On-the-Job Search and Inflation Under the Microscope *

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Abstract

We develop a HANK model with a job ladder and endogenous on-the-job search (OJS). In this setup, the search decisions of the employed matter for inflation, as they elicit interfirm wage competition to hire or retain workers. To validate the mechanism, we study the effects of the 2012 Danish tax reform, which increased the incentives to search on the job differentially across the income ditribution. The predictions of the model on employment-to-employment transition rates and wage growth for both leavers and stayers are in line with those estimated in the microdata, both qualitatively and quantitatively. In a version of the model calibrated to the Danish economy, policies that affect OJS incentives simultaneously decrease inflation and unemployment, while raising welfare.

JEL Codes:

Keywords: Job ladder models; inflation; Danish microdata

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1 Introduction

Phillips (1958) was the first to show that reductions in wage growth are associated with increases in unemployment. Successive refinements of the Phillips Curve have emphasized the relevance of gaps, for instance drawing attention to unemployment gaps rather than rates, but the essence has not changed. Unemployment, or unemployment gaps are still considered the key determinant of inflation. Yet, the relationship between unemployment rates—or gaps—and inflation is not very strong in the data, as testified by a large literature pointing to very small slopes of the Phillips curve. Yellen (2014), among others, has observed that labor market slack appears to be a multifaceted object, where inflationary pressures stemming from the labor market can hardly be traced down to a single determinant.

In this paper we turn the focus away from the search behavior of that tiny fraction of the workforce that is unemployed, and zoom in on the search decisions of that vast majority that is employed. We show that the search decisions of the employed matter too, and that higher OJS leads to higher wage inflation. Because employed and unemployed job seekers produce opposite effects on wage inflation, the composition of the pool of job seekers matters. We illustrate this point by means of a Heterogeneous Agents New Keynesian (HANK) job ladder model with sequential auction bargaining in the spirit of Postel-Vinay and Robin (2002), and endogenous on-the-job search (OJS). We use this framework as a laboratory to study the Danish tax reform of 2012, which changed the incentives to search for higher wages differentially across the income distribution. To validate the mechanism whereby changes in on-the job search affects wage inflation, we compare key model outcomes across the income distribution with those estimated in the Danish microdata.

The mechanism works as follows. The tax reform increased the income tax-threshold at which the highest tax rate applies, thereby lowering the marginal tax rate faced by some high-income workers. In the model, a fall in the marginal tax rate increases the incentives to search on the job, as it increases the after-tax returns from search. Higher OJS increases interfirm competition to hire or retain workers. As a result, wages increase either directly, as the employed accept higher wage offers, or indirectly, when the incumbent employer matches the offer of the poaching firm.[1] The ensuing increase in unit labor costs increases price

[1] Wage bidding wars to retain staff, as modelled by Postel-Vinay and Robin (2002), have long been a feature of an extensive literature in macro labor, which we partially review below. The assumptions embedded in this bargaining protocol align well with anecdotal and survey evidence, documenting the importance of wage bidding wars to retain staff in explaining the recent dynamics of inflation and its decoupling with conventional measures of labor market slack. According to Financial Times (2023), a survey-study by the Chartered Institute for Personnel and Development (CIPD) shows that bidding wars to retain staff are becoming increasingly prevalent in the UK job market, and over the year to August 2023 40% of surveyed firms made counter offers. According to CIPD (2023), this could explain the puzzling high pace of wage growth in the face of an apparently cooling labor market.

inflation. Concurrently, the increase in the share of employed workers among the job seekers raises the expected wage of a new hire, given that the employed workers are more expensive to hire than the unemployed. Intuitively, their "bargaining position" is stronger since they can spark a wage-bidding war between incumbent and prospective employers. As a result, vacancies fall and unemployment rises, while wage and price inflation increase. So in the model, policies that affect the incentives to search on the job, can elude the traditional Phillips-curve tradeoff.

A permanent increase in the high-income tax rate produces an *a-priori* ambiguous impact on welfare. In the long run, it increases the efficient allocation of workers on the ladder as well as the costs of OJS. In transition to the new steady state, it persistently reduces employment. At the calibrated equilibrium, this policy increases welfare in the long run, but decreases it persistently in the transition. In the model, OJS produces a negative externality on job creation, as it increases the share of the employed in the pool of job seekers, thereby raising the expected cost of entry. By lowering the value of searching on the job, a higher income tax allows the employed to internalize this negative externality on job matches.

The impact of a change in the tax threshold on the incentives to search on the job and therefore on wage inflation, differs across the income distribution. Intuitively, the original threshold does not constrain workers with earnings far below it, since the higher earnings associated with a job-to-job transition would still be taxed at the lower marginal rate; hence workers in lower income bins should not exhibit a differential on-the-job search behavior before and after the reform. Similarly, workers above the new threshold would be subject to the same high-income-tax rate both before and after: so also for the workers in these high income bins, there should be no differential effect of the reform on job search behavior. The income group that is most strongly affected by the tax reform lies in between these two polar cases and, more precisely, around the old income-tax threshold. For these workers, the entire additional wage growth from a transition is taxed at a lower marginal rate after the reform, compared to the period that precedes it. These differential effects of the tax reform on the returns to on-the-job search across the income distribution lead to an inverse-V shape response in the share of employed job seekers.

In the model, the inverse V-shape response in the share of employed job seekers induces an inverse V-shape pattern in EE transition rates. Concurrently, it also induces a similar pattern in the differential rates of wage growth—before and after—but only among the stayers, not among the leavers. Indeed, conditional on changing jobs, the rate of growth in gross wages does not change with the change in the tax schedule, given that the workers still derive gross wage offers from the same underlying distributions. On the other hand, the share of the employed who are able to renegotiate wages upwards at their current employer increases with the share of employed workers who are able to get a valuable outside offer. Because the response of this variable across the income distribution is proportional to the share of employed job seekers, the wage growth for the stayers must also experience a similar inverse-V-shape pattern.

We apply to the Danish microdata a difference-in-differences research design to retrieve the causal effects elicited by the HANK model. The results produce an inverse V-shaped response for EE rates and wage inflation of the stayers, and no response for wage inflation of the leavers, exactly in line with the model's predictions. Quantitatively, the empirical results are strong: we find that both EE transition rates and the wage growth of the stayers are higher by about 10% in the proximity of the 2012 threshold. It is worth noting that the stayers represent that 80% of the workers' population who do not experience changes in jobs and the associated match-productivity in a year. Through the lenses of the model, these wage increases are akin to a pure cost-push shock at the firm-level. Finally, we run placebo experiments to show that the inverse V-shaped responses of EE transition rates can no longer be retrieved when comparing years in which the threshold did not change.

The result that an increase in OJS increases the wages of the stayers arises in the model from the assumption of sequential auction bargaining, and is in sharp contrast with the result that would arise under Nash bargaining. With Nash bargaining, wages depend positively on the ratio of vacancies to job seekers, and the composition of the pool of job seekers does not matter: wages would fall independently of whether it is the unemployed or the employed that search more.

Our work is directly related to the very recent literature that provides new insights on inflation dynamics from job ladder models of the labor market. Moscarini and Postel-Vinay (2023) have shown that the state of cyclical labor misallocation has implications for the propagation of shocks on inflation. In their setup the OJS rate is constant, so the authors abstract from the channel that is the specific object of interest in this paper. Faccini and Melosi (2023) derive a model-based indicator of labor market slack that accounts for time variation in the rate of on-the-job search and illustrate how that can provide an explanation for the missing inflation observed prior to the pandemic recession and part of the subsequent surge that has characterized the Great Resignation (Alves (2020) and Birinci et al. (2023) reach similar conclusion within a heterogeneous households setup). Pilossoph and Ryngaert (2024) provide evidence that employed workers who expect higher inflation are more likely to search on the job and experience an EE transition in the short term. They produce a model where this link between inflation expectations and search decisions can produce wage-price spirals.

Relative to this literature, we make a theoretical and an empirical contribution. On theoretical side, we illustrate how, when wage and price inflation are driven by the search decisions of the employed, economic policies can potentially reduce inflation without necessarily raising unemployment. On the empirical side, we test the effects of taxes on EE transitions and wages using the Danish microdata. In looking explicitly at the wages of the stayers, we provide causal evidence supporting the primitives of this class of models.

Our paper is closely connected to the literature on fiscal and monetary interactions. Over the past decade, much policy and academic interest has focused on the question of whether——and if so, how——alternative policy tools could be used to replicate monetary stimulus when nominal interest rates are constrained by an effective lower bound (Wolf, 2024). More generally, Blanchard (2024) advocates for fiscal policy to complement monetary policy in the quest for the joint stabilization of output and inflation, also beyond periods when the effective lover bound is binding. This is necessary, in his view, due to the lack of divine coincidence. Our paper builds on this strand of literature, by illustrating a novel mechanism for the transmission of fiscal policy that operates by affecting the incentives for job mobility, and which can potentially circumvent the policy trade-offs faced by the monetary authority.

Our paper is also related to a vast literature that investigates the effects of income taxes on labor market outcomes. Over several decades, this literature has typically been focusing on the effects of taxes on the intensive and extensive margin of labor supply (see Keane (2011) and Chetty et al. (2013) for a survey of the literature). Our model abstracts entirely from these channels to elucidate a different propagation mechanism, which relies on the response of on-the-job search, i.e., an alternative and much under-explored aspect of labor supply decisions. Perhaps closer to our paper is the work by Bagger et al. (2021), which looks into the effect of income taxes in a job ladder model with endogenous on-the-job search and makes use of danish microdata to estimate the model. In their paper as in ours, income taxation reduces the returns to on-the-job search. Their work focuses on the effects of taxation on the equilibrium allocation of labor and on the elasticity of taxable income; it entirely abstract from inflation, which is the object of our investigation.

The paper is organised as follows. Section 2 presents a HANK job-ladder model with endogenous on-the-job search and taxes. Section 3 describes the datasets used in the empirical analysis, and Section 4 presents the calibration of the HANK model. Section 5 illustrates the effects on EE rates and wages across the income distribution of a shift in the tax threshold for high income earners, both in the theoretical model and in the data. Section 6 presents the general equilibrium effects of this policy on the macroeconomic aggregates. Section 7 concludes.

2 The Model

We present a HANK model extended to include a job ladder, endogenous on-the-job search and taxes. This setup allows us to investigate how tax shocks affect macroeconomic dynamics when on-the-job search responds to incentives. In Section 5, we show that in this model, a change in the income-threshold for the marginal tax rate produces differential responses in the rate of on-the-job search, EE transitions and wage growth across the income distribution. These theoretical predictions highlight a key mechanism, whereby income taxes affect wage inflation through the impact on on-the-job search.We then test for these results making use of administrative Danish microdata.

2.1 The environment

The economy comprises a unit measure of ex-ante identical individuals facing a discrete and infinite time horizon. All of them participate to the labor market until they retire. While active in the labor market, workers can be either employed or unemployed. The pool of job seekers comprises the entire measure of the unemployed, and an endogenous share of the employed. Every period, an employed worker draws a cost of search from a stochastic distribution and optimally decides to search provided that the expected return is larger than the cost. By searching on the job, the workers can move up the ladder to more productive matches. Employers compete *à la* Bertrand to hire or retain workers, which implies that workers have the opportunity to renegotiate their wages upwards with the arrival of outside offers. As a result, income processes evolve endogenously in this model, stemming from individual reallocation decisions, which lead to better matches and wage renegotiations. This marks a difference with respect to the standard HANK setup, where income processes are assumed to evolve exogenously.

Each period, households decide how much to consume and save. The savings decision takes into account the income risk that arises from the existence of frictions in the labor market. In the asset market, the individuals trade shares of a mutual fund, which owns all government bonds and firms in the economy, redistributing all profits as dividends.

We assume two types of firms. Service sector firms produce a homogeneous good, using labor as the only factor of production. This is sold to monopolistically competitive producers, which differentiate the good and sell it to the households facing nominal price rigidities *à la* Rotemberg. Finally, a monetary authority is in charge of the nominal interest-rate policy, while the fiscal authority levies income taxes and administers lump-sum transfers.

2.2 Labor market and wage negotiations

The labor market is governed by a standard meeting function that brings together vacancies and job seekers. This implies that the rates at which job seekers meet a vacant job, $\phi(\theta)$, and the rate at which vacant jobs meet a job seekers, $q(\theta)$, only depends on labor market tightness θ, defined as the ratio of the aggregate measure of vacancies and job seekers, i.e. $\theta = \frac{v}{S}$. Homotheticity of the meeting function implies that $d\phi(\theta)/d\theta > 0$ and $dq(\theta)/d\theta < 0$. Upon contact, the worker draws match productivity from a distribution G^x, defined over the support $[\underline{x}, \overline{x}]$ and receives a wage offer (details below) that can be accepted or rejected. Every period, matches can be terminated either because of an exogenous shock that hits with probability δ, or because workers endogenously reallocate to other firms.

The bargaining protocol follows Bagger et al. (2014) and assumes that firms Bertrand compete on the share of output they are willing to pay as wages. Workers hired from unemployment cannot spark wage competition between employers, and are assumed to receive a wage equal to the full production of the least productive firm in the economy, $\underline{x}$.

To understand wage determination for the employed workers who receive an outside wage offer, it is useful to distinguish between two different cases. Let the wage schedule be denoted by $w(x, \alpha) = \alpha\zeta x$, where $\zeta \in (0, 1)$ represents the maximum share of output that a worker with piece-rate $\alpha = 1$ can capture as wage. Consider first the case of a worker employed with productivity x, who meets with a firm with productivity $x' > x$. This is the case where the poaching firm is more productive than the incumbent. The maximum wage that the incumbent can offer is $w(x, 1)$. This offer can be outbid by the poacher, by offering $w(x, 1) + \epsilon$, where $\epsilon \approx 0$ is an arbitrarily small value. Bertrand competition implies that the worker will switch employer, and receive the wage schedule $w(x', x/x')$, where $\alpha' = x/x'$ is the updated piece-rate.

