Taxing Entrepreneurs and Workers

A Linear Optimization Approach for Multidimensional Screening*

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Abstract

We study optimal taxation in economies with general equilibrium market clearing, where agents with privately known labor skills and entrepreneurial abilities choose between deterministic labor income and risky firm operation. The government observes labor income and realized dividends but not effort, or technology shocks. We formulate the multidimensional screening problem as a lottery-based linear optimization, accounting for global incentive constraints, fixed costs and other non-convexities. Optimal policies exhibit tax breaks, which can render net taxes negative, for agents with intermediate entrepreneurial abilities and labor skills above a threshold. General equilibrium strengthens this effect under decreasing returns, as labor-market clearing requires sufficient entry into entrepreneurship, further increasing subsidies for agents with high worker options. In a calibrated U.S. economy, optimal taxes are lower and can be negative for low-profit realizations. Subsidies rise when risk declines and when the frequency of high-ability entrepreneurs in the population diminishes.

JEL Codes: D82, E60, H21, H24, H25, L26.

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1 Introduction

Entrepreneurial income is both risky and highly concentrated, with a small share of founders earning large rewards. At the same time, entrepreneurial choices have significant general-equilibrium effects. Firm entry and expansion raise labor demand, affect equilibrium wages and aggregate output. This creates a policy tension that is muted in partial equilibrium. A government that values redistribution and insurance may wish to tax high business incomes aggressively, implement transfers financed by the proceeds, and provide substantial protection against entrepreneurial risk. Yet such policies can also weaken entry incentives for high-productivity entrepreneurs, reducing labor demand and — through equilibrium wage adjustments — lowering aggregate labor income. Because labor skill, entrepreneurial ability, and effort are privately known, and fiscal instruments can condition only on realized labor income and realized firm profits, optimal tax and transfer design must balance redistribution and entry incentives, not only to affect entrepreneurs' behavior, but because the occupational allocation of talent is itself a determinant of economy-wide earnings and welfare.

This paper studies optimal taxation in economies where individuals differ along two dimensions—labor skill and entrepreneurial ability—choose ex ante between paid work and operating a firm, and entrepreneurs face idiosyncratic productivity risk. The government observes labor income and realized dividends, but not effort, entrepreneurial skills, or shock realizations. Optimal policies can be characterized only after solving mechanism-design problems with multidimensional private information, occupational choice, and risk-contingent observables. A key implication is that redistribution and insurance cannot be designed independently of the allocation of talent across occupations: taxes affect entry and firm operation, which in turn affect labor demand, wages, and output in general equilibrium.

Our main results shed light on reasons why a utilitarian planner may tolerate—and even engineer—pockets of high entrepreneurial payoffs. First, the optimal allocation exhibits sharp sorting in the two-dimensional type space. For each labor skill level in the population, there is a threshold in entrepreneurial ability above which agents choose to run firms. This threshold increases with the labor skill, because being capable of generating more labor income requires larger informational rents to be induced to run a firm. As a result, optimal taxes make entrepreneurship locally attractive at the extensive margin. The planner grants information rents to marginal entrepreneurs to prevent high-ability agents from selecting into labor and thus sustain socially valuable entry into entrepreneurship.

Second, these entry incentives have significant general-equilibrium effects. With decreasing returns, even the most productive firms, run by the most skilled entrepreneurs, cannot absorb the entire aggregate labor supply on their own. The labor market clearing must generate a sufficiently large extensive margin of entrepreneurship, in order to stimulate entry by the most talented agents. This amplifies the entry-margin tax break for agents who are close to indifferent between working and operating a firm.

Third, the distribution of entrepreneurial talent and labor skills in the population determines who receives these subsidies. The largest entry subsidies accrue to agents with the highest options as workers. Although the planner is utilitarian, the presence of private information requires granting rents to these marginal entrepreneurs, which must then be financed by higher net taxes elsewhere. Put differently, in order to support job creation and stimulate output the planner must treat agents with intermediate entrepreneurial skills more favorably than others.

Our analysis is organized in three steps. First, we introduce the model. Agents differ along two dimensions, labor skill and entrepreneurial ability, face fixed participation costs in both occupations, and choose between paid work and operating a firm. Entrepreneurs hire labor at the market clearing wage and are subject to idiosyncratic productivity shocks that are realized after all occupational choices are made. The government observes only realized labor income and realized dividends. Therefore any feasible tax policy cannot be based on any productivity shock realizations, or the agents' types, their efforts as workers and their choices as entrepreneurs. The planner's problem is a two-dimensional screening problem with risk-contingent observables, where the wage is determined endogenously by the labor market clearing. We compute optimal tax policies using a discrete-lottery linear optimization framework that accommodates a two-dimensional type space and entrepreneurial risk. Unlike approaches that rely on smoothness or convexity, our linear optimization framework handles nonconvexities—such as fixed entry costs—and delivers a characterization of allocations and tax schedules in environments where first-order methods are unreliable. Using our framework we can quantify the impact that features of the environment – such as the curvature of the production function, the joint distribution of abilities, and the magnitude of entrepreneurial risk – have on optimal tax policies.

Second, to build intuition we analyze a tractable special case that highlights the mechanism behind the optimal entry subsidy. We show that, with only two entrepreneurial skill levels and no heterogeneity on the labor side, when the high-talent incentive constraint binds, the planner must distort the occupational menu in favor of entrepreneurship. Productive entrepreneurs receive positive information rents, consuming more than workers. These rents are not merely redistributive; they also weaken entrepreneurial effort. In our general-equilibrium setting, this reduces productivity, output, and wages and requires higher labor supply from workers. The analysis of this special case thus delivers a transparent trade-off: relaxing screening constraints requires shifting resources toward marginal entrepreneurs, but the same shifts propagate through labor-market clearing and can lower aggregates. It also yields sharp comparative statics. Screening distortions are smaller when production is more labor-intensive, and are larger when high-talent entrepreneurs become scarce.

Third, we calibrate the model to U.S. moments on labor force participation, entrepreneurship, firm size, and the dispersion of business income, and solve for optimal taxes and transfers in general equilibrium. The calibrated optimum features sharp occupational sorting and a highly localized entry-margin subsidy: tax liabilities fall discretely—and often turn negative—right where individuals are close to switching into entrepreneurship. Entrepreneurial risk matters both because the planner values insurance and because state-contingent taxes relax incentive constraints. Taxes are higher in high-profit states, while subsidies expand in low-profit states where the temptation to select into labor is strongest. As highlighted in the analysis of the special case, when high-ability entrepreneurs are scarcer, subsidies extend deeper into entrepreneurship (even in high-profit states) to sustain labor demand. We also find that when labor and entrepreneurial skills are positively correlated (the "superstars" scenario), subsidies concentrate at the very top, whereas in the opposite scenario (negative correlation) subsidies shift toward entrepreneurs with weaker worker outside options.

Related literature. This paper contributes to the literature on optimal taxation and mechanism design in environments with multidimensional private information and endogenous occupational choice. Multidimensional screening problems have been studied first by Armstrong (1996), Rochet and Choné (1998), and Manelli and Vincent (2007). These papers highlight that in multidimensional screening problems without any specific structure, the optimal mechanism tends to be highly sensitive to the belief of the designer. More recent work in public finance includes Kleven et al. (2009), Scheuer (2014), Rothschild and Scheuer (2013, 2014), Bergstrom and Dodds (2021), Boerma et al. (2022), Dodds (2024), Bierbrauer et al. (2024), Spiritus et al. (2025), and Golosov and Krasikov (2025).[1] These papers examine different economic problems (e.g., taxation of couples or multi-sector abilities), and identify analytically tractable solutions. The main distinguishing feature of our work is the numerical analysis of the general model, which relies on a linear formulation that can account for global incentive constraints as well as non-convexities generated by the economic primitives. Most closely related to our work are Prescott and Townsend (1984), Scheuer (2014), and Boerma et al. (2022).

