ii) applying the shifter transition probabilities.

We compute the invariant distribution μ by power iteration on the transpose:

$$\mu^{(k+1)} = P^{\top} \mu^{(k)}, \qquad \mu^{(k+1)} \leftarrow \mu^{(k+1)} / \sum \mu^{(k+1)}.$$

A.5 Fixed-point system and solver

Let $\mathrm{AggL} = \int l_j dj$ and $\mathrm{AggD} = \int d_j dj$ denote aggregates implied by the stationary distribution and policy-induced levels. The logit denominators implied by the stationary cross-section are

$$Z^L = \int e^{V_j^L}\, dj, \qquad Z^D = \int e^{V_j^D}\, dj.$$

Under the scaled formulation, we solve for $(L/Z^L, D/Z^D)$ using the fixed-point conditions

$$\frac{\text{AggL}}{Z^L} = \frac{L}{Z^L}, \qquad \frac{\text{AggD}}{Z^D} = \frac{D}{Z^D},$$

implemented in logs for numerical stability:

$$\log\left(\frac{\text{AggL}}{Z^L}\right) - \log\left(\frac{L}{Z^L}\right) = 0, \qquad \log\left(\frac{\text{AggD}}{Z^D}\right) - \log\left(\frac{D}{Z^D}\right) = 0.$$

We solve this two-equation system using `fsolve` (Levenberg–Marquardt), with warm starts for (Q^L, Q^D, V) across iterations.