Now consider the case where a worker employed in a match with productivity x and piece-rate α meets with a firm with productivity $x' < x$. In this case the poacher is less productive than the incumbent. In this case, the worker stays with the incumbent, but the wage is still renegotiated upwards if the maximum wage that the poacher is willing to pay is higher than the pay the worker is currently receiving. That is, the outcome of the auction is a wage that satisfies $max\{w(x, \alpha), w(x, x'/x)\}$.

Let $\mu_0^U(e)$ and $\mu_0^E(e, x, \alpha)$ denote the beginning-of-period distribution of the unemployed and the employed workers, respectively. Let $\xi(e, x, \alpha)$ denote the share of workers in the state space defined by the vector (e, x, α) who optimally decides to search. Then the measure of workers looking for jobs at the beginning of a period is given by:

$$S = \int d\mu_0^U(e) + \int \xi(e, x_0, \alpha)\, d\mu_0^E(e, x_0, \alpha). \tag{1}$$

2.3 Timing of events

The timing of events is as follows: first, the aggregate tax shock hits the economy. Then both the unemployed and the employed workers search for jobs. Subsequently, reallocation takes place: some unemployed find jobs and some employed move to a different employer. Next, production takes place, wages, interest rates, dividends and government transfers are paid, taxes are levied and consumption decisions are taken. At the end of the period, idiosyncratic, retirement and death shocks take place.

In what follows, we denote by the time subscript 0 the value of a variable defined at the beginning of the period, i.e. at the stage where the search decision is taken. We denote by the subscript 1 the value at the end of a period, i.e. when the consumption/savings decision is taken.

2.4 Workers

We let U, V and Γ denote the value function associated with the states of unemployment, employment and retirement, respectively. All individuals derive utility from the consumption of a homogeneous good c as dictated by the function $u(c)$. The price of the consumption good, which is the *numeraire* in this economy, is denoted by P. Workers receive an amount D of dividend payments for each share they hold of the mutual fund. Shares are noted by e, and their unit price by P^e. They also receive wage payment $w(x,\alpha)$ if employed, unemployment benefits b if unemployed and pensions T^R if retired. All income from the labor market is taxed at the rate τ, where τ is a function of income, which we specify below. Finally, all workers receive the same amount of transfers T from the government.

Consider an unemployed worker who did not manage to find a job within a given time period. At the end of the period, the value of unemployment is

$$U(e)=u\left(c\right)+\left(1-\psi^{R}\right)\beta\left[f\left(\theta'\right)E_{x}V_{1}\left(e',x,\frac{x}{\underline{x}}\right)+\left(1-f\left(\theta'\right)\right)U(e')\right]+\beta\psi^{R}\Gamma\left(e'\right), \qquad (2)$$

subject to the budget constraint

$$Pc+P^{e}e'=P\left(1-\tau(b)\right)b+(P^{e}+D)e+T,$$

where $\beta\in(0,1)$ is the discount factor, E denotes the expectation operator, ψ^R is the probability that at the end of a period a worker retires and a $'$ superscript denotes next period values. The above maximization problem shows that an unemployed workers chooses current consumption and savings e' taking into account the probabilities associated with being in the three different labor market states next period. Specifically, if the worker does not retire at the end of the period with probability ψ^R, she will be either employed or unemployed at the end of the following period with probabilities $f(\theta)$ and $1-f(\theta)$, respectively. And if the worker ends-up employed in a match with productivity x, she will only be able to make a first step on the wage ladder, starting with a salary equal to $\underline{x}\zeta$, which correspond to a piece-rate $\alpha = \underline{x}/x$.

The problem of an employed worker is separated in two parts. First, she choose whether to search. Next, after reallocation has taken place and wages have been rebargained, she choose consumption and savings. So the problem of search is solved at the beginning of the period (intra-time 0), while the consumption-savings problem is solved at the end (intra-time 1). Let's proceed by backward induction and start from the end-of-period problem. The end-of-period value of employment is:

$$V_1(e, x_1, \alpha) = \max_{e' \geq 0, c} \left\{ u(c) + \beta \left(1 - \psi^R\right) \left[(1-\delta) V_0(e', x_1, \alpha) + \delta U(e')\right] + \psi^R \Gamma(e') \right\} \tag{3}$$

subject to

$$Pc + P^e e' = P\left[1 - \tau(w)\right] w_1(x_1, \alpha) + (P^e + D)e + T$$

where $V_0(e, x_0, \alpha)$ is the value function of employment at the beginning of the period, i.e., before the search cost is drawn from the i.i.d. stochastic distribution G^ϕ. The solution to this problem is a policy function that characterizes the optimal savings decision: $e' = g^E(e, x, \alpha)$.

The search decision maximizes the expected value:

$$V_0(e, x_0, \alpha) = \int_\phi \widetilde{V}_0(e, x_0, \alpha, \phi)\, G^\phi(d\phi), \tag{4}$$

where

$$\widetilde{V}_0(e, x_0, \alpha, \phi) = \max\left\{ -\phi + V_0^S(e, x_0, \alpha), V_0^{NS}(e, x_0, \alpha) \right\}, \tag{5}$$

and where V^S and V^{NS} denote the value of an employed worker searching and not searching, respectively. In turn, these are given by:

$$V^{NS}(e, x_0, \alpha) = V_1(e, x_0, \alpha)$$

$$\begin{aligned} V^S(e, x_0, \alpha) = {} & f(\theta) E_{\widetilde{x}} \max\left\{ V_1\left(e, \widetilde{x}, \frac{x_0}{\widetilde{x}}\right), V_1\left(e, x_0, \max\left\{\alpha, \frac{\widetilde{x}}{x_0}\right\}\right)\right\} \\ & + (1 - f(\theta)) V_1(e, x_0, \alpha). \end{aligned} \tag{6}$$

The first term inside the curly brackets is the value of a worker who has met with another firm with probability $f(\theta)$, and will be employed next period in another firm at productivity $\widetilde{x} > x$; the second term is the value of a worker who has got in touch with another firm with probability $f(\theta)$, and has re-bargained his wage with the current employer at the new wage: $\max\{\alpha, \frac{\widetilde{x}}{x}\}\zeta F x_0$. This case occurs whenever the productivity of the incumbent firm is greater than the productivity of the poacher, i.e. $x > \widetilde{x}$. With probability $(1 - f(\theta))$ the worker does not meet with any vacancy and therefore gets the same value she would have got without searching. Opening the expectation operator, the above equation can be rewritten

$$V^S(e,x_0,\alpha)=f(\theta)\left\{\int_{\widetilde{x}=x_0}^{\overline{x}} V_1\left(e,\widetilde{x},\frac{x_0}{\widetilde{x}}\right)G^x(d\widetilde{x})\right.$$
$$\left.+\int_{\widetilde{x}=\underline{x}}^{x_0} V_1\left(e,x_0,\max\left\{\alpha,\frac{\widetilde{x}}{x_0}\right\}\right)G^x(d\widetilde{x})\right\}+(1-f(\theta))V_1(e,x_0,\alpha).$$

We can define a threshold search cost $\phi^T(e,x_0,\alpha)$ such that the employed worker is indifferent between searching and not searching:

$$-\phi^T+V^S(e,x_0,\alpha)=V^{NS}(e,x_0,\alpha). \tag{7}$$

The solution to this problem is a rule, which can be expressed by the indicator function $I_{\phi<\phi^T}(e,x_0,\alpha)=1$, which means that the worker searches if and only if $\phi<\phi^T$. For future convenience, it is helpful to denote by $\xi(e,x,\alpha)$ the ex-ante probability (i.e. before the fixed cost of search is drawn) that a worker defined by the state vector $\{e,x_0,\alpha\}$ ends up searching. By the law of large numbers, this will be given by the share of workers searching in every bin over $\{e,x,\alpha\}$.

The value of retirement is

$$\Gamma(e)=\max u(c)+\beta\left(1-\psi^D\right)\Gamma(e') \tag{8}$$

s.t

$$Pc+P^e e'=\left[1-\tau\left(T^R\right)\right]T^R+(P^e+D)e+T,$$

where ψ^D is the probability that a retired worker dies, and T^R denotes pension income.

2.5 Labor service firms

The end-of-period value of a filled job is given by:

$$J(e,x,\alpha)=p^l x-w(x,\alpha)+\frac{1}{1+r}\left(1-\psi^R\right)(1-\delta)$$
$$\times\left\{\left[(1-\xi(e',x,a))+\xi(e',x,a)(1-f(\theta'))\right]J(e',x,\alpha)\right.$$
$$\left.+\xi(e',x,a)f(\theta')\int_{\underline{x}}^{x} J\left(e',x,\max\left\{\alpha,\frac{\widetilde{x}}{x}\right\}dG^x(\widetilde{x})\right)\right\}, \tag{9}$$

where e' satisfies the savings policy function of the workers, i.e., $e'=g^E(e,x,\alpha)$. The above expression relates the present value of a match to current period profits and expected future values. The profits are given by the value of production x, measured in terms of the consumption good, p^l, minus the real wage. If the match is not dissolved at the end of the period at rate δ, and if the worker does not retire at rate ψ^R, the firm gets the continuation value of the relationship. This value depends on whether the worker will search or not, in the following period. In turn, the probability of searching depends on the assets of the worker, its current productivity and the piece-rate wage that he is able to command. If the worker does not search, with probability $1-\xi\left(e^{\prime}, x, a\right)$, or if the worker searches but does not meet a vacancy, with probability $\xi\left(e^{\prime}, x, a\right)\left(1-f(\theta^{\prime})\right)$, the match will continue with the same productivity x and piece-rate α. If the worker instead searches and finds a job, with probability $\xi\left(e^{\prime}, x, a\right) f(\theta^{\prime})$, the match will continue only if the worker meets with a firm with lower productivity than the incumbent, i.e. for any $\tilde{x}<x$, where $\tilde{x}$ is the poacher's productivity. In this case, the wage will be renegotiated upwards with the incumbent whenever $\tilde{x}/x>\alpha$.

Vacancies are opened at the beginning of the period at a flow cost κ. An additional fixed cost κ^f is paid if a match is formed. We assume that vacancies are matched at random with the workers in the pool of job seeker, who are either employed or unemployed. The free entry condition, which equates the expected costs and returns from a match, is:

$$\kappa^f+\frac{\kappa}{q\left(\theta\right)}=\frac{1}{S_t}\left[\int_e\int_{\widetilde{x}} J\left(e,\widetilde{x},\frac{\underline{x}}{\widetilde{x}}\right)dG^x\left(\widetilde{x}\right)d\mu_0^U\left(e\right)\right.$$
$$\left.+\int_{e,x,\alpha}\int_x^{\overline{x}} J\left(e,\widetilde{x},\frac{x}{\widetilde{x}}\right)dG^x\left(\widetilde{x}\right)\xi\left(e,x,\alpha\right)d\mu_0^E\left(e,x,\alpha\right)\right] \tag{10}$$

On the LHS, the expected cost is given by the flow cost κ times the number of periods that a vacancy is expected to remain open before a match is found, $1/q(\theta)$, plus the fixed cost.

On the RHS, the expected return is broken down into two terms: the first (second) integral expression inside the squared brackets characterizes the expected return of meeting with an unemployed (employed) worker. The value of a match with an unemployed depends on the stochastic productivity draw, and assumes that all unemployed workers start at the bottom of the wage ladder. The value of meeting with a worker employed depends not only on the productivity draw, but also on the productivity of her employer, the piece rate of her current wage contract, as well as the distribution of on-the-job search across the state-space.

The free entry equation (10) is key to understand the mechanism. The value to the firm of meeting with a worker unemployed is higher, everything else equal, than the value of meeting with a worker employed, because unemployed workers are cheaper to hire, given that they are not able to elicit wage competition between employers. A fall in the share of job seekers that are employed will increase the chances of meeting with an unemployed worker, reducing in expectation the wage payments of a new hire and increasing the surplus of a match on the RHS. With flexible prices, labor market tightness is the only variable that adjusts to restore the equilibrium in the labor market. Namely, tightness would increase, reducing the vacancy filling rate and increasing the expected vacancy costs required to meet a worker on the LHS.

With nominal price rigidities though, there is a second variable that can adjust to restore the equilibrium: it is the relative price of the labor service p^l, which appears inside the expression for the value function J in equation (9). Namely, when the share of employed job seekers falls, the value of the labor service also falls, reducing the value of the expression on the RHS, which compensates for the expected increase in match surplus. Intuitively, lower expected wage payments in the service sector are passed through as a lower cost of the homogeneous service that is provided to the intermediate fringe of producers. Hence, in this model, the current and future path of real marginal costs p^l is directly affected by the composition of the pool of job seekers; specifically, it is related positively to how many employed workers decide to search on the job and negatively to the rate of unemployment.

2.6 Price setting firms

Intermediate goods producers purchase one unit of the homogeneous labor service and transform it into one unit of a differentiated good, subject to the demand function from the workers. Under the standard assumption that workers minimize the expenditure required to consume a CES bundle of differentiated products, the demand for an individual variety is given by

$$y_i = \frac{p_i}{P}^{-\eta}, \tag{11}$$

where η is the elasticity of substitution across varieties.

The problem of the price setters is to maximize current and expected profits subject to the demand constraint in equation (11) and quadratic price adjustment costs *à la* Rotemberg. The value function of the price setters solves:

$$\Omega(p_{i,-1}) = \max_{p_i}(p_i - p^l)y_i - \frac{\eta}{2\vartheta} log\left(\frac{p_i}{p_{i,-1}}(1+\pi) - \pi^*\right)^2 Y + \frac{1}{1+r}\Omega(p_i), \tag{12}$$

where ϑ is a price adjustment cost parameter and π^* is the stady-state rate of inflation.