It is well known that optimal allocations in multidimensional environments may exhibit global phenomena such as bunching and binding nonlocal incentive constraints which cannot be handled by local first-order methods. Solving large-scale problems of this kind requires identifying all binding incentive constraints, which is analytically challenging and computationally demanding, especially so in settings with nonconvexities. Prescott and Townsend (1984) introduced pioneering techniques to represent nonlinear incentive problems as linear optimization problems using lottery-based transformations (see, e.g., Phelan and Townsend 1991, Doepke and Townsend 2006, and Myerson 2013). However, this approach requires introducing a large number of control variables. Our paper combines this discrete-lottery formulation with well-known constraint generation techniques, and leverages modern large-scale linear programming solvers (such as Gurobi or CPLEX), to solve the multidimensional screening problem by accounting for all incentive constraints numerically. As such, our analysis provides a tractable way to enforce full global incentive compatibility in multidimensional type spaces, while allowing for non-smooth primitives and non-convexities that naturally arise in economic applications, such as endogenous occupational choices with entry costs.[2]

Our approach is also related to recent work by Boerma et al. (2022) who obtain an alternative linear representation to compute optimal tax mechanisms in multidimensional settings. Their

[1]Beyond these closest connections, our paper relates to the broader literature on taxing entrepreneurial income under one-dimensional private information; see, e.g., Shourideh (2012); Piketty et al. (2014); Ales et al. (2017).

[2]Carlier et al. (2024) also develops a solution algorithm applicable to all (including non-convex) quasi-linear multidimensional screening problems.

representation hinges on Legendre-type transformations of underlying convex functions. On the one hand, our discrete-lottery formulation does not rely on any underlying convexities, allowing us to incorporate empirically relevant features such as fixed costs.[3] On the other hand, the computational cost grows faster with the problem size in our setting than in theirs, as our lottery representation requires introducing a number of controls that grows proportionally to the grid size.

Finally, Scheuer (2014) studies optimal taxation in a general-equilibrium model with endogenous occupation choice by agents who are privately informed about their labor productivity and their cost of entering entrepreneurship. Our theoretical framework builds on his seminal model, with the following differences. First, the multidimensional screening problem in Scheuer (2014) is simplified by the assumption that the two components of each agent's privately known type enter in the payoff function separably.[4] This implies that all meaningful incentive constraints depend only on a one-dimensional privately known variable. By contrast, due to single-crossing conditions on both labor and entrepreneurial payoffs implied by the multiplicative interaction between efforts and abilities, the incentive constraints in our model involve explicitly both privately-known skill types.[5] Second, as is common in the heterogeneous firm macroeconomic literature (e.g. Khan and Thomas 2013), we introduce idiosyncratic productivity shocks in the production function. Finally, as described earlier, we use a lottery-based linear optimization representation of the planner's problem.

Structure. The remainder of the paper is organized as follows. Section 2 presents the full model. Section 3 develops the discrete-lottery linear formulation. Section 4 analyzes a tractable special case of the model to clarify the mechanism with analytical results. Section 5 describes the calibration and reports the quantitative results. Section 6 concludes.

2 Model

In this section we describe the model. We allow for heterogeneity in labor skills and entrepreneurial abilities, fixed participation costs in both occupational choices, and idiosyncratic firm-level productivity shocks.

2.1 Overview and notation

Types. There is a unit mass of agents with privately known type

$$(s, z) \in \mathcal{S} \times \mathcal{Z} \subset \mathbb{R}^2_+,$$

[3] While it may be possible to extend the approach of Boerma et al. (2022) to solve the particular application that we study here, we view our paper as a proof of concept for a larger class of economic problems.

[4] Specifically, the entrepreneurial characteristic that is privately known is the fixed cost of operating a firm, which enters in an additive separable manner in the payoff function.

[5] Rothschild and Scheuer (2013, 2014) analyze models of endogenous sorting across tasks or sectors; but these papers do not focus explicitly on entrepreneurial income.

distributed with joint density $f(s, z)$. We interpret s as idiosyncratic labor productivity ("skill") and z as idiosyncratic entrepreneurial productivity.

Labor market and wage. There is a competitive labor market with wage $w > 0$ per efficiency unit of labor. Labor supplied by workers and labor hired by firms are measured in efficiency units.

Occupational choice and private actions (exclusive occupations). Each agent chooses *one* occupation *ex ante*, before the realization of entrepreneurial risk. The agent either (i) works as a worker by choosing labor effort $\ell \geq 0$, or (ii) operates a firm by choosing entrepreneurial effort $e \geq 0$ and labor demand $n \geq 0$ (efficiency units) to hire in the competitive labor market. Occupations are mutually exclusive:

$$\ell > 0 \;\Rightarrow\; (n, e) = (0, 0), \qquad \text{and} \qquad n > 0 \;\Rightarrow\; \ell = 0.$$

Labor effort ℓ (if working), entrepreneurial effort e (if operating), and labor demand n (if operating a firm) are privately chosen and unobservable to the government.

Fixed participation costs. Working requires a fixed participation cost $\kappa_1 \geq 0$ whenever $\ell > 0$. Operating a firm requires a fixed cost $\kappa_2 \geq 0$ whenever $n > 0$.

Preferences. Preferences are separable in consumption and effort:

$$u(c, \ell, e) = U(c) - V(\ell) - V(e), \tag{1}$$

where $U' > 0$, $U'' < 0$ and $V' > 0$, $V'' > 0$. The concavity of U implies risk aversion, so insurance against entrepreneurial risk is valued.

Observables and policy instruments. The government observes realized labor income y and realized entrepreneurial dividend/profit d, but does *not* observe (s, z), ℓ, e, n, nor the entrepreneurial shock separately. A tax/transfer schedule $T(y, d)$ is feasible. Equivalently, the government can use a direct mechanism that assigns consumption as a function of the observable outcomes (y, d).

2.2 Technology, income, and entrepreneurial risk

Labor income. If the agent works (i.e. $\ell > 0$), observed labor income is

$$y \;=\; w \cdot s \cdot \ell \;-\; \kappa_1 \cdot \mathbf{1}[\ell > 0]. \tag{2}$$

If the agent does not work ($\ell = 0$), then $y = 0$.

Entrepreneurial technology. If the agent operates a firm ($n > 0$), output is

$$q \;=\; z \cdot \varepsilon \cdot e^{\alpha} \cdot n^{1-\alpha}, \qquad \alpha \in (0,1), \tag{3}$$

where $e \geq 0$ is entrepreneurial effort and $\varepsilon > 0$ is an idiosyncratic productivity shock. Dividends/profits (observable) are

$$d \;=\; q - w \cdot n - \kappa_2 \cdot \mathbf{1}[n > 0] \;=\; z \cdot \varepsilon \cdot e^{\alpha} \cdot n^{1-\alpha} - w \cdot n - \kappa_2 \cdot \mathbf{1}[n > 0]. \tag{4}$$

If the agent does not operate ($n = 0$), then $d = 0$.

Shock timing and information. The entrepreneurial shock ε is realized after the occupational choice and reporting. The government does not observe ε separately; it only observes its implications through realized dividends d.