The solution of the maximization problem is the standard Phillips curve:

$$\begin{aligned}\frac{\log\left(1+\pi-\pi^*\right)\left(1+\pi\right)}{1+\pi-\pi^*} &= \vartheta\left(p^l - \frac{\eta-1}{\eta}\right) \\ &+ \frac{1}{1+r}\frac{\log\left(1+\pi'-\pi^*\right)\left(1+\pi'\right)}{1+\pi'-\pi^*}\frac{Y'}{Y}.\end{aligned}$$

2.7 Fiscal and monetary authorities

The fiscal authority levies income taxes and administers lump sum transfers to ensure that the budget balances period-by-period. Define two threshold levels of real income w_L and w_H, with $w_L < w_H$. The tax schedule is such that the marginal tax rate is equal to: (i) τ_0 for any income below w_L; (ii) $\tau_L > \tau_0$ for any share of income above w_L and below w_H; $\tau_H > \tau_L$ for any share of income above w_H. The government budget constraint is given by:

$$\begin{aligned} B_{-1} + T + P\int b d\mu_1^U(e) + P\int T^R d\mu_1^R(e) = {} & \frac{B}{1+i} \\ & + P\int b\tau(b)\, d\mu_1^U(e) \\ & + P\int w(e,x,\alpha)\,\tau(w(e,x,\alpha))\, d\mu_1^E(e,x,\alpha) \\ & + P\int T^R\tau\left(T^R\right) d\mu_1^R(e), \end{aligned} \tag{13}$$

where the LHS and RHS denote the allocation and funding of the public administration, respectively. Namely, the government revenues on the RHS are given by the new emissions of public debt in present value, $B/(1+i)$, and by the taxes levied on the unemployed, the employed and the retirees. These funds can be used to repay outstanding government debt, transfers, unemployment benefits and pensions. It is assumed that Transfers T are distributed equally across the entire population of workers, including the employed, the unemployed, and the retirees. In equilibrium, it is assumed that government bonds are in zero net supply, i.e. $B = 0$.

The monetary authority is assumed to set the nominal interest rate i following the Taylor rule:

$$i = i^* + \Phi_\pi(\pi - \pi^*) + \Phi_U(u - u^*), \tag{14}$$

where an asterisk superscript over a variable denotes its the steady-state value. The link between nominal and real interest rates is governed by the Fisher equation:

$$1 + i \equiv E(1+\pi')(1+r). \tag{15}$$

2.8 Mutual fund

The mutual fund owns all government bonds and firms in the economy. The no arbitrage condition across these two assets implies that the returns on investing in bonds and ownership of firms are equalized:

$$\frac{P^{e\prime} + D'}{P^e} = 1 + i.$$

It is assumed that all balances are redistributed as dividends to the shareholder, including profits and changes in the value of bond holdings i.e.,

$$D = B_{-1} - \frac{B}{1+i} + P\Pi^I + P\Pi^S,$$

where Π^I and Π^S denote the profits of the price setters and the firms operating in the service sector, respectively. Specifically, the profits of the intermediate producers are given by:

$$\Pi^I = \left(1 - p^l - \frac{\eta}{2\vartheta} \log\left(1 + \pi - \pi^*\right)^2\right) Y,$$

where $1 - p^l$ is the real marginal profits obtained from selling one unit of the differentiated product purchased at the relative price p^l, net of the Rotemberg costs of price adjustment. The profits of service sector are given by the period profits integrated across the employment distribution:

$$\Pi^S = \int \left[p^l x - w\left(x, \alpha\right)\right] d\mu_1^E\left(e, x, \alpha\right).$$

2.9 Market clearing and equilibrium

The goods market clearing condition requires that the aggregate demand of labor services from the intermediate producers equals supply

$$\int_0^1 y_i di \equiv Y = \int x d\mu_1\left(e, x, \alpha\right). \tag{16}$$

Moreover, the total demand for shares of the mutual fund, which is obtained by aggregating the optimal savings decisions across the workers distribution, must equal supply, which is normalized to unity:

$$\int g^U\left(e\right) d\mu_1^U\left(e\right) + \int g^E\left(e, x, \alpha\right) d\mu_1^E\left(e, x, \alpha\right) + \int g^R d\mu_1^R\left(e\right) = 1, \tag{17}$$

where g denotes the saving policy functions, i.e., the optimal choice of e' for every combination of $\{e, x, \alpha\}$ defined for each of the three labor market states, unemployment, employment and participation, respectively.

Finally, labor market clearing requires that the sum of the employed, unemployed and retirees equals unity, both at the beginning and at the end of a period:

$$\int d\mu_j^E\left(e, x, \alpha\right) + \int d\mu_j^U\left(e\right) + \int d\mu_j^R\left(e\right) = 1, \quad \text{for} j \in \{0, 1\}. \tag{18}$$

2.10 Computational strategy

In Appendix E, we describe in details the algorithms we use to solve for both the stationary equilibrium and the transitional dynamics.

3 Data

We combine various administrative records provided by Statistics Denmark. At the heart of our analysis are two data sets.

Wage payment data. The *Beskæftigelse for lønmodtagere* (BFL) registry contains the universe of wage payments. We use these to create employment spells. Each record contains the hours registered for a period and the gross paid earnings, together with a firm and worker identifier.

Social security data. *Ikke lønmodtagerdata fra E-Indkomst* (ILME) contains the universe of social security payments. We use these to create unemployment spells, and to compute unemployment and pension benefits. Each record contains a person identifier, a period, a benefit-type code and the corresponding payments. Individuals might receive multiple payments simultaneously.

Education data. *Uddanelser* (UDDA) contains for each individual and year the highest obtained degree. We exclude workers from our analysis that have not yet reached their highest obtained degree.

Job spells and job-to-job transitions. Consecutive wage payments within a worker-firm pair are considered a job spell.[2] We use unemployment benefit payments to define an unemployment spell.[3]

We measure job-to-job transitions as follows. Denote the month in which a worker-firm spell ends as t. If the worker has any other job spell that begins within $[t-1, t+1]$, we define that transition to be a job-to-job transition, as long as (i) the worker actually changes physical work places, and (ii) the worker receives no unemployment benefits within $[t-1, t+1]$. We allow for both overlapping transitions (where the next job begins before the previous has ended) and separated transitions (where there is a gap of up to one month between the two spells). Through the lens of the model, we consider the former job-to-job transitions since the worker found the subsequent job during the previous job (and thus the previous

[2] A gap in payments of a year or longer is considered a new spell. We consider single wage payments that are made more than 3 months after the previous payments “clearing payments” that could contain residual benefit or holiday payments, and remove them from the data so as to not incorrectly extend the duration of the spell.

[3] Here we follow closely the methodology established by Henning citation and since applied in a series of research on Danish employment spells, for example recently Rune paper on fiscal and Rune and Antoine RED.

job's earnings influenced the acceptance decision). Separated transitions might either be spells that actually went through unemployment/nonemployment (and the worker's outside option being considerably lower), or situations where the worker found the new job while still employed (and with the higher outside-option), but chose to time the beginning of the new spell in a manner that allows for additional leisure in between the two spells.[4] We decided to count these transitions as job-to-job transitions, but additionally use unemployment benefits (restriction (ii)) to limit counting spurious transitions as much as possible. Restriction (i) ensures that firm restructuring, mergers, and similar are not falsely measured as a job-to-job transition.

4 Calibration

We calibrate the stationary equilibrium of the model to the Danish economy at quarterly frequencies. Some parameters are assigned using conventional values in the literature, others are fitted directly from the data while the remaining ones are calibrated to match a number of moments from the Danish micro data.

With regards to functional forms, we assume a CES matching function, which ensures that the contact rates of both workers and vacancies do not exceed unity, i.e. $f(\theta) = \theta(1+\theta^{\xi})^{(-1/\xi)}$ and $q(\theta) = (1+\theta^{\xi})^{(-1/\xi)}$, where ξ is an elasticity parameter. The utility function is assumed to be logarithmic in consumption. The distribution of idiosyncratic productivity shocks is assumed to be lognormal, and defined by the mean and dispersion parameters ω_x and σ_x, with the restriction $\omega_x = -\sigma_x^2/2$, which implies that the mean of the distribution is normalized to one. The distribution of search costs is assumed to be uniform over the support $\left[\vartheta^l, \vartheta^u\right]$, where the lower bound ϑ^l is normalized to zero. The parameters governing the probability of dying and moving to the retirement state, ψ^D and ψ^R respectively, are chose in order to match an expected duration of retirement of twenty years and an expected duration of work life of forty, as in Birinci et al. (2023).

The elasticity of substitution between goods, η, is set to 6, which implies a markup of 25%, as estimated by Adam et al. (2024) for the Danish economy. The discount factor is set to .9875, as in Faccini et al. (2024). The marginal tax rates τ_L and τ_H are set to 0.4226 and 0.5606, which are the income tax rates in force in Denmark in 2012. The annual inflation target is set to 2%. The elasticity of the matching function, ξ, is set to 1.6, in line with estimates by Schaal (2017) on the US economy.

This leaves us with nine parameters to calibrate: the maximum share of output paid as

[4] For a fuller discussion we refer to **?** , who show that Danish workers expect time off after a voluntary separation, consistent with the notion that households plan additional leisure between job-to-job transitions.

Calibration			
Parameters	Description	Value	Target/source
β	Discount factor	0.9875	Faccini et al. (2024)
χ	Elasticity of substitution	6.0000	25% markup
ξ	Elasticity of CES matching function	1.6000	Schaal (2017)
ψ^D	Death probability	0.0125	40 years of work life
ψ^R	Retirement probability	0.00625	20 years of retirement
τ^H	High marginal tax rate	0.5606	Danish data
τ^L	Low marginal tax rate	0.4226	Danish data
τ^0	Low marginal tax rate	0.0800	Danish data
ζ	Scale parameter wage function	0.6850	Calibrated
δ	Job separation rate	0.0400	Calibrated
w^H	High income tax threshold	0.7200	Calibrated
b	Unemployment benefits	0.1500	Calibrated
T^R	Pension income	0.3000	Calibrated
κ	Flow cost of vacancy	0.0769	Calibrated
κ^f	Fixed cost of hiring	1.2800	Calibrated
ω_x	Mean parameter productivity dist.	-0.011	Normalization
σ_x	Dispersion parameter productivity dist.	0.1500	Calibrated
ϑ^l	Lower bound cost-search distribution	0.0000	Normalization
ϑ^u	Upper bound cost-search distribution	0.5000	Calibrated
φ	Slope of Phillips Curve	0.05	Hansen and Hansen (2007)
π^*	Steady-state gross inflation rate	1.0500	Inflation rate 2% p.a.
ϕ_π	Taylor rule response to inflation	1.8000	Conventional
ρ_τ	Autocorrel. tax shock	0.9500	Arbitrary
$100\sigma_\tau$	St. dev. tax shock	1.0000	Normalization
Variable	Description	Model	Target
	Steady-state calibration targets		
$\frac{c}{q(\theta)}/c^f$	Ratio of variable to fixed cost	0.0771	0.0780
$EE \equiv \frac{\int \xi(e,x,\alpha) f(\theta) d\mu_0^E(e,x,\alpha)}{\int \mu_0^E(e,x,\alpha)}$	EE transition rate	0.0270	0.0250
$\int w(x,\alpha) d\mu_1^E(e,x,\alpha)/Y$	Labor share of income	0.6123	0.6300
$\int \mathcal{I}_{w(x,\alpha)<w^H} d\mu_1^E(e,x,\alpha)$	Share of workers earning less than w^H	0.5358	0.5300
$f(\theta)$	UE transition rate	0.5000	0.2000
$\int \frac{w(x',\alpha')-w(x,\alpha)}{w(x,\alpha)} \mathcal{I}_{EE=1} d\mu_1(e,x,\alpha)$	Wage growth for leavers	0.0809	0.0800
$b/\int w(x,\alpha) d\mu_1^E(e,x,\alpha)$	Average unempl.- over empl.-income	0.4000	$0.yyyy$
$T^R/\int w(x,\alpha) d\mu_1^E(e,x,\alpha)$	Average pension- over empl.-income	0.4900	$0.yyyy$
$\int w(x,\alpha) d\mu_1^E(e,x,\alpha)/w^H$	Average empl. income over w^H	0.9649	1.1000

Table 1: Calibrated values for model parameters. Notes: EE stands for employment-to-employment.

wages, ζ; the job separation rate δ; the high income tax threshold w^H; the unemployment benefits parameter b; the transfer to retired workers T^R; the variable and fixed cost of posting a vacancy, κ, and κ^f, respectively, the upper bound of the uniform search-cost distribution ϑ^u, and the dispersion parameter σ_x of the idiosyncratic productivity shock process. These are calibrated in order to match: *(i)* A labor-income share of 63%; *(ii)* An unemployment rate of 6%; *(iii)* A UE transition rate of 50%; *(iv)* A ratio of total variable costs of hiring to fixed costs $\frac{c}{q(\theta)}/c^f$ equal to 0.078 as in Faccini and Melosi (2023)[5]; *(v)* An average increase

[5]This value is the ratio of pre-match recruiting, screening, and interviewing costs to post-match training

of wages of 8% conditional on experiencing an EE transition[6]; *(vi)* An EE transition rate of 2.5%; *(vii)* A ratio of unemployment income over average income of 33%; *(viii)* A ratio of average pensions to the average wage paid to the employed equal to 0.67; *(ix)* Average income equal to $0.96 \cdot w^H$.

As for the parameters that do not affect the stationary equilibrium of the model, we set the parameter governing the response to inflation in the Taylor rule to 1.5. The slope of the Phillips curve is set to 0.05.

5 The effects of a Danish tax reform: model vs. data

The model of Section 2 was designed to study how changes in the incentives to search on the job, triggered by tax changes, affect negotiated wages and inflation. In this Section, we engineer in the model a change in the high-income-tax threshold, akin to the one produced by the Danish tax reform of 2012, and illustrate the implications for EE transition rates and wage growth across the income distribution. We then compare model predictions across this set of untargeted moments with those that we estimate from the Danish microdata as a way to test for the direct mechanism of the model, whereby higher OJS induces higher wage pressures.

Studying tax brackets. Through the lens of our model, a decrease in the marginal tax rate raises the incentives to search on the job, as it increases the expected after-tax return to search, i.e., net wage growth. A shift in tax thresholds can significantly affect the marginal tax rate for some workers, while holding them constant for the rest of the population, ensuring that the change in tax brackets predominantly affects a small subset of the population, thus without raising worries about general equilibrium effects.[7]

Job search behavior changes starkly with income. To isolate effects stemming from the change in tax brackets, one cannot simply compare workers close to the threshold to those further away, since differences in job-search behavior might be driven by differences in income and not the differential effect through the effective marginal tax rate. Since the tax threshold affects all workers in a given year in the same manner, we resort to comparing workers before and after the change to tease out the causal effects of the change in marginal tax rates on worker's job search decisions.

costs in the U.S., following the analysis of Silva and Toledo (2009).

[6]Because in our model transitions are only up the ladder, we condition on workers experiencing an increase in wage upon changing job when computing this moment.

[7]Indeed, we will still consider general equilibrium effects throughout the analysis in this section.

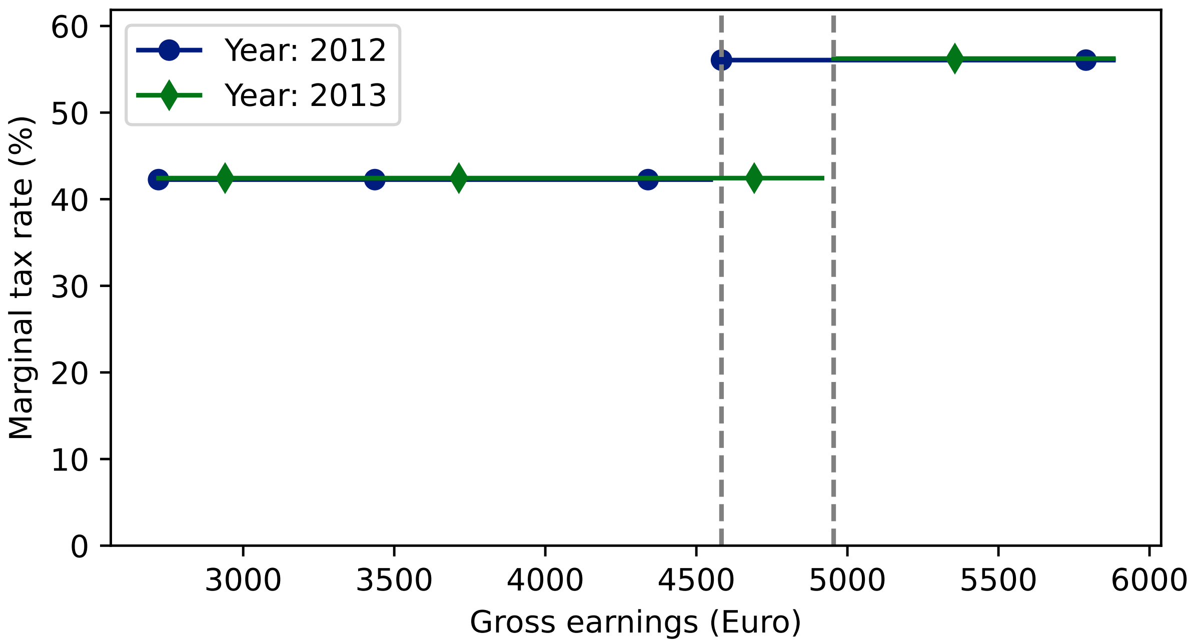

Figure 1: Top tax threshold - earnings are monthly

Danish tax schedules. Two income tax thresholds were present in Denmark from 2009 to 2012. In our analysis, we focus changes of the top tax threshold, which raises the marginal tax rate by 13.8%.[8] Before 2012, this threshold has undergone slight increases in most years, mostly to reflect (low) inflation. This top tax threshold underwent a major shift in 2012, when it was moved from 423,804 DKK to 457,609 DKK, a shift of about 8%.[9] Figure 1 illustrates this shift – denominated in monthly Euros. The dynamics around this threshold shift in 2013 make it attractive for studying job search shifts for three reasons. First, this threshold had been stable for several years in real terms. To the extent that threshold shifts induce job search and thus a change in the joint distribution over search intensity and annual income, previous stationarity of the threshold implies stationary of that joint distribution. Second, the large magnitude of the shift, which ensured high salience and a substantial increase in the returns to job-search.[10] Third, the period during which it happened. Our identification is based on comparing workers prior and post reform: if those workers exhibit

[8] The jump in the gross marginal tax rate for high income earners is 15%, but effectively becomes 13.8% after deducting labor market contributions.

[9] Information on tax rates and thresholds is available at: https://skm.dk/tal-og-metode/satser/tidsserier/centrale-beloebsgraenser-i-skattelovgivningen-2018-2024

[10] Searching for jobs might be associated with fixed costs – mental, household coordination, updating one's CV or references. Our model abstracts away from these since they are not irrelevant to the mechanisms studied, but such fixed costs might tamper the response of job-search to smaller changes in marginal tax rates.

differential behavior due to aggregate conditions unrelated to the change in tax threshold, identification would be threatened. The Danish economy was in a moderate upswing from 2011 to 2015, and we can confidently compare workers after the introduction of the reform to those before. Since the economy was steady for several years, we can also pool several years of data before and after the introduction in order to increase statistical power. Our analysis will show results for both shorter and longer time horizons.

Data. The goal is to study year-on-year changes in job-to-job transitions and annual wage growth around the top tax threshold. We use job spells and job-to-job transitions as described in section 3.

Our analysis will compare workers in the pre- and post reform period, where periods can be of either one, two, or three years. We focus on workers aged 25 to 65. Additionally, for each period, we include only workers that are full-time employed.[11]

For each worker, we compute annual labor earnings as total labor earnings in a given year across all job spells, including both wages and bonuses. Annual wages (hourly income) is computed by dividing annual earnings by annual hours.

5.1 Job search response to tax threshold shift

What changes in job search should we expect in response to the shift in the tax threshold? The top panel of Figure 2 illustrates model-based EE transition rates across the income distribution for the years 2012 and 2013, respectively. Focusing first on the 2012 cohort (the blue line), transition rates decrease as they approach the higher tax threshold (the vertical blue bar) from the left. Quite intuitively, increased proximity to this tax threshold decreases the after-tax returns to job search, since a higher share of the expected gains will be taxed at the higher marginal rate. To the right of the tax threshold, transition rates keep falling, albeit at a slower pace, as both the incentives to search on the job and the likelihood of reallocating to better matches tend to fall with income.

We can observe a very similar pattern for the 2013 cohort: EE rates fall as annual earnings approach the 2013 bracket from the left, and slowly recover to the right of that bracket. To a large extent, the pattern of 2013 transition rates mimics those of 2012, just translated upwards. The effect of the tax-bracket shift on EE rates is highlighted in the bottom panel, where we compute the percentage differences across the two cohorts: the outwards shift of

[11] We require annual hours worked that are within 5% of 1927. This restriction is consistent with Statistics Denmarks' definition of full-time employment, monthly hours of 160.6. Being non-employed for only one month in a given year already leads to extreme fluctuations in annual wage growth – focusing on full-time employment effectively deals with this.

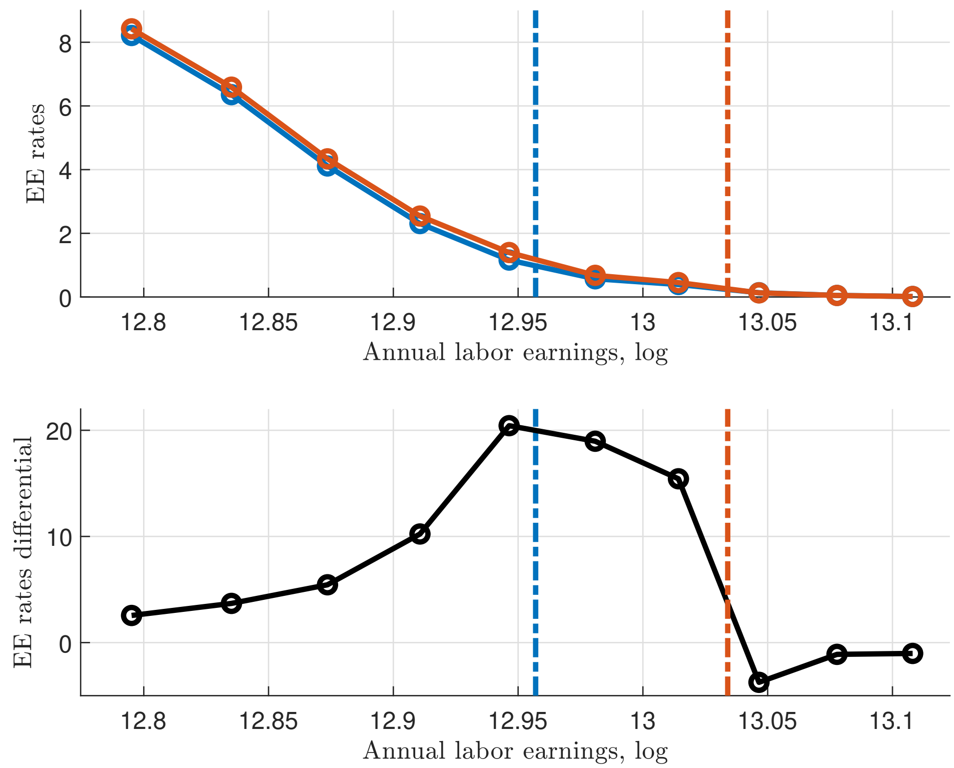

Figure 2: Effects of a shift of the high-income tax threshold in the HANK model on employment-to-employment (EE) transition rates. The upper panel shows EE rates across the income distribution for the years 2012 and 2013 (the blue and orange lines, respectively). These EE rates are computed in the stationary equilibrium of the model, and the only difference between the two calibrations comes from the increase in the tax threshold of 2013. The tax threshold for 2012 and 2013 are represented by the vertical bars in both panels. The lower panel shows the differential effects between the two EE schedules plotted in the upper panel.

the tax bracket increases EE transition rates (in percentage changes) when approaching the 2012 tax threshold from the left and then decreases them to its right, producing an inverse-V-shaped pattern. To the right of the 2013 brackets, the difference between EE transition rates in 2013 and 2012 falls into negative territory. This pattern for the response of EE rates is driven by the response of the share of employed workers searching on the job, as reported in Figure 9 in Appendix.

These results are quite intuitive. Any change in the high-income threshold would be irrelevant for workers with earnings far below it, since the higher earnings associated with a job-to-job transition would still be taxed at the lower marginal rate; hence workers in lower income bins should not exhibit a differential on-the-job search behavior before and after the reform. Similarly, workers who already in 2012 had incomes above the 2013 threshold would be subject to the same high-income-tax rate both before and after: so also for the workers in these high income bins, there should be no differential effect of the reform on job search behavior. The income group that is most strongly affected by the tax reform lies in between these two polar cases and, more precisely, around the old income-tax threshold. For these workers, the entire additional wage growth from a transition is taxed at a lower marginal rate after the reform, compared to the period that precedes it. These differential effects of the tax reform on the returns to on-the-job search across the income distribution lead to an inverse-V shape response in the share of employed job seekers.

The reason why differential EE rates move into negative territory to the right of the 2013 threshold is due to a change in the distribution of the employed workers over the state space. Due to lower marginal tax rates to the left of the high-income threshold, the income bin to the right of the 2013 threshold experience a higher EE inflow of workers from the bins to the left, which leads to a relatively lower concentration of low-income workers and therefore a lower share of employed job seekers.

To compute the data equivalent, we estimate

$$y_{i,t} = \text{after}_t + \sum_g \beta_g \mathbf{1}_{(\text{income } = \ g),i,t} + \gamma_g \mathbf{1}_{(\text{income } = \ g),i,t} \times \text{after}_t + X_{i,t} + \epsilon_{i,t}, \tag{19}$$

where $y_{i,t}$ is an outcome variable for individual i, after is a dummy variable which equals one in the period that follows the change in threshold of the first of January 2013, $\mathbf{1}_{income=g}$ is a dummy for the income bin g and X is a potential vector of control variables.

Let $y_{i,t}$ be a dummy whether worker i had a job-to-job transition in year t. The estimated $after_t$ dummies would then be the data equivalent to estimating the job-to-job transition rate along the income distribution.

Figure 3 shows the effects of a shift in the tax threshold on EE transition rates (the panels in the first row) and hourly income growth for the stayers (the panels on the second row). In each column of the figure, we report results for symmetric time windows centered around the first of January 2013, which is when the shift in the tax threshold took place. The one-year window results compare both outcome variables in 2013 relative to 2012. The two- (three-) year windows instead report the effects on the outcome variables averaged over 2013-2014 (2013-2015), relative to the period 2011-2012 (2010-2012). Results are presented for log-income bins in the range of 10% above and below the 2012 threshold along with 95% confidence-interval bands.