Shock distribution. Entrepreneurial risk is i.i.d. across agents and independent of (s, z). We assume ε is lognormal:

$$\log \varepsilon \sim \mathcal{N}(\mu_\varepsilon, \sigma_\varepsilon^2), \qquad \varepsilon > 0, \qquad \mathbb{E}[\varepsilon] = 1,$$

so that $\mu_\varepsilon = -\frac{1}{2}\sigma_\varepsilon^2$.

Mutual exclusivity in observables. Exclusive occupations imply that an agent cannot simultaneously have positive labor income and positive entrepreneurial dividends. Hence implementable outcomes satisfy

$$y > 0 \;\Rightarrow\; d = 0, \qquad \text{and} \qquad d > 0 \;\Rightarrow\; y = 0. \tag{5}$$

We restrict attention to allocations satisfying (5).

Risk and insurance. Because d is observed, the tax/transfer schedule can condition transfers on realized dividends. Hence the planner can provide insurance against entrepreneurial risk, subject to incentive constraints.

2.3 Implementable allocations and constraints

Direct mechanism. A (deterministic) direct mechanism specifies, for each report $(\hat{s}, \hat{z})$, an intended occupational outcome and a mapping from realized observables to consumption:

$$c = c(y, d).$$

Under truthful reporting, a type (s, z) receives the allocation intended for (s, z); under a misreport $(\hat{s}, \hat{z})$ it receives the allocation intended for $(\hat{s}, \hat{z})$.

Implied labor effort from a labor-income assignment. If the intended outcome assigns positive labor income $y(\hat{s}, \hat{z}) > 0$ (hence, by (5), $d = 0$), then a true type (s, z) must supply effort

$$\ell \;=\; \frac{y(\hat{s}, \hat{z}) + \kappa_1 \mathbf{1}[y(\hat{s}, \hat{z}) > 0]}{w \cdot s}. \tag{6}$$

If $y(\hat{s}, \hat{z}) = 0$, then $\ell = 0$.

Implied entrepreneurial effort from a dividend realization. If the intended outcome assigns entrepreneurial activity (i.e. $d > 0$ and hence $y = 0$), then for a true type (s, z) and realized shock ε the observed dividend d must be generated by some pair of private inputs (e, n) satisfying (4), or equivalently

$$z \cdot \varepsilon \cdot e^{\alpha} n^{1-\alpha} \;=\; d + wn + \kappa_2, \qquad e \geq 0,\; n \geq 0. \tag{7}$$

For any given $n > 0$, (7) uniquely pins down the entrepreneurial effort required to deliver dividend d:

$$e(n; d, z, \varepsilon, w) \;=\; \left(\frac{d + \kappa_2 + wn}{z \cdot \varepsilon \cdot n^{1-\alpha}} \right)^{1/\alpha}. \tag{8}$$

Hence many (e, n) pairs can rationalize the same observed d. To obtain a unique "implied effort" mapping from dividends to disutility, we impose a selection rule: among all feasible (e, n) satisfying (7), the agent chooses the pair that minimizes entrepreneurial effort e (and thus minimizes the effort disutility $V(e)$).[6] Minimizing (8) over $n > 0$ yields the least-effort labor demand

$$n^*(d; w) \;:=\; \frac{1-\alpha}{\alpha} \cdot \frac{d + \kappa_2}{w} \quad \text{for } d > 0, \tag{9}$$

and substituting $n^*(d; w)$ into (7) gives the associated implied entrepreneurial effort

$$e^*(d; z, \varepsilon, w) \;:=\; \left(\frac{d + \kappa_2 + w \cdot n^*(d; w)}{z \cdot \varepsilon \cdot \big(n^*(d; w)\big)^{1-\alpha}} \right)^{1/\alpha}. \tag{10}$$

We set $e^*(0; z, \varepsilon, w) = 0$ and $n^*(0; w) = 0$.

Implied effort costs. Define implied worker effort from labor income y as

$$\ell(y; s, w) := \begin{cases} \dfrac{y + \kappa_1}{ws}, & y > 0, \\ 0, & y = 0, \end{cases}$$

and implied entrepreneurial effort from dividends d as

$$e(d; z, \varepsilon, w) := \begin{cases} e^*(d; z, \varepsilon, w), & d > 0, \\ 0, & d = 0. \end{cases}$$

[6] Because V is strictly increasing, minimizing $V(e)$ is equivalent to minimizing e.

Aggregate resource constraint. With a continuum of agents, cross-sectional averages coincide with expectations. In what follows, we therefore use expectation notation over the entrepreneurial shock interchangeably with cross-sectional averages. The aggregate resource constraint is

$$\int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon\big[c\big(y(s,z), d(s,z,\varepsilon)\big)\big]\ f(s,z)\,ds\,dz \ \le\ \int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon[y(s,z) + d(s,z,\varepsilon)]\ f(s,z)\,ds\,dz. \tag{11}$$

where g denotes the density of ε and where $d(s,z,\varepsilon) = 0$ for agents who do not operate a firm.

Labor market clearing. Total effective labor supplied equals total labor hired:

$$\int_{\mathcal{S}\times\mathcal{Z}} s\cdot\ell(s,z)\cdot f(s,z)\,ds\,dz = \int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon\big[n^*(d(s,z,\varepsilon); w)\big]\cdot f(s,z)\,ds\,dz. \tag{12}$$

Incentive compatibility (ex ante). Truthful reporting must be optimal. Fix a true type (s,z) and consider any report $(\hat{s},\hat{z})$. Under a report $(\hat{s},\hat{z})$, the induced observables are $(y(\hat{s},\hat{z}), d(\hat{s},\hat{z},\varepsilon))$, and the true type incurs the corresponding effort costs implied by $\ell(\cdot; s, w)$ and $e(\cdot; z, \varepsilon, w)$.

The incentive constraints require that for all (s,z) and all reports $(\hat{s},\hat{z})$,

$$\begin{aligned} &\mathbb{E}_\varepsilon\big[U\big(c(y(s,z), d(s,z,\varepsilon))\big) - V\big(e(d(s,z,\varepsilon); z,\varepsilon,w)\big)\big] - V\big(\ell(y(s,z); s,w)\big)\ \ge \\ &\mathbb{E}_\varepsilon\big[U\big(c(y(\hat{s},\hat{z}), d(\hat{s},\hat{z},\varepsilon))\big) - V\big(e(d(\hat{s},\hat{z},\varepsilon); z,\varepsilon,w)\big)\big] - V\big(\ell(y(\hat{s},\hat{z}); s,w)\big). \end{aligned} \tag{13}$$

2.4 Planner's problem

Let $W(s,z) \ge 0$ be welfare weights. The planner chooses a tax/transfer schedule (equivalently, a direct mechanism) to maximize expected social welfare, anticipating that the wage w adjusts to clear the labor market.

$$\begin{aligned} \max_{\{c(\cdot),y(\cdot),d(\cdot)\},w}\quad \int W(s,z)\cdot\Big[\ &\mathbb{E}_\varepsilon\big[U\big(c(y(s,z), d(s,z,\varepsilon))\big) - V\big(e(d(s,z,\varepsilon); z,\varepsilon,w)\big)\big] \\ &- V\big(\ell(y(s,z); s,w)\big)\ \Big]\cdot f(s,z)\,ds\,dz \end{aligned} \tag{14}$$

s.t. (5), (11), (12), (13) $\forall (s,z), (\hat{s},\hat{z})$.

3 Discrete Lottery Formulation and LP Characterization

In this section, we describe our approach for quantitative analysis. We define a finite grid on the continuum of feasible allocations, and optimize over the set of all lotteries with support contained in the grid. This construction yields a finite linear problem (LP) which approximates the original planner's problem. The approximation becomes more accurate as the grid increases in size.