In line with the results of the HANK model, EE rates exhibit an inverse V-shape pattern, which peaks in the proximity of the 2012 income threshold. We note that the increase in EE transition rates is statistically significant across the three panels and that the accuracy of the inverse V-shape pattern increases with the accuracy of the estimates, as larger estimation windows exploit a larger sample of data.We also notice a tendency in the estimates to move towards the negative territory to the right of the 2013 threshold, which is also in line with the model results. Altogether, these results indicate that a lower rate of taxes increases

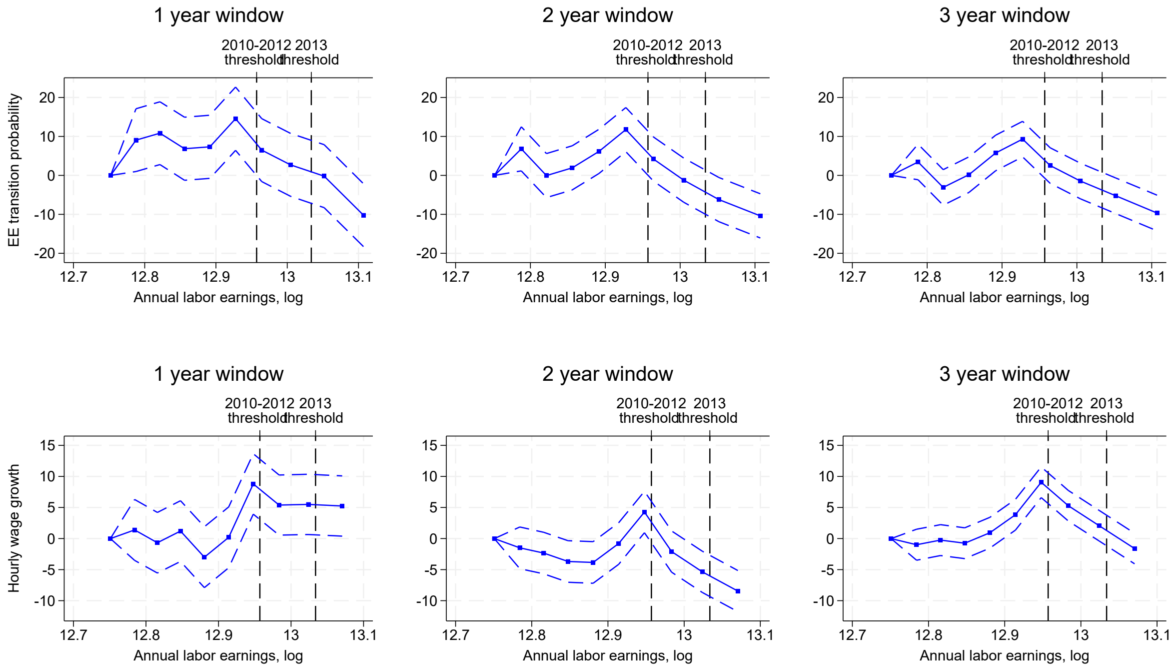

Figure 3: Difference-in-difference effect of a shift in the tax threshold on EE transition rates and hourly income for the stayers. Notes: the panels in the first row of the figure plot the response of employment-to-employment transition rates. The panels in second row plot the response of hourly wages. The one year window results in the first column report the differential behavior of the outcome variables in 2013 relative to 2012. The two (three) year window results report differential outcomes for the outcome variables averaged of 2013-2014 (2013-2015) relative to 2011-2012 (2010-2012). Confidence interval represent 95% error bands. Standard errors are clustered at the level of earning bins.

on-the-job search and EE transitions. Quantitatively, this effect is strong: for the income bin where the effect is strongest, EE rates increase by 10%, i.e., from about 4% to 4.4% a year.

5.2 Wage growth of stayers

What does the rightward shift in the tax threshold imply for wage growth across the distribution? A lower threshold affects wage growth through two channels. On the one hand, the increase in EE transitions shown in Figure 2 implies a proportionally higher share of workers leaving to higher paying jobs. On the other hand, wage growth increases also for the stayers, since a higher share of on-the-job search implies a higher arrival rate of wage offers that are matched by the current employer. Figure 4 shows that the effect of a shift in the tax threshold produces an inverse-V shaped response in the wage growth of the stayers, which, as in the case of EE rates in Figure 2, peaks around the 2012 threshold and turns in negative territory to the right of the 2013 threshold.

It is worth noting that while the a higher share of EE transitions contributes positively

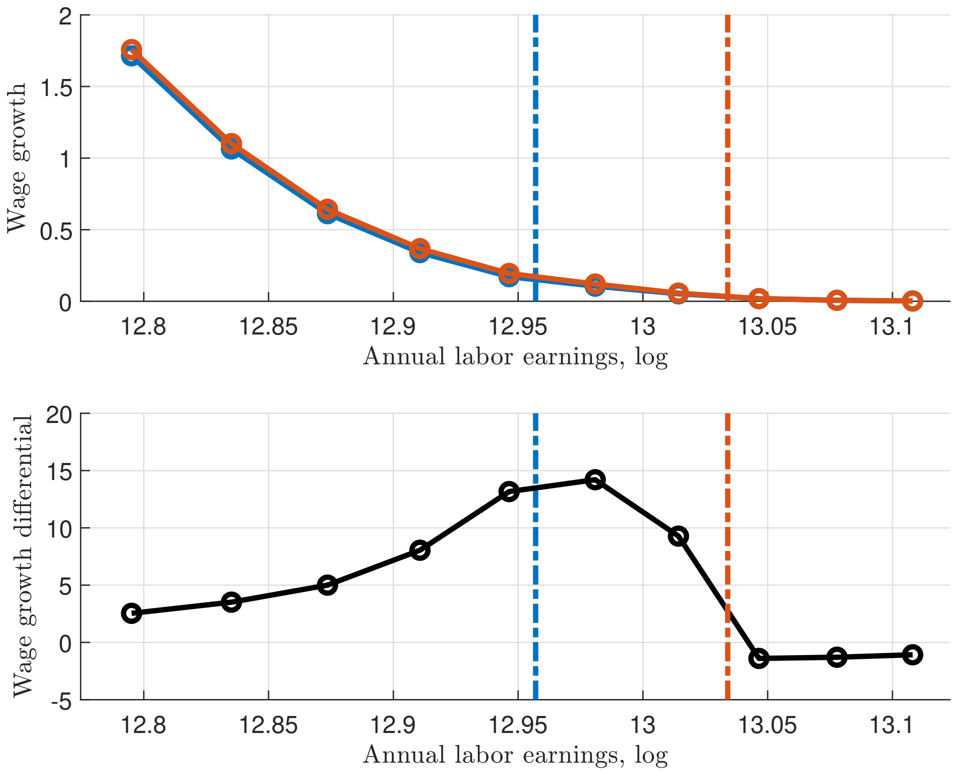

Figure 4: Effects of a shift of the high-income tax threshold in the HANK model on the wage growth of the stayers. The upper panel shows wage growth across the income distribution for the years 2012 and 2013 (the blue and orange lines, respectively). These rates of wage growth are computed in the stationary equilibrium of the model, and the only difference between the two calibrations comes from the increase in the tax threshold of 2013. The tax threshold for 2012 and 2013 are represented by the vertical bars in both panels. The lower panel shows the differential effects between the two wage growth schedules plotted in the upper panel.

to wage growth, conditional on experiencing an EE transition, the leavers exhibit the same rate of wage growth in 2013 as in 2012, given that they draw from the same productivity distribution. So for the leavers, the causal effect of a shift in the tax threshold across the income distribution is just a flat line (see Figure 10 in Appendix). We also notice that the inverse-V shaped pattern of responses for the wages of the stayers produced by the model would not arise under the alternative assumption of Nash bargaining. Because a fall in taxes raises match surplus, gross wages would have to fall for firms to keep their surplus share constant, thereby giving rise to an opposite response of wages to taxes.

The second row of Figure 3 reports results for the hourly wage growth of the stayers. These estimates reflect relatively closely the responses observed for EE rates. Namely, the patterns tend to increase significantly in the proximity of the 2012 threshold, and the accuracy of the inverse V-shaped pattern increases with the size of the windows. Quantitatively, the results are also strong: the peak effect on wage growth is also estimated to be around 10%, i.e. wage inflation increases from $xx\%$ to $xx\%$. We emphasize that this increase in

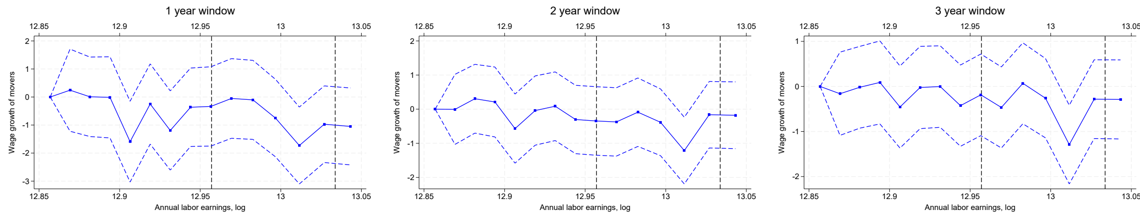

Figure 5: Difference-in-difference effect of a shift in the tax threshold on EE transition rates for the leavers. Notes: the one year window results in the first column report the differential behavior of the outcome variables in 2013 relative to 2012. The two (three) year window results report differential outcomes for the outcome variables averaged of 2013-2014 (2013-2015) relative to 2011-2012 (2010-2012). Confidence interval represent 95% error bands. Standard errors are clustered at the level of earning bins.

wages applies to the stayers, i.e., that 80% of the workers' population who did not experience changes in jobs and the associated match-productivity in a given year. Hence, according to our identification strategy, the tax threshold induced a material cost-push shock on the workers employed in the vicinity of the tax-threshold.

The results of Figure 3 are the outcome of estimates that do not control for any observable. As such, the estimated differences across the two periods may conflate variation in other variables. To assuage potential concerns, we have estimated the same regressions after residualizing both EE rates and wages for age groups, gender, education, industry and occupation. The results, reported in Figure 11 in the Appendix reveals very similar patterns to those estimated without controls. Namely, EE rates and wages for the stayers exhibit inverse V-shaped patterns peaking in the proximity of the 2012 threshold. These results indicate that treatment and control groups in each bin are relatively similar.

5.3 Wage growth of movers

Does the tax change affect the wage growth of workers who undergo a job-to-job transition?

We compute differential effects for the wages of the workers who experience an EE transition. The results, reported in Figure 5, show no significant effect, in line with the outcomes of the model, where, conditional on leaving, workers search from the same distribution.

5.4 Placebo exercise

To ensure that our results are indeed due to the 2013 change in the tax threshold and not due to other factors that may correlate with the income distribution, we create placebo experiments on neighboring years. Here we expect no significant findings around the tax threshold since it remained constant throughout these placebo periods.

Figure 6 reports the results for these placebo experiments, whereby we compare the response of EE transition rates in 2011 relative to 2010 (panels in the first row) and 2012

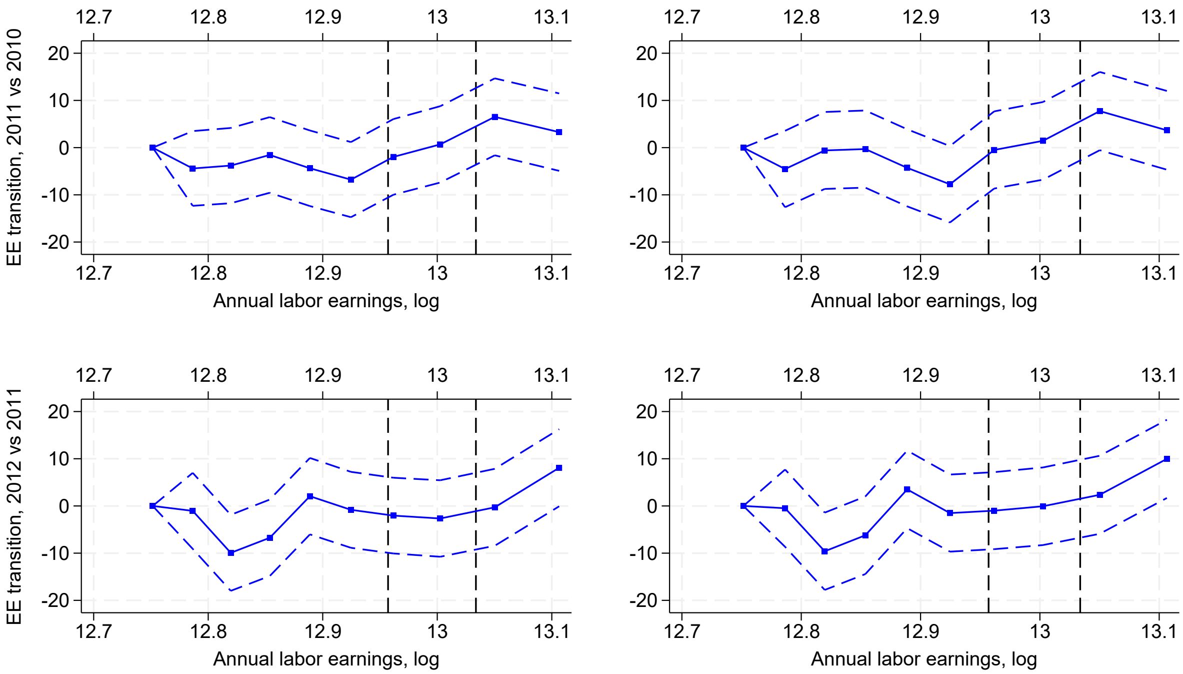

Figure 6: Placebo experiment: difference-in-difference effect of a shift in the tax threshold on EE transition rates. The vertical bars represent the high-income tax thresholds for the years 2012 and 2013. The upper and lower panels refer to changes over the years 2011 vs 2010 and 2012 vs. 2011, respectively. The panels in the first (second) column report results not controlling (controlling) for household characteristics, respectively.

relative to 2011 (panels in the second row). Panels to the left and right of each row report results for the raw and residualized EE data, respectively. Because the threshold tax rate for high income earners remained unchanged over the 2010-2012 period, the difference-in-difference results should show no differential outcomes across the treatment and control periods, which is precisely what the figure illustrates.

The computation of the effects of a change in the tax threshold based on the HANK model, and reported in Figure 7, implicitly assumes that changes in the tax threshold affects workers' incentives to search for jobs only in 2013 and not already in 2012, i.e., that responses to the change in threshold were not anticipated. However, the tax reform was already announced in May 2012, so it is indeed possible that workers responded to the announcement well before the beginning of 2013. Yet, it takes time to process new tax information and take decisions to change jobs. Moreover, even after one decides to look for jobs, it takes time before finding a suitable offer. In Denmark, the average duration of unemployment is about xx months. Arguably, it should take even longer to find jobs for the employed, given that they have less time available to search and they would selectively consider only those vacancies that offer at least a similarly valuable match. Hence, it is reasonable to believe that most of the workers seeking to increase their earnings to take advantage of lower marginal

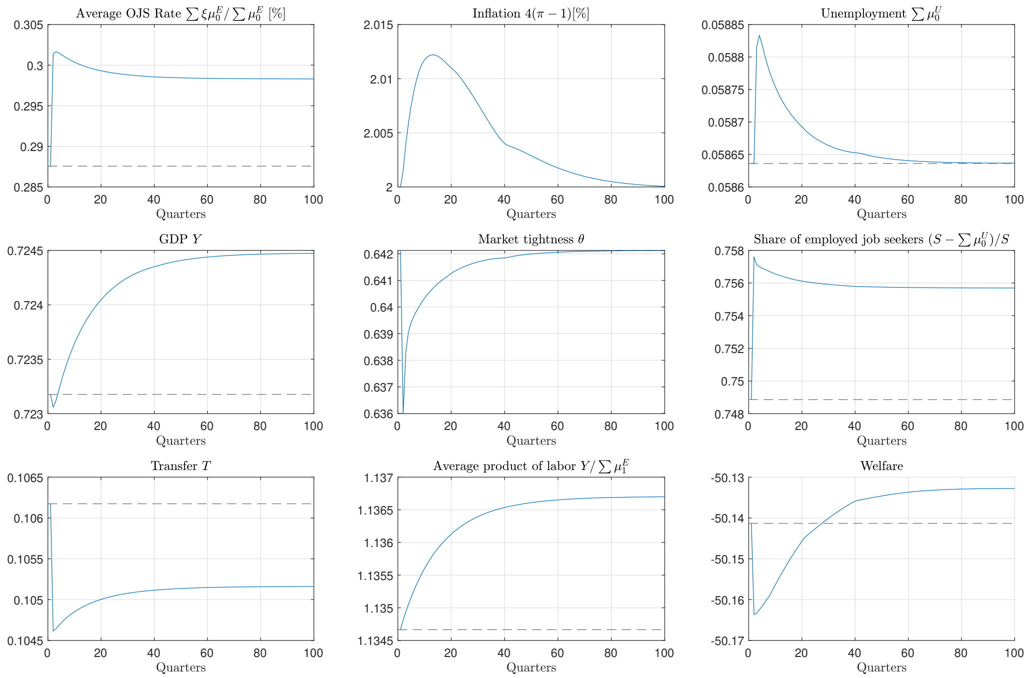

Figure 7: Impulse responses to a 8% increase in the tax threshold for high-income earners.

tax rates, would have been able to do so only in 2013. That said, it is still possible that some workers managed to shift jobs and get higher earning already before the end of 2012. To the extent that that is the case, our estimated increase of earnings for the stayers is biased downwards, and hence should be regarded as conservative.

6 General equilibrium effects

6.1 A shock to the high-income tax threshold

The results presented in the previous section indicate that the model mechanism through which the tax shock propagates to wage inflation via OJS produces quantitatively reasonable responses across the income distribution. We now turn to investigate the effects of the tax reform on macroeconomic aggregates such as unemployment, output and price inflation, as well as the welfare implications. We do so by computing the transition path to the new stationary equilibrium featuring an eight percent permanent increase in the high-income tax threshold w^H, as dictated by the 2012 tax reform. The responses to this policy are reported in Figure 7.

The tax reform lowered the marginal tax rate faced by the workers with incomes close to the 2012 high-income tax threshold, leading to a permanent increase in the average rate of OJS. In turn, this generates a persistent increase in both unemployment and inflation, as shown in the first row of Figure 7.

We notice that the shift in the tax threshold induces a fall in average government revenues. Because of the assumption that the government budget is balanced in every period, lower revenues are offset in the model by lower transfers to households. Because transfers are identical across all workers, while taxes are reduced only for the rich, this income redistribution contributes to a fall in aggregate demand, due to the higher marginal propensities to consume of the poor. These contractionary effects on aggregate demand are, however, not of first-order importance for the main propagation, as shown quite clearly by the inflation response.

The mechanism leading to a simultaneous rise in both inflation and unemployment works as follows. The increase in OJS raises the share of employed job seekers. The expected return to posting a vacancy falls, given that firms are able to extract a lower share of surplus when meeting an employed worker, relative to an unemployed one. Intuitively, the employed are more expensive to hire, given that their bargaining position is higher; unlike the unemployed, employed workers can spark wage competition between poachers and incumbent employers. Wage pressures rise, and higher expected wage payments in the labor market are reflected into higher real marginal costs for the price-sector firms, which in turn are passed through to higher prices, increasing the rate of inflation persistently over the impulse response horizon.

Output falls on the impact of the shock, driven by the fall in employment. Over time though, the increase OJS rate leads to a more efficient allocation of workers up the ladder, raising the average product of labor in a way that more than compensates for the persistent fall in employment.

A permanent increase in the high-income tax rate produces an *a-priori* ambiguous impact on welfare. In the long run, it increases the efficient allocation of workers on the ladder as well as the costs of OJS. In transition to the new steady state, it persistently reduces employment. At the calibrated equilibrium, this policy increases welfare in the long run, but decreases it persistently in the transition. In the model, OJS produces a negative externality on job creation, as it increases the share of the employed in the pool of job seekers, thereby raising the expected cost of entry. By lowering the value of searching on the job, a higher income tax allows the employed to internalize this negative externality on job matches.

The impulse responses in Figure 7 imply that in the calibrated equilibrium, the opposite policy to the one considered above, i.e., a lowering of the high-income tax threshold, would simultaneously reduce unemployment and inflation, while temporarily increasing welfare. Quantitatively though, the effects of this particular policy are negligible. This is not surpris-

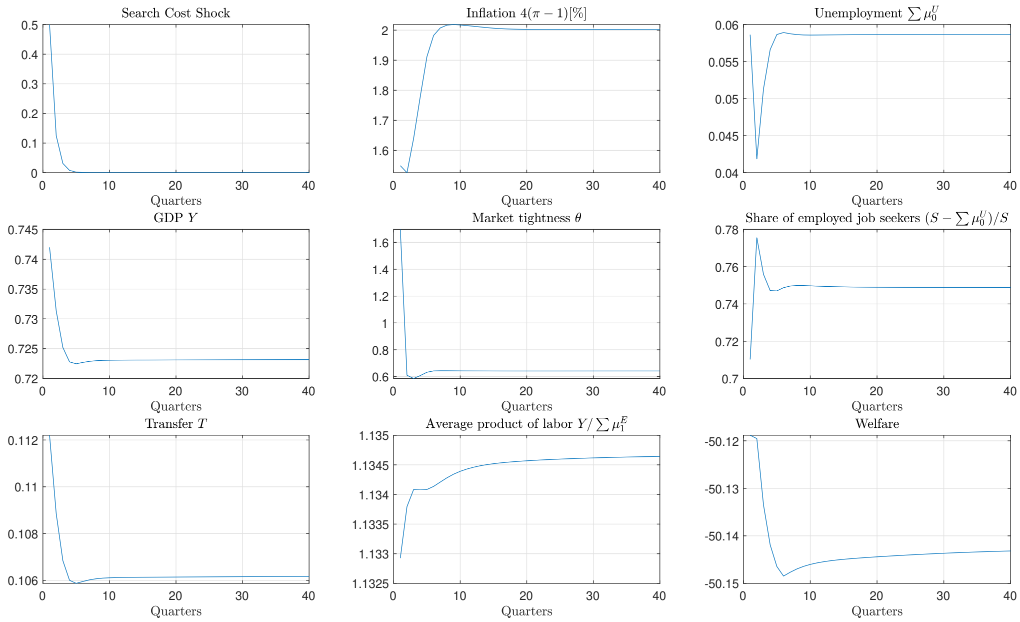

Figure 8: Impulse responses to an increase in the upperbound parameter of the search-cost distribution.

ing, given that this policy only affects a small share of high-income workers located close to the threshold, so the average rate of OJS increases only marginally. The general equilibrium effect of this policy are naturally small, and too small to be retrieved in the microdata. In the next section, we show that an alternative policy, which increases OJS costs for all the workers, has the potential to generate quantitatively strong results.

6.2 A shock to the cost of on-the-job search

We now study the effects of a large temporary shock to the cost of searching on the job. We think of this shock as a simple way of capturing a temporary regulatory restriction to OJS. We keep the lowerbound of the cost-shock distribution $\vartheta^l = 0$ and assume that the upperbound follows the process $\vartheta^u_t = \rho_\vartheta \vartheta^u_{t-1} + \epsilon_t$, where we set the autocorrelation coefficient $\rho_\vartheta = 0.25$ and the shock on impact to equal 0.5, which doubles the value of ϑ^u relative to steady-state.

The impulse responses to a positive cost shock are reported in Figure 8. As shown by the panel in the top-left corner, the shock is short-lived, and almost entirely gone by the end of the fourth quarter. The higher search cost produces a simultaneous fall in unemployment and inflation. Inflation decreases, reflecting the fall in hiring costs, and hence a cheaper labor service. At the same time, the fall in the share of job seekers, by lowering the expected cost of hiring, increases labor market tightness, and reduces unemployment. The resulting increase in employment, more than compensates for the decline in productivity, leading to an increase in output. In turn, higher production increases government revenues, which are transferred as a lump-sum to the households in order to maintain the budget balanced. Welfare increases, driven by the increase in employment.

Even though the fall in the share of employed job seekers is short-lived, its effects on inflation and unemployment are quantitatively large, as they fall by about 0.5 and 1.5 percentage points, respectively. We note that these results are derived from a relatively simple model that was designed to highlight OJS as a transmission channel to price inflation. As such, any quantitative result, and in particular any welfare consideration, should be simply taken as indicative of the potency of the channel within the model.

7 Conclusions

We have developed a HANK model with a job ladder and endogenous OJS search to study how the search decisions of the employed respond to tax incentives and what are the implications for wage and price inflation. We have produced impulse responses of EE rates and wages across the income distribution and compared model outcomes with estimates based on Danish microdata to validate the mechanism of the model. Our findings that higher OJS increases negotiated wages not just for the leavers but also for the stayers provides evidence in favor of the sequential auction bargaining protocol. Moreover, the strong response of EE rates and wage growth for the stayers, both in the model and in the microdata, suggests that the search behavior of the employed matters for inflation dynamics. The general equilibrium dynamics generated by the model suggest that policies targeting the incentives to search on the job may induce a positive comovement between inflation and unemployment rates, eluding the traditional Phillips-curve tradeoff.

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A Laws of motion

Define $\mathcal{E}_t^E(e'; e, x, \alpha) = \left\{e \in \mathcal{E} : g^E(e, x, \alpha) = e'\right\}$, $\mathcal{E}_t^U(e'; e) = \left\{e \in \mathcal{E} : g^U(e) = e'\right\}$ and $\mathcal{E}_t^R(e'; e) = \left\{e \in \mathcal{E} : g^R(e) = e'\right\}$ denote the set of period-t share holdings e that map into a given level of next-period share holdings e' by employment status, through the policy functions g.

Intertemporal law of motion for the employed

$$\mu_{0,t+1}^E(e', x', \alpha') = \left(1 - \psi^R\right)(1 - \delta)\,\mu_{1,t}^E(e', x', \alpha'), \tag{20}$$

Intratemporal law of motion for the employed

$$\begin{aligned}
\mu_{1,t}^E(e', x', \alpha') = &\sum_{e \in \mathcal{E}_t^E} \mu_{0,t}^E(e, x', \alpha')\left[\left[1 - \xi(e, x', \alpha') f(\theta)\right] + \xi(e, x', \alpha') f(\theta) \sum_{\widetilde{x} < x'\alpha'} G^x(\widetilde{x})\right] \\
&+ \sum_{\alpha}\sum_{e \in \mathcal{E}_t^E} \mu_{0,t}^E(e, x', \alpha)\,\xi(e, x', a) f(\theta) G^x(x'\alpha')\,\mathbf{1}_{x'\alpha' > x'\alpha} \\
&+ \sum_{\alpha}\sum_{e \in \mathcal{E}_t^E} \mu_{0,t}^E\left(e, \underbrace{\alpha' x'}_{x}, \alpha\right)\xi(e, \alpha' x', a) f(\theta) G^x(x') \\
&+ \sum_{e \in \mathcal{E}_t^U} \mu_{0,t}^U(e) f(\theta) G^x(x')\,\mathbf{1}_{\alpha' = \frac{x}{x'}}
\end{aligned} \tag{21}$$

The first raw in the above expression refers to the employed workers who do not search for jobs, or, if they search and find a job, they get an outside offer that is too low to renegotiate the wage with the current employer.

The second row refers to the employed workers who find a job leading to renegotiate their wage at the current employer such that they extract a share α' of the incumbent's productivity x.

The third row refers to workers who are employed in some job with productivity x, search for a job and find one that leads them to shift to a different employer of productivity x', and such that they extract exactly a share α' of the poacher's productivity.