Wage normalization. For the LP representation it is also convenient to rescale all monetary variables in wage units. Formally, we redefine the following key variables[7]

$$\tilde{c} := \frac{c}{w}, \qquad \tilde{y} := \frac{y}{w}, \qquad \tilde{d} := \frac{d}{w}, \qquad \tilde{\kappa}_m := \frac{\kappa_m}{w}\ (m = 1, 2), \qquad \tilde{z} := \frac{z}{w}.$$

The normalized problem is identical to the original one with $w \equiv 1$, after the original variables $(c, y, d, \kappa_1, \kappa_2, z)$ are replaced by their tilded counterparts. Because w is the shadow price on labor market clearing, setting $w = 1$ amounts to resetting the numeraire. The associated Lagrange multiplier adjusts so that labor market clearing holds. Accordingly, in the LP below we interpret the variables $(\bar{y}_a, \bar{d}_b, \bar{c}_k, \kappa_1, \kappa_2, z_i)$ as wage-normalized objects.

3.1 Finite sets

In order to set up the LP formulation we need to define: (i) type and shock spaces, and (ii) grids for allocations.

Types. Let N denote the number of possible types indexed by $i \in \{1, \dots, N\}$. Each type (s_i, z_i) is given weight $\omega_i \geq 0$ with $\sum_{i=1}^{N} \omega_i = 1$.

Shock states. Discretize entrepreneurial risk into R shock states $r = 1, \dots, R$:

$$\varepsilon_r > 0, \qquad p_r \geq 0, \qquad \sum_{r=1}^{R} p_r = 1,$$

with (ε_r, p_r) common across types.

Grids for allocations. Let

$$\mathcal{C} = \{\bar{c}_k\}_{k=1}^{K}, \quad \mathcal{Y} = \{\bar{y}_a\}_{a=1}^{A_y}, \quad \mathcal{D} = \{\bar{d}_b\}_{b=1}^{B_d},$$

with $\bar{c}_k > 0$ and with $\bar{y}_a, \bar{d}_b$ including 0. Precompute $U_k := U(\bar{c}_k)$.

Exclusive occupations (discrete admissible set). At the discrete level, restrict attention to admissible pairs

$$\mathcal{J} \subset \{1, \dots, A_y\} \times \{1, \dots, B_d\},$$

defined by

$$(a, b) \in \mathcal{J} \quad \Longleftrightarrow \quad \Big(\bar{y}_a > 0 \Rightarrow \bar{d}_b = 0\Big) \text{ and } \Big(\bar{d}_b > 0 \Rightarrow \bar{y}_a = 0\Big). \tag{15}$$

Equivalently, $\mathcal{J}$ contains (i) worker bundles $(\bar{y}_a > 0, \bar{d}_b = 0)$, (ii) entrepreneur bundles $(\bar{y}_a = 0, \bar{d}_b > 0)$, and (iii) the inactive point $(0, 0)$, but excludes all $(\bar{y}_a > 0, \bar{d}_b > 0)$.

[7] All formal details are described in Appendix B.1.

3.2 Precomputations for the LP

(i) Worker effort disutility. Given a labor-income gridpoint $\bar{y}_a$, a type-i worker must supply effort

$$\ell_{i,a} := \frac{\bar{y}_a + \kappa_1 \mathbf{1}[\bar{y}_a > 0]}{s_i},$$

and we precompute

$$V^{\ell}_{i,a} := V(\ell_{i,a}). \tag{16}$$

Under exclusivity, this term is relevant only for worker bundles $(a,b) \in \mathcal{J}$ with $\bar{d}_b = 0$; for entrepreneur bundles with $\bar{y}_a = 0$, $\ell_{i,a} = 0$.

(ii) Implied inputs and entrepreneurial effort disutility at a dividend node. Under the least-effort selection rule in the continuous model, a dividend realization pins down a least-effort labor demand and the associated entrepreneurial effort. With $w \equiv 1$, for each dividend node $\bar{d}_b > 0$ define

$$n^*_b := \frac{1-\alpha}{\alpha} \cdot (\bar{d}_b + \kappa_2), \qquad e^*_{i,b,r} := \left(\frac{\bar{d}_b + \kappa_2 + n^*_b}{z_i \cdot \varepsilon_r \, (n^*_b)^{1-\alpha}} \right)^{1/\alpha}. \tag{17}$$

If $\bar{d}_b = 0$, set $n^*_b = 0$ and $e^*_{i,b,r} = 0$ for all (i,r). We precompute entrepreneurial effort disutility

$$V^{e}_{i,b,r} := V(e^*_{i,b,r}). \tag{18}$$

(iii) Total effort disutility coefficient. For each type i, admissible bundle $(a,b) \in \mathcal{J}$, and shock state r, define

$$\mathcal{V}_{i,a,b,r} := V^{\ell}_{i,a} + V^{e}_{i,b,r}. \tag{19}$$

By exclusivity, exactly one of the two components is nonzero for worker or entrepreneur bundles, and both are zero at $(0,0)$. Then $U_k - \mathcal{V}_{i,a,b,r}$ is the per-state utility coefficient used in the LP.

3.3 Decision variables

The numerical implementation uses a lottery representation. For each reported type i and shock state r, the mechanism chooses:

- a lottery over consumption nodes $\mathcal{C}$:

$$\lambda_{i,k,r} \geq 0, \qquad k = 1, \ldots, K;$$

- a lottery over admissible income/dividend pairs $(a,b) \in \mathcal{J}$:

$$\mu_{i,a,b,r} \geq 0, \qquad (a,b) \in \mathcal{J}.$$

The planner's objective and constraints depend only on these marginals because preferences are additively separable $(U(c) - \mathcal{V})$ and the feasibility constraints in the implementation depend on c only through its expectation, and on (y,d) only through their expectations and pre-computed coefficients. Any pair of marginals (λ, μ) can be coupled into a joint lottery over (c, y, d) conditional on (i, r), so this marginal representation is without loss for the discretized problem.

3.4 LP representation

Fix $(s_i, z_i, \omega_i, W_i)_{i=1}^N$, grids $(\mathcal{C}, \mathcal{Y}, \mathcal{D})$, the admissible set $\mathcal{J}$ in (15), the discretized shock system $(\varepsilon_r, p_r)_{r=1}^R$, and the precomputations (16)–(19) and (17). The objective and constraints below are linear in the decision variables (λ, μ).