The fourth row refers to the unemployed workers who match with a job with productivity x', and such that the share of output paid as wages is exactly $\alpha' = \underline{x}/x'$.

Intertemporal law of motion for the unemployed

$$\mu^U_{0,t+1}(e') = \left(1-\psi^R\right)\mu^U_{1,t}(e') + \left(1-\psi^R\right)\delta\sum_{\alpha}\sum_{x}\sum_{e\in\mathcal{E}^U_t}\mu^E_{1,t}(e,x,\alpha) + \psi^D\sum_{e\in\mathcal{E}^R_t}\mu^R_{1,t}(e) \tag{22}$$

Intratemporal law of motion for the unemployed

$$\mu^U_{1,t}(e') = \sum_{e\in\mathcal{E}^U_t}\mu^U_{0,t}(e)\left[1-f(\theta)\right] \tag{23}$$

Intertemporal law of motion for the retirees

$$\mu^R_{0,t+1}(e') = \left(1-\psi^D\right)\sum_{e\in\mathcal{E}^R_t}\mu^R_{1,t}(e) + \psi^R\sum_{e\in\mathcal{E}^U_t}\mu^U_{1,t}(e) + \psi^R\sum_{x,\alpha,e\in\mathcal{E}^E_t}\mu^E_{1,t}(e,x,\alpha) \tag{24}$$

Intratemporal law of motion for the retirees

$$\mu^R_{1,t}(e') = \mu^R_{0,t}(e') \tag{25}$$

B Growing net earnings

Because of log utility, the marginal utility of consumption is dC/C, and because consumption equals after tax wages, the returns from search depend on the percentage increase in net earnings. Let w, T and $\hat{w}$ denote gross earnings, the tax bill, and net earnings, respectively. We study a given tax threshold $\overline{w}$, which increases the marginal tax rate from τ_0 to τ_1. We focus on the area area above the tax threshold, $w > \overline{w}$.

$$T(w) = \tau_0\overline{w} + (w-\overline{w})\tau_1$$
$$\hat{w}(w) = w - \tau_0\overline{w} - (w-\overline{w})\tau_1,$$

Noting that the marginal tax rate is indeed $\hat{w}'(w) = 1-\tau_1$. But what is the percentage change in net wage $N(w,p)$, as we increase gross wage by factor p?

$$
\begin{aligned}
N(w,p) &\equiv \frac{\hat{w}(pw) - \hat{w}(w)}{\hat{w}(w)} \\
&= \frac{pw - \tau_0\overline{w} - (pw - \overline{w})\tau_1 - [w - \tau_0\overline{w} - (w - \overline{w})\tau_1]}{w - \tau_0\overline{w} - (w - \overline{w})\tau_1} \\
&= \frac{(p-1)w - (p-1)w\tau_1}{w - \tau_0\overline{w} - (w - \overline{w})\tau_1} \\
&= \frac{(p-1)(1-\tau_1)w}{w - \tau_0\overline{w} - (w - \overline{w})\tau_1}
\end{aligned}
$$

How does this change with w?

$$
\begin{aligned}
\frac{\partial N(w,p)}{\partial w} &= \frac{(p-1)(1-\tau_1)[w - \tau_0\overline{w} - (w - \overline{w})\tau_1] - (p-1)(1-\tau_1)w(1-\tau_1}{[w - \tau_0\overline{w} - (w - \overline{w})\tau_1]^2} \\
&= \frac{(p-1)(1-\tau_1)w(1-\tau_1) + (p-1)(1-\tau_1)[-\tau_0\overline{w} + \tau_1\overline{w}] - (p-1)(1-\tau_1)(1-\tau_1)}{[w - \tau_0\overline{w} - (w - \overline{w})\tau_1]^2} \\
&= \frac{(p-1)(1-\tau_1)\overline{w}(\tau_1 - \tau_0)}{[w - \tau_0\overline{w} - (w - \overline{w})\tau_1]^2},
\end{aligned}
$$

which is strictly positive since $p > 1$, $\tau_1 < 1$, $\tau_1 > \tau_0$.

C Additional figures HANK model

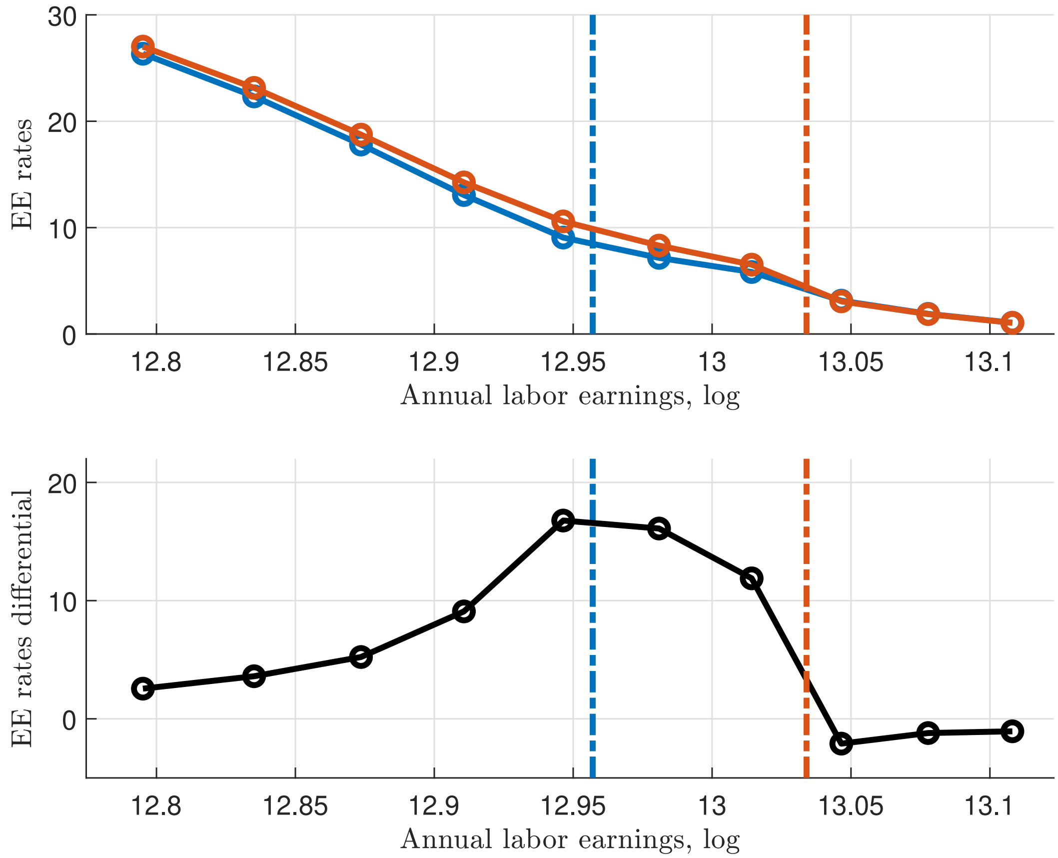

Figure 9: Effects of a shift of the high-income tax threshold in the HANK model on the rate of on-the-job search (OJS). The upper panel shows the OJS rate across the income distribution for the years 2012 and 2013 (the blue and orange lines, respectively). These rates of search are computed in the stationary equilibrium of the model, and the only difference between the two calibrations comes from the increase in the tax threshold of 2013. The tax threshold for 2012 and 2013 are represented by the vertical bars in both panels. The lower panel shows the differential effects between the two search rate schedules plotted in the upper panel.

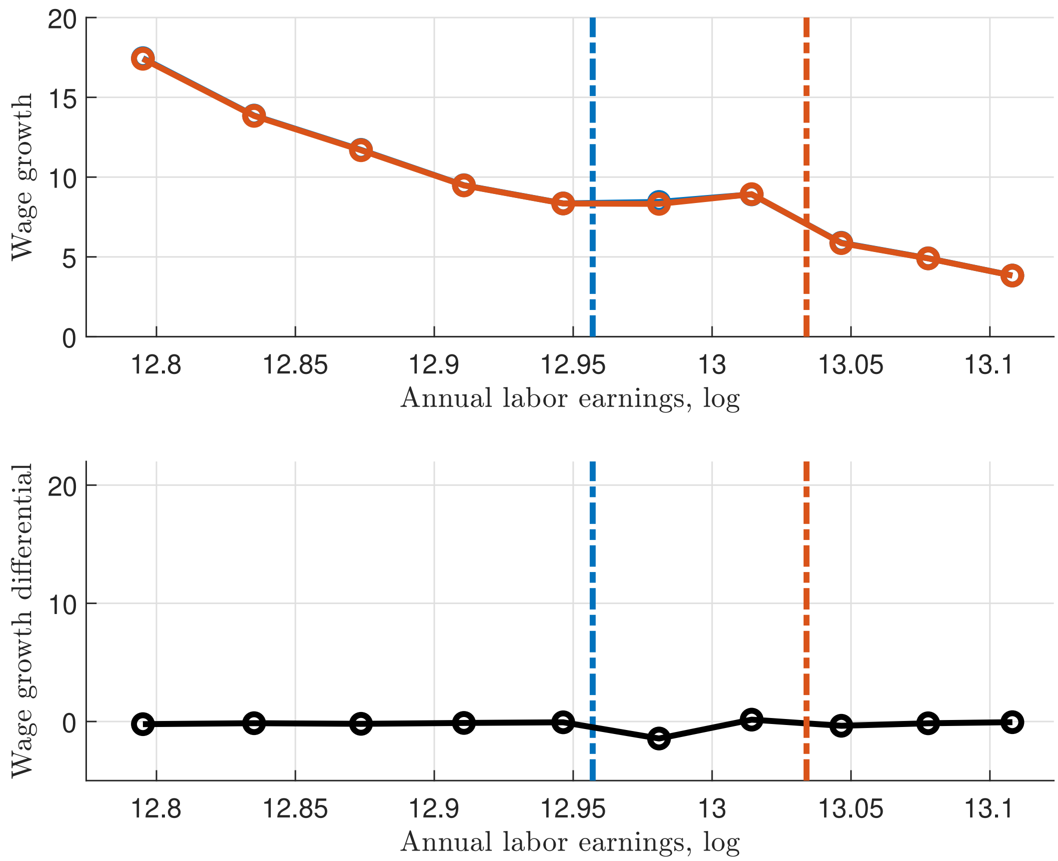

Figure 10: Effects of a shift of the high-income tax threshold in the HANK model on the wage growth of the leavers. The upper panel shows wage growth across the income distribution for the years 2012 and 2013 (the blue and orange lines, respectively). These rates of wage growth are computed in the stationary equilibrium of the model, and the only difference between the two calibrations comes from the increase in the tax threshold of 2013. The tax threshold for 2012 and 2013 are represented by the vertical bars in both panels. The lower panel shows the differential effects between the two wage growth schedules plotted in the upper panel.

D Additional figures empirical

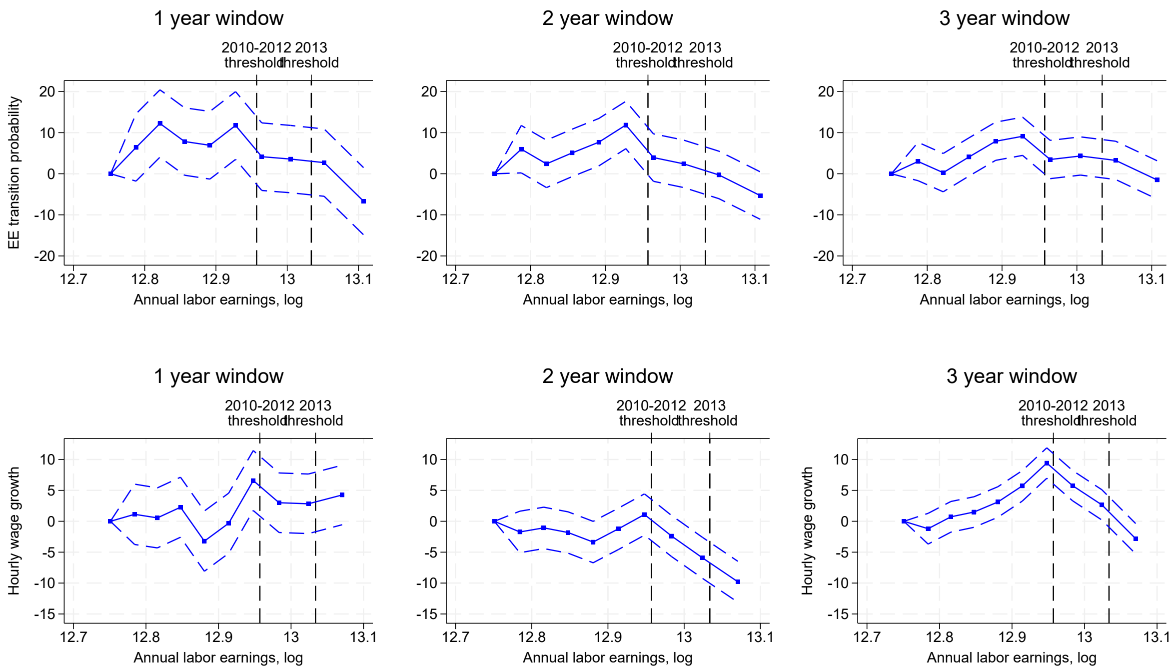

Figure 11: shift in the tax threshold - top: transitions. bottom: wage growth of stayers

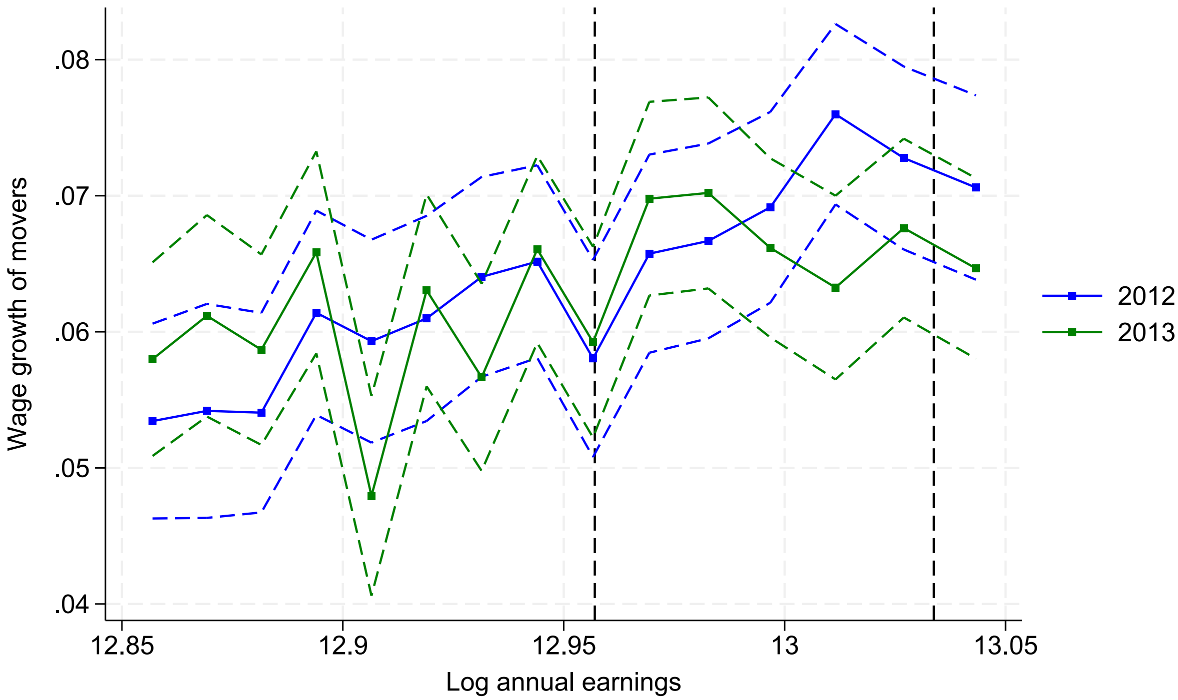

Figure 12: movers single year

E Computational Appendix

In this section, we describe the algorithms we use to solve for the stationary equilibrium and the transitional dynamics.

E.1 Solution algorithm for the stationary equilibrium

We create the following three grids. Namely, the exponentially scaled assets grid $A = [\underline{e}, e_1, ..., \bar{e}]$, the log-normally distributed productivity grid $\mathcal{X} = [\underline{x}, x_1, ..., \bar{x}]$, and the linearly scaled piece rate grid $P = [\underline{\alpha}, \alpha_1, ..., 1]$, where $\underline{\alpha}$ is the minimum possible piece rate $\underline{x}/\bar{x}$. The population density distributions are $\mu^U_{p,t}(e), \mu^R_{p,t}(e)$, and $\mu^E_{p,t}(e, x, \alpha)$ for period $p \in \{0, 1\}$. We use 17 nodes on each of the three grids, for a total of 17^3 nodes. We use piece-wise linear interpolation to evaluate both policy and value functions outside of the nodes of the grids. We do not use a grid for the search cost ϕ since, given a uniform probability distribution function, we can compute all the relevant integrals analytically using a continuous support.

We compute a wage grid $w = \zeta \cdot P \times \mathcal{X}$, where ζ is the maximum share of output as wages and $\times$ indicates the Cartesian product. We use the three taxation brackets τ_0, τ_L, and τ_H to create a measure of average taxation in function of income w:

$$\tau = \begin{cases} \tau_0, & \text{if } w \leq w_L \\ \dfrac{w_L \cdot \tau_0 + (w - w_L) \cdot \tau_L}{w}, & \text{if } w_L \leq w \leq w_H \\ \dfrac{w_L \cdot \tau_0 + (w_H - w_L) \cdot \tau_L + (w - w_H) \cdot \tau_H}{w}, & \text{otherwise.} \end{cases} \tag{26}$$

The algorithm works as follows.

1. Create an iterator z and set $z = 0$. Guess initial values for the real rate of interest r^z.

2. Create a second iterator j and set $j = 0$. Guess the initial transfer T^j.

    - Create a third iterator w and set $w = 0$.

        (a) Use guesses for all value functions: $\Gamma^w(e)$ for retired, $U^w(e)$ for the unemployed, $V_0^w(e, x, \alpha)$ for start-of-period, and $V_1^w(e, x, \alpha)$ for end-of-period value of employment to solve the associated optimization problems (8), (2), (3), and (4) and find $\Gamma^{w+1}(e), U^{w+1}(e), V_0^{w+1}(e, x, \alpha)$, and $V_1^{w+1}(e, x, \alpha)$.

        (b) Update the job search policy function for the employed population, $I^{w+1}_{\phi<\phi^T}(e, x, \alpha)$ using equations (6) and (7) and evaluate the job search probability $\xi^{w+1}(e, x, \alpha)$ based on the job search decisions for the employed.

        (c) Using $e'_E(e, x, \alpha), r^z$, and $\xi^{w+1}(e, x, \alpha)$, calculate the value of a filled job $J^{w+1}(e, x, \alpha)$ using equation (9).

(d) If all value functions converged (i.e. $\max(\sup|\Gamma^{w+1}(e)-\Gamma^{w}(e)|, \sup|U^{w+1}(e)-U^{w}(e)|, \sup|V^{w+1}(e,x,\alpha)-V^{w}(e,x,\alpha)|, \sup|J^{w+1}(e,x,\alpha)-J^{w}(e,x,\alpha)|) < \epsilon$), exit the loop. Otherwise, set $w = w + 1$ and restart from step (a).

- Create an iterator t and set $t = 0$. This step uses the policy functions to solve for the asymptotic distributions. We simulate using the Young (2010) lottery method when the policy functions contains value outside of the nodes of the grids.

  (a) Use the intratemporal laws of motion, calculate the population distribution density for period $p = 1$, $\mu^{E}_{1,t}(e^{'}, x^{'}, \alpha^{'}), \mu^{U}_{1,t}(e^{'})$, and $\mu^{R}_{1,t}(e^{'})$ from the guess for period $p = 0$, $\mu^{E}_{0,t}(e, x^{'}, \alpha^{'}), \mu^{U}_{0,t}(e)$, and $\mu^{R}_{0,t}(e)$ using equations (21), (23), and (25).

  (b) Using the results from step (a), $\mu^{E}_{1,t}(e, x, \alpha), \mu^{U}_{1,t}(e)$, and $\mu^{R}_{1,t}(e)$ and the intertemporal laws of motion, calculate the population distribution function for period 0 for $t + 1$, $\mu^{E}_{0,t+1}(e^{'}, x^{'}, \alpha^{'}), \mu^{U}_{0,t+1}(e^{'})$, and $\mu^{R}_{0,t+1}(e^{'})$ using equations (20), (22), and (24).

  (c) If the population distributions converge (i.e. $\max(\sup|\mu^{U}_{0,t+1}-\mu^{U}_{0,t}|, \sup|\mu^{R}_{0,t+1}-\mu^{R}_{0,t}|, \sup|\mu^{E}_{0,t+1}-\mu^{E}_{0,t}|, \sup|\mu^{U}_{1,t+1}-\mu^{U}_{1,t}|, \sup|\mu^{R}_{1,t+1}-\mu^{R}_{1,t}|, \sup|\mu^{E}_{1,t+1}-\mu^{E}_{1,t}|) < \epsilon$), exit the loop. Otherwise, set $t = t + 1$ and restart from step (a).

- Calculate transfer T^{j+1} using the values for wages w and the population density distributions $\mu^{U}_{1}(e'), \mu^{R}_{1}(e')$, and $\mu^{E}_{1}(e', x', \alpha')$ using the government budget constraint (13). If transfers converged (i.e. $|T^{j+1} - T^{j}| < \epsilon$), then exit the loop. Otherwise, set $j = j + 1$, update the value of T^{j} towards T^{j+1} using a dampening parameter and restart. We use a dampening parameter of 0.9, i.e. $T^{j+1} \leftarrow 0.9 \cdot T^{j+1} + (1 - 0.9) \cdot T^{j}$.

3. Calculate the savings aggregated across all workers and evaluate the asset market clearing condition (17). If the asset market clearing condition is satisfied then exit the loop. Otherwise, set $z = z + 1$ and restart from step (2). Use a bisection algorithm to find the value of real interest rate r that clears the asset market.

E.2 Solution algorithm for the dynamic equilibrium

The economy is initially in a stationary equilibrium when all agents experience a sudden tax shock $\Delta\tau_{t=0}$ at time $t = 0$. This tax shock reverts back to zero over time with a constant persistence, P. We solve for the transition numerically, allowing a sufficiently high number of periods $\bar{t}$ for the shocks to fade away and economy converge to the stationary equilibrium. In particular, we use $\bar{t} = 200$. In order to calculate the equilibrium dynamics, we need to find sequences of: (i) government transfer, $\{T_t\}_{t=0}^{\bar{t}}$, (ii) market tightness parameter, $\{\theta_t\}_{t=0}^{\bar{t}}$, and (iii) real interest rates, $\{r_t\}_{t=0}^{\bar{t}}$.

1. Create an iterator j and set $j = 0$. Guess an interest rate path $\{r_t^j\}_{t=0}^{\bar{t}}$. Using the Taylor Rule (14), calculate the associated inflation path $\{\pi_t^j\}_{t=0}^{\bar{t}}$.

2. Create an iterator t and set $t = \bar{t} - 1$. Hence, use projection with backward time iteration from $t = \bar{t} - 1$ to $t = 0$. The policy functions at $t = \bar{t}$ are the ones associated with the ending stationary equilibrium as previously calculated. At each time $t = 0$, we proceed similarly as before in the case of stationary equilibrium. Start from guessed paths $\{T_t^j\}_{t=0}^{\bar{t}}$ and $\{\theta_t^j\}_{t=0}^{\bar{t}}$ using the stationary equilibrium values.

   - Calculate consumption for unemployed, employed, and retired population, $C_t^U(e), C_t^E(e, x, \alpha)$, and $C_t^R(e)$ after having update the average taxation level generated by the tax shock $\Delta\tau_t$ calculated, at each time t, from equation (26).
   - Start from the stationary equilibrium value functions and iterate backward on the optimization problems (8), (2), (3), and (4) to find $\{\gamma^t(e)\}_{t=0}^{\bar{t}}$ for retired, $\{U^t(e)\}_{t=0}^{\bar{t}}$ for the unemployed, $\{V_0^t(e, x, \alpha)\}_{t=0}^{\bar{t}}$ for start-of-period, and $\{V_1^t(e, x, \alpha)\}_{t=0}^{\bar{t}}$ for end-of-period value of employment.

3. Now, start from $t = 0$ and iterate forward up to $t = \bar{t}$. Start at $t = 0$ from the $p = 0$ distributions of the initial stationary equilibrium $\mu_{0,0}^E(e, x, \alpha), \mu_{0,0}^U(e)$, and $\mu_{0,0}^R(e)$.

   (a) Use the intratemporal laws of motion to calculate the population distribution density for period $p = 1$, $\mu_{1,t}^E(e, x, \alpha), \mu_{1,t}^U(e)$, and $\mu_{1,t}^R(e)$ from the $p = 0$, $\mu_{0,t}^E(e, x, \alpha), \mu_{0,t}^U(e)$, and $\mu_{0,t}^R(e)$ using equations (21), (23), and (25).

   (b) Use the results from step (a), $\mu_{1,t}^E(e, x, \alpha), \mu_{1,t}^U(e)$, and $\mu_{1,t}^R(e)$ and the intertemporal laws of motion to calculate the population distribution functions for $p = 0$ for $t + 1$, $\mu_{0,t+1}^E(e, x, \alpha), \mu_{0,t+1}^U(e)$, and $\mu_{0,t+1}^R(e)$ using equations (20), (22), and (24).

4. Iterate backward again from $t = \bar{t} - 1$ to $t = 0$.

   - Retrieve stored policy decisions and population distributions generated in the previous steps to calculate the value of filled job $\{J^t(a, x, \alpha)\}_{t=0}^{\bar{t}}$ at each time t using equation (9).

5. Iterate forward again from $t = 0$ to $t = \bar{t}$.

   - Calculate transfers $\{T_t^{j+1}\}_{t=0}^{\bar{t}}$ from wages and the population density distributions using equation (13).

- Evaluate the market clearing condition (17) at each time t. Update $\{r_t^j\}_{t=0}^{\bar{t}}$ to get $\{r_t^{j+1}\}_{t=0}^{\bar{t}}$ using the residuals on all asset market clearing conditions.
- Calculate the market tightness path $\{\theta_t^{j+1}\}_{t=0}^{\bar{t}}$ using equation (10).

6. If all market clearing conditions are satisfied and the government transfer and market tightness paths converged, and real interest rates (i.e. $\max(\sup|\{r_t^{j+1}\}_{t=0}^{\bar{t}} - \{r_t^j\}_{t=0}^{\bar{t}}|, \sup|\{T_t^{j+1}\}_{t=0}^{\bar{t}} - \{T_t^j\}_{t=0}^{\bar{t}}|, \sup|\{\theta_t^{j+1}\}_{t=0}^{\bar{t}} - \{\theta_t^j\}_{t=0}^{\bar{t}}) < \epsilon)$, stop. Otherwise, set $j = j + 1$, shift the values for $\{r_t^{j+1}\}_{t=0}^{\bar{t}}$, $\{T_t^{j+1}\}_{t=0}^{\bar{t}}$, and $\{\theta_t^{j+1}\}_{t=0}^{\bar{t}}$ using a dampening parameter and restart from step (2).