LP representation

The planner maximizes weighted expected utility:

$$\max_{\lambda,\mu} \quad \sum_{i=1}^{N} \omega_i \cdot W_i \cdot \left[\sum_{r=1}^{R} p_r \cdot \left(\sum_{k=1}^{K} \lambda_{i,k,r} \cdot U_k - \sum_{(a,b)\in\mathcal{J}} \mu_{i,a,b,r} \cdot \mathcal{V}_{i,a,b,r} \right) \right]. \tag{20}$$

subject to:

(i) Simplex constraints. For every reported type i and shock state r,

$$\sum_{k=1}^{K} \lambda_{i,k,r} = 1, \qquad \lambda_{i,k,r} \geq 0, \tag{21}$$

$$\sum_{(a,b)\in\mathcal{J}} \mu_{i,a,b,r} = 1, \qquad \mu_{i,a,b,r} \geq 0. \tag{22}$$

(ii) Resource feasibility. Expected consumption cannot exceed expected total (observable) income:

$$\sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{k=1}^{K} \lambda_{i,k,r} \cdot \bar{c}_k \;\leq\; \sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{J}} \mu_{i,a,b,r} \cdot (\bar{y}_a + \bar{d}_b). \tag{23}$$

(iii) Labor market clearing. Define effective labor supplied at a labor-income node as

$$L_a^s := \bar{y}_a + \kappa_1 \cdot \mathbf{1}[\bar{y}_a > 0].$$

Labor demanded at dividend node $\bar{d}_b$ (under the least-effort rule) is n_b^* from (17). Labor market clearing is:

$$\sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{J}} \mu_{i,a,b,r} \cdot L_a^s \;=\; \sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{J}} \mu_{i,a,b,r} \cdot n_b^*. \tag{24}$$

(iv) Incentive compatibility. For any true type i and any report j, truthful reporting must yield weakly higher expected utility. Under misreport j, the agent receives the lottery designed for report j, but effort disutility is evaluated using the true (s_i, z_i) (via $\mathcal{V}_{i,a,b,r}$). Thus, for all i, j,

$$\sum_{r=1}^{R} p_r \left(\sum_{k=1}^{K} \lambda_{i,k,r} \cdot U_k - \sum_{(a,b)\in\mathcal{J}} \mu_{i,a,b,r} \cdot \mathcal{V}_{i,a,b,r} \right) \;\geq\; \sum_{r=1}^{R} p_r \left(\sum_{k=1}^{K} \lambda_{j,k,r} \cdot U_k - \sum_{(a,b)\in\mathcal{J}} \mu_{j,a,b,r} \cdot \mathcal{V}_{i,a,b,r} \right). \tag{25}$$

(v) Ex-ante occupational choice (no switching across shocks). The implementation imposes that occupational status does not depend on the entrepreneurial shock realization. Let

$$\mathcal{J}^W := \{(a,b) \in \mathcal{J} : \bar{y}_a > 0\}, \qquad \mathcal{J}^0 := \{(a,b) \in \mathcal{J} : \bar{y}_a = 0,\ \bar{d}_b = 0\}.$$

Then for each reported type i and each $r \geq 2$,

$$\sum_{(a,b)\in\mathcal{J}^W} \mu_{i,a,b,r} = \sum_{(a,b)\in\mathcal{J}^W} \mu_{i,a,b,1}, \tag{26}$$

$$\sum_{(a,b)\in\mathcal{J}^0} \mu_{i,a,b,r} = \sum_{(a,b)\in\mathcal{J}^0} \mu_{i,a,b,1}. \tag{27}$$

Since $\mathcal{J}$ excludes $(\bar{y}_a > 0, \bar{d}_b > 0)$, these equalities imply that the remaining probability mass on entrepreneur bundles $(\bar{y}_a = 0, \bar{d}_b > 0)$ is also constant across r.

4 A Tractable Special Case

Before turning to the quantitative analysis in Section 5, we analyze a tractable special case of the model in Section 2 with common labor skill and two entrepreneurial types. We compare a "first-best" benchmark, in which there is no private information and thus taxes can be contingent on entrepreneurs' characteristics, with a "second-best" setting, in which entrepreneurial talent is privately known and thus taxes can depend only on observed dividends.

The analysis delivers three main insights. First, with private information the planner must make the entrepreneurship option sufficiently attractive to deter high-talent agents from becoming workers. Specifically, when the high-type incentive constraint binds, entrepreneurs must be given positive information rents: they consume more than workers and, relative to the full-information benchmark, exert less entrepreneurial effort. In our general-equilibrium setting, this reduces productivity, output, and wages and requires higher labor supply from workers. The planner therefore faces the following trade-off: relaxing the incentive constraint requires shifting resources toward entrepreneurs, but doing so weakens entrepreneurial effort incentives and tends to depress aggregates. Under private information, entrepreneurs are strictly better off both relative to workers and relative to the first-best.

Second, as the production technology becomes more labor-intensive, private information generates smaller distortions in wages, output, and welfare. When the share of high-talent entrepreneurs decreases, the incentive problem becomes more severe, so private information has a larger impact on aggregates and welfare.

Finally, the gap in entrepreneurial talent affects the economy primarily through a scale channel, not through the severity of the incentive problem. The tightness of the incentive constraint—that is, the shadow cost of separation—depends on technology and population composition, rather than on the level (or gap) of entrepreneurial productivity itself. A larger entrepreneurial talent advantage raises equilibrium wages, dividends, and consumption for both occupations while leaving equilibrium effort choices unchanged. As a result, the welfare gains from greater entrepreneurial talent accrue uniformly, benefiting both workers and entrepreneurs at the same rate in both the first-best benchmark and under private information.

4.1 Environment

The setting is as in Section 2, except for the following specializations. Entrepreneurial risk is shut down ($\varepsilon \equiv 1$) and participation costs are zero ($\kappa_1 = \kappa_2 = 0$). A unit mass of agents has common labor productivity $s = 1$ and privately known entrepreneurial productivity $z \in \{z_L, z_H\}$ with $z_H > z_L$ and population shares (ω_L, ω_H), with $\omega_L + \omega_H = 1$. Agents choose ex ante between working (choosing labor effort $\ell \geq 0$) and operating a firm (choosing entrepreneurial effort $e \geq 0$ and labor demand $n \geq 0$). Preferences are $u(c, \ell, e) = \log c - \frac{1}{2}\ell^2 - \frac{1}{2}e^2$. A competitive labor market clears at wage $w > 0$. If working, observable income is $y = w\ell$ and $d = 0$; if operating, output is $q = ze^\alpha n^{1-\alpha}$ and observable dividends are $d = q - wn$, with $y = 0$.

As in Section 2, the planner conditions allocations only on observables (y, d) and we restrict attention to exclusive outcomes ($y > 0 \Rightarrow d = 0$ and $d > 0 \Rightarrow y = 0$). Given (w, z) and a target dividend $d \geq 0$, we adopt the least-effort implementation rule: among all (e, n) delivering d, the entrepreneur chooses the feasible pair minimizing e. For $d > 0$ this yields

$$n^*(d; w) = \frac{1-\alpha}{\alpha}\frac{d}{w}, \qquad q^*(d; w) = \frac{d}{\alpha}, \tag{28}$$

and implied effort

$$e^*(d; z, w) = \Gamma(\alpha)\, w^{\frac{1-\alpha}{\alpha}} \frac{d}{z^{1/\alpha}}, \qquad \Gamma(\alpha) := \frac{1}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}. \tag{29}$$

4.2 Planner's occupational menu and the separating allocation

Policy instrument. The planner offers an occupational menu with two options: a worker contract $\mathcal{W} = (c^W, y^W)$ and an entrepreneur contract $\mathcal{E} = (c^E, d^E)$. Given wage w, a worker who selects $\mathcal{W}$ supplies $\ell^W = y^W/w$ and attains indirect utility

$$u^W = \log c^W - \tfrac{1}{2}\left(\frac{y^W}{w}\right)^2. \tag{30}$$

A type-z entrepreneur who selects $\mathcal{E}$ chooses the inputs needed to deliver the targeted dividend d^E and attains

$$u^E(z) = \log c^E - \tfrac{1}{2}\big(e^*(d^E; z, w)\big)^2. \tag{31}$$

Sorting. In this environment, any competitive equilibrium with production features *occupational sorting by entrepreneurial productivity*: low-z agents choose to become workers and high-z agents operate firms. The reason is that, for any given dividend target $d > 0$, the optimal entrepreneurial $e^*(d; z, w)$ is strictly decreasing in z; hence higher-z agents can implement any profitable scale at a lower disutility level than lower-z agents. Because workers have no comparative advantage in operating firms and entrepreneurial output is increasing in z, efficiency dictates that entrepreneurial activity is allocated to the high type, while the low type supplies labor.[8] Finally, labor-market clearing requires that both occupations be present: whenever dividends are positive ($d^E > 0$), firm operation entails a positive measure of workers supplying labor and a positive measure of entrepreneurs demanding labor.

Incentive constraints. Any incentive compatible outcome must satisfy:

$$(\mathrm{IC}_H): \quad \log c^E - \tfrac{1}{2}\big(e^*(d^E; z_H, w)\big)^2 \;\geq\; \log c^W - \tfrac{1}{2}\left(\frac{y^W}{w}\right)^2, \tag{32}$$

$$(\mathrm{IC}_L): \quad \log c^W - \tfrac{1}{2}\left(\frac{y^W}{w}\right)^2 \;\geq\; \log c^E - \tfrac{1}{2}\big(e^*(d^E; z_L, w)\big)^2. \tag{33}$$

Constraint (IC_H) requires that any high-talent agent prefers the entrepreneur option to the worker option. By becoming a worker, she would receive consumption c^W and would have to supply labor effort $\ell = y^W/w$. Constraint (IC_L) requires any low-talent agent prefers to become a worker rather than an entrepreneur. If he selected the entrepreneur option, he would receive consumption c^E and, given his type z_L, would need to exert the higher effort level $e^*(d^E; z_L, w)$ to generate the same observable dividend d^E.

Market clearing conditions. With low types supplying labor and high types demanding labor, labor-market clearing equation can be written as

$$\omega_L \cdot \frac{y^W}{w} \;=\; \omega_H \cdot \overbrace{\frac{1-\alpha}{\alpha} \cdot \frac{d^E}{w}}^{n^*(d^E;w)}. \tag{34}$$

or equivalently

$$y^W = s \cdot d^E, \qquad \text{where } s := \frac{\omega_H}{\omega_L} \cdot \frac{1-\alpha}{\alpha}. \tag{35}$$

and the aggregate resource feasibility can be written as

$$\omega_L \cdot c^W + \omega_H \cdot c^E \;=\; \omega_H \cdot \overbrace{d^E/\alpha}^{q^*(d^E;w)}. \tag{36}$$

The aggregate amount of labor and the total consumption are thus jointly determined by the dividend target d^E, while the wage w is determined by equilibrium conditions (and remains explicit in the expressions below).

[8] Formally, among allocations satisfying feasibility and the occupational observability restriction, any allocation with z_L operating firms and z_H working can be improved by swapping occupations, holding fixed the observable outcomes (y, d): the high type can replicate the same dividend with strictly lower effort cost, while the worker contract depends only on y.

Planner problem. The planner chooses (c^W, y^W, c^E, d^E, w) to maximize utilitarian welfare subject to incentive compatibility and feasibility:

$$\max_{c^W>0,\, y^W\geq 0,\, c^E>0,\, d^E\geq 0,\, w>0} \quad \omega_L \cdot \left[\log c^W - \tfrac{1}{2}\left(\tfrac{y^W}{w}\right)^2\right] + \omega_H \cdot \left[\log c^E - \tfrac{1}{2}\big(e^*(d^E; z_H, w)\big)^2\right] \tag{37}$$

$$\text{s.t.} \quad (\text{IC}_H)\ (32), \qquad (\text{IC}_L)\ (33),$$

$$\omega_L \cdot \frac{y^W}{w} = \omega_H \cdot n^*(d^E; w), \tag{38}$$

$$\omega_L \cdot c^W + \omega_H \cdot c^E = \omega_H \cdot \frac{d^E}{\alpha}. \tag{39}$$

Let $\lambda \geq 0$ denote the multiplier on (IC_H) and $\mu \geq 0$ the multiplier on (IC_L). In the closed-form characterization below we focus on the empirically relevant setting in which

$$\lambda > 0, \qquad \mu = 0,$$

i.e., (IC_H) binds and (IC_L) is slack.

4.3 Closed form when the high-type incentive constraint binds

We now focus on the case where (IC_H) binds and (IC_L) is slack. In words, the planner must deter the high-z agent from wanting to select the worker contract, while the low type finds the entrepreneurship option too costly because doing so would require a substantially higher effort in order to generate the same observable dividend. In this case, the analysis delivers a sharp closed-form characterization of the optimal allocation, and a transparent mapping from the information friction, represented by the magnitude of the multiplier λ, into the differences in consumption and tax burdens the two agent types. Equations (28) and (29) can be rewritten as

$$n^*(d; w) = \beta \cdot \frac{d}{w}, \qquad e_H(d, w) := e^*(d; z_H, w) = \Gamma(\alpha) \cdot w^\beta \cdot \frac{d}{z_H^{1/\alpha}},$$

where

$$\beta := \frac{1-\alpha}{\alpha}, \qquad s := \frac{\omega_H}{\omega_L}\beta, \qquad \Gamma(\alpha) := \frac{1}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}$$

and labor-market clearing implies $y^W = s \cdot d^E$. The solution to the planner's problem can now be characterized in detail.

Closed-form representation (endogenous wage; (IC_H) binds, (IC_L) slack)

If the solution of (40) below satisfies $\lambda > 0$, then (IC_H) binds, hence the high type is indifferent between the entrepreneur and worker options. The other incentive constraint (IC_L) is automatically satisfied with slack.
If instead (40) yields $\lambda \leq 0$, (IC_H) is slack. By complementary slackness, the relevant

multiplier is $\lambda = 0$, and the allocation is obtained by evaluating (41)–(43) at $\lambda = 0$.

Step 1: scalar equation for λ. Imposing (IC_H) as equality implies the following equation in λ

$$\boxed{\log\left(\frac{\omega_L \cdot (\omega_H + \lambda)}{\omega_H \cdot (\omega_L - \lambda)}\right) \;=\; \frac{1}{2} \cdot \left[\frac{\alpha}{\omega_H + \lambda} - \frac{1-\alpha}{\omega_L - \lambda}\right].} \tag{40}$$

Step 2: wage $w(\lambda)$. The wage is pinned down by the planner's FOC with respect to w

$$\boxed{w(\lambda) \;=\; z_H \cdot \left(\frac{(\omega_L - \lambda) \cdot \left(\frac{\omega_H}{\omega_L}\right)^2 \cdot \beta}{(\omega_H + \lambda) \cdot \Gamma(\alpha)^2}\right)^{\alpha/2}.} \tag{41}$$

Step 3: allocations, given $w(\lambda)$. Dividends and labor income are given by

$$\boxed{d^E(\lambda) \;=\; \frac{w(\lambda)}{s} \cdot \sqrt{\frac{1-\alpha}{\omega_L - \lambda}}, \qquad y^W(\lambda) \;=\; s \cdot d^E(\lambda).} \tag{42}$$

and the consumption levels are

$$\boxed{c^E(\lambda) \;=\; (\omega_H + \lambda) \cdot \frac{d^E(\lambda)}{\alpha}, \qquad c^W(\lambda) \;=\; \frac{\omega_L - \lambda}{\omega_L} \cdot \omega_H \cdot \frac{d^E(\lambda)}{\alpha}.} \tag{43}$$

Step 4: Effort and Labor, given $w(\lambda)$. The equilibrium labor and entrepreneurial effort simplify to:

$$\ell^W(\lambda) = \frac{y^W(\lambda)}{w(\lambda)} = \sqrt{\frac{1-\alpha}{\omega_L - \lambda}}, \qquad e_H(\lambda) = \sqrt{\frac{\alpha}{\omega_H + \lambda}}. \tag{44}$$

4.4 Information rents and the distributional tilt toward entrepreneurs

If (IC_H) is binding and (IC_L) is slack, the closed-form solution implies the consumption and effort wedges

$$G(\lambda) := c^E(\lambda) - c^W(\lambda) = \frac{d^E(\lambda)}{\alpha} \cdot \lambda \cdot K, \quad N(\lambda) := e_H(\lambda) - \ell^W(\lambda) = \sqrt{\frac{\alpha}{\omega_H + \lambda}} - \sqrt{\frac{1-\alpha}{\omega_L - \lambda}}, \tag{45}$$

where $K := 1 + \omega_H/\omega_L > 0$.

Distribution and welfare implications. Equation (45) summarizes the redistribution induced by private information. When (IC_H) binds, we have $\lambda > 0$ and $d^E(\lambda) > 0$ (by (42)), hence $G(\lambda) > 0$: entrepreneurs consume more than workers, so the separating allocation necessarily tilts resources toward entrepreneurship. This redistribution is especially stark in the two-type economy: the government budget constraint implies that net taxes must balance, so the information rent is financed one-for-one by workers. In particular, workers pay positive net taxes while entrepreneurs receive an equal net transfer (a subsidy).[9]

To assess the welfare implications, it is useful to compare effort across occupations. There are two cases. If $\alpha \le \omega_H$, then effort gap $N(\lambda) < 0$ for all $\lambda > 0$. Thus entrepreneurs exert less effort than workers while consuming more, so they are strictly better off in utility terms. Instead, if $\alpha > \omega_H$, which is arguably the more relevant case (e.g., the high-talent share is small, say $\omega_H \approx 5\%$–10%). Then, we have

$$N(\lambda) \begin{cases} > 0, & \text{iff } 0 \le \lambda < \alpha - \omega_H, \\ = 0, & \text{iff } \lambda = \alpha - \omega_H, \\ < 0, & \text{iff } \lambda > \alpha - \omega_H. \end{cases} \tag{46}$$

For moderate wedges ($\lambda < \alpha - \omega_H$), entrepreneurs still exert more effort than workers, but private information shifts the allocation in their favor: they receive higher consumption and face weaker effort requirements than under the full-information benchmark. As λ increases, the allocation becomes progressively more tilted: entrepreneurial effort falls more relative to worker effort, while the consumption advantage persists. In all cases, entrepreneurs are better off in utility terms relative to the first best.

Information rents and general-equilibrium feedback. A useful way to unpack (45) is to differentiate the rent wedge $G(\cdot)$ with respect to the incentive distortion λ:

$$\frac{dG}{d\lambda} = \frac{K}{\alpha}\Big(\underbrace{d^E(\lambda)}_{\substack{\text{direct effect:} \\ \text{higher rent weight}}} + \underbrace{\lambda \cdot \frac{d}{d\lambda} d^E(\lambda)}_{\substack{\text{general-equilibrium effect:} \\ \text{dividends adjust via } w(\lambda)}} \Big). \tag{47}$$

The first term represents the *direct* screening force: holding aggregate dividends fixed, a higher λ increases the entrepreneurs' consumption to relax (IC_H), thus widening the consumption gap. The second term reflects *general-equilibrium* feedbacks, due to the endogeneity of both dividends (hence GDP) and wages. Using $d^E(\lambda) = \frac{w(\lambda)}{s} \cdot \ell^W(\lambda)$, we obtain the decomposition

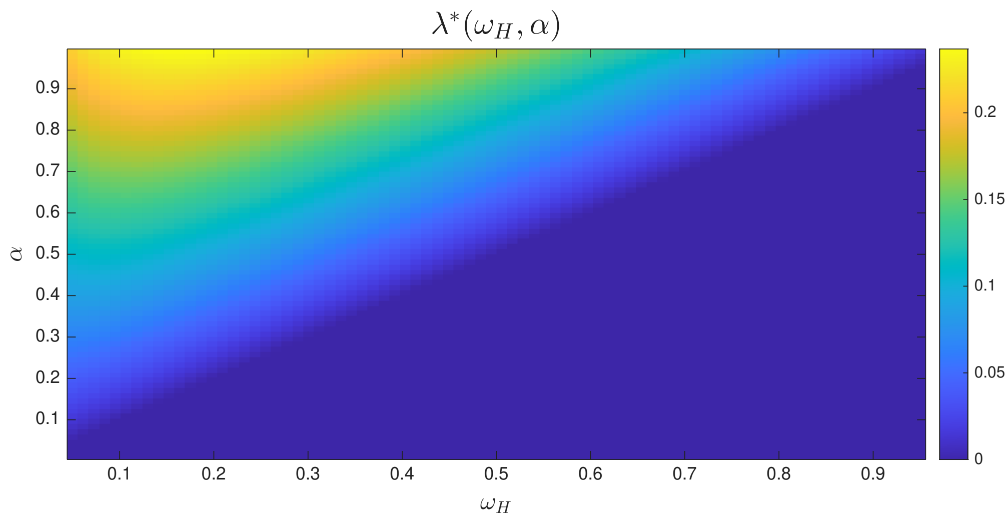
$$\frac{d}{d\lambda} d^E(\lambda) = \frac{1}{s}\left(\underbrace{\frac{dw(\lambda)}{d\lambda} \cdot \ell^W(\lambda)}_{\text{wage channel}} + \underbrace{w(\lambda) \cdot \frac{d\ell^W(\lambda)}{d\lambda}}_{\text{labor-supply channel}} \right). \tag{48}$$

The equations (41) and (44) imply $\frac{dw(\lambda)}{d\lambda} < 0$ and $\frac{d\ell^W(\lambda)}{d\lambda} > 0$. The wage channel pushes dividends *down* (a higher λ is associated with lower entrepreneurial effort and, through the wage formula, a lower equilibrium wage), while the labor-supply channel pushes dividends — and therefore GDP, $\omega_H \alpha^{-1} d^E(\lambda)$ — *up*. Overall, $G(\lambda)$ rises with λ when the direct rent effect and the labor-supply

[9] With no private information, the self-selection motive is absent; in the present representation this corresponds to $\lambda = 0$, which collapses the rent wedge to $c^E(0) = c^W(0)$.

channel dominate the wage decline; if instead $w(\lambda)$ falls sufficiently steeply as λ increases, then $d^E(\lambda)$ and GDP can fall enough so that the consumption gap need not be monotone in λ, even though it remains tilted toward entrepreneurs whenever $\lambda > 0$.

Figure 1: Comparative statics of the Lagrange multiplier λ



Notes: The figure plots the numerical solution for λ as a function of the high-type share ω_H and the technology parameter α, obtained by solving equation (40) for λ for each parameter pair.

When are rents largest? Figure 1 shows that $\lambda^*(\omega_H, \alpha)$ increases with α, and is largest at an interior and relatively small value of ω_H.[10] Three remarks are in order.

First, holding the population share ω_H fixed, a higher α shifts the production technology toward being more effort-intensive and less labor-intensive, and induces the planner to increase the attractiveness of the entrepreneurship option by increasing their equilibrium payoff relative to the workers. Second, holding technology (α) fixed, a smaller population share ω_H makes it harder for the planner to induce the high type to choose entrepreneurship and thus generate output. As a result, the shadow value of (IC_H) increases, which in turn amplifies information rents.

Finally, z_H does not affect the shadow value of the incentive constraint: the multiplier λ is pinned down only by α and ω_H. By contrast, higher entrepreneurial productivity raises equilibrium wages, dividends, and thus consumption levels for both entrepreneurs and workers. The implemented worker labor supply and high-type entrepreneurial effort depend on $(\omega_H, \alpha, \lambda)$ only, not on z_H. As a result, the welfare gains from higher z_H operate entirely through a scale effect on consumption. In particular, welfare for both entrepreneurs and workers increases at rate $1/z_H$ in both the first-best benchmark and under private information (see Appendix A.2 for a formal proof). This result suggests that policies aimed at stimulating entrepreneurial talent would be uniformly beneficial for all agents in the economy.

[10] In Appendix A we show that $\partial\lambda^*/\partial\alpha > 0$.

5 Calibration and Quantitative Results

This section describes how we parameterize the model and presents the main quantitative findings. We calibrate five key parameters to match five empirical moments from U.S. data on labor force participation, entrepreneurship, firm size, and business income dispersion. The quantitative analysis delivers three main results: (i) occupational choice exhibits sharp sorting across the two-dimensional type space, with general equilibrium effects playing a central role; (ii) the optimal allocation features distinct tax treatment across occupation-specific margins; and (iii) entrepreneurial risk shapes the mechanism by providing additional screening power to the planner.

5.1 Calibration

The model is static and interpreted as a yearly cross-section. All monetary variables are measured in *wage units*: we normalize the competitive wage per efficiency unit to $w \equiv 1$. Under this normalization, labor income, entrepreneurial dividends, consumption, and fixed participation costs are all expressed relative to the wage.

Preferences. We assume CRRA utility over consumption and a standard power disutility of effort:

$$U(c) = \begin{cases} \dfrac{c^{1-\gamma} - 1}{1-\gamma}, & \gamma \neq 1, \\ \log c, & \gamma = 1, \end{cases} \qquad V(\ell) = \frac{\ell^{1+\psi}}{1+\psi}.$$

We set $\gamma = 1$ (log utility) and $\psi = 1$ (unit Frisch elasticity), both standard values in the quantitative public finance literature.

Technology and productivity risk. The firm production function is $q = z \cdot \varepsilon \cdot e^{\alpha} n^{1-\alpha}$ with $\alpha \in (0, 1)$. The parameter α governs firm scale: lower values imply higher labor demand per unit of entrepreneurial effort, hence larger firms on average. We calibrate α to match the average number of employees per operating firm in the U.S.

Idiosyncratic entrepreneurial risk is multiplicative, i.i.d. across agents, and independent of (s, z). We assume a lognormal shock

$$\varepsilon \sim \log\mathcal{N}\left(-\tfrac{1}{2}\sigma_\varepsilon^2, \sigma_\varepsilon^2\right), \qquad \mathbb{E}[\varepsilon] = 1.$$

We calibrate σ_ε to match the coefficient of variation of business income among entrepreneurs, following DeBacker et al. (2021).

Type distribution. We discretize (s, z) on an $N_1 \times N_2 = 50 \times 50$ grid. The two dimensions follow independent lognormal distributions:

$$\log s \sim \mathcal{N}(m_s, \sigma_s^2), \qquad \log z \sim \mathcal{N}(m_z, \sigma_z^2), \qquad \log s \perp \log z.$$

Table 1: Calibrated parameters

Parameter	Symbol	Value	Target moment	Source
Preferences (set externally)				
Risk aversion	γ	1.00	Log utility	Standard
Frisch elasticity	$1/\psi$	1.00	Unit elasticity	Standard
Calibrated parameters				
Firm curvature	α	0.201	Avg. employees/firm	U.S. Census
Worker fixed cost	κ_1	0.378	Labor participation	BLS
Entrepreneur fixed cost	κ_2	0.0871	Entrepreneurship rate	BLS
Shock volatility	σ_ε	0.267	CV of business income	SCF/PSID
Entrep. ability dispersion	σ_z	0.280	p90/p50 business income	DeBacker et al. (2021)
Type distribution (set externally)				
Labor skill dispersion	σ_s	0.318	Wage skill premium	Heathcote et al. (2014)
Ability correlation	ρ	0.00	Independence	Baseline

We normalize $m_s = m_z = -\frac{1}{2}\sigma_j^2$ so that the means equal one in levels. The dispersion σ_s is set to match the wage skill premium from Heathcote et al. (2014). The dispersion σ_z is calibrated to match the 90th-to-50th percentile ratio of business income.

Extensive margins. We discipline participation through fixed costs (κ_1, κ_2). The fixed cost κ_1 governs the labor force participation rate, and κ_2 governs the entrepreneurship rate.

Calibration strategy. We jointly calibrate five parameters—$(\kappa_1, \kappa_2, \alpha, \sigma_\varepsilon, \sigma_z)$—to match five moments. Table 1 reports the calibrated parameter values, and Table 2 compares model-implied moments to their data counterparts.

The calibration matches the targeted moments reasonably well. The model slightly overpredicts labor force participation (66.4% vs. 62%) and underpredicts entrepreneurship (8.7% vs. 10%), while matching the business income dispersion almost exactly. The coefficient of variation of entrepreneur income is somewhat higher in the model (1.50 vs. 1.20), reflecting the substantial idiosyncratic risk faced by entrepreneurs. The low value of $\alpha \approx 0.201$ implies that entrepreneurial production is labor-intensive, generating substantial labor demand and hence sizable general equilibrium effects through wage determination. The fixed cost of working ($\kappa_1 \approx 0.378$) is larger than the fixed cost of entrepreneurship ($\kappa_2 = 0.0871$), consistent with the interpretation that entrepreneurship is a selective, high-return activity.

Numerical method. We discretize the two-dimensional type space on a 50×50 grid, giving $N = 2{,}500$ types. Each type is assigned a lottery over $K = 70$ consumption levels and $|\mathcal{A}| = 49$ admissible income pairs across R shock states, yielding approximately 17 million decision variables. Incentive compatibility requires that no type prefer the allocation of any other, generating $N^2 \approx 6.25$ million pairwise IC constraints.

Table 2: Targeted moments: model vs. data

Moment	Model formula	Model	Data
Labor participation	$\int \mathbf{1}\{y_1 > 0 \vee y_2 > 0\}\, dF(s,z)$	0.628	0.620
Entrepreneurship rate	$\int \mathbf{1}\{y_2 > 0\}\, dF(s,z)$	0.082	0.100
Avg. employees/firm	$\frac{\int n(s,z)\cdot \mathbf{1}\{y_2>0\}\, dF}{\int \mathbf{1}\{y_2>0\}\, dF}$	8.8	10.0
CV of business income	$\frac{\text{Std}(y_2 \mid y_2 > 0)}{\mathbb{E}[y_2 \mid y_2 > 0]}$	1.48	1.20
log(p90/p50) biz income	$\log\left(\frac{y_2^{p90}}{y_2^{p50}}\right)\Big\vert_{y_2>0}$	2.5	2.64

Notes: $n(s,z) = \frac{1-\alpha}{\alpha} \cdot \frac{y_2+\kappa_2}{w}$ is firm labor demand. Data sources: labor participation from Bureau of Labor Statistics (2024b); entrepreneurship rate from Bureau of Labor Statistics (2024a); average employees from U.S. Census Bureau (2021); business income dispersion from DeBacker et al. (2021).

As described in Subsection 4.3, the planner's problem admits a linear optimization formulation over marginal lotteries. The LP reformulation transforms the original non-convex mechanism design problem—which is generally ill-behaved and difficult to solve with standard nonlinear methods—into a well-posed linear problem with a unique global optimum. However, even in its LP form the sheer number of IC constraints makes the full problem computationally infeasible at this grid resolution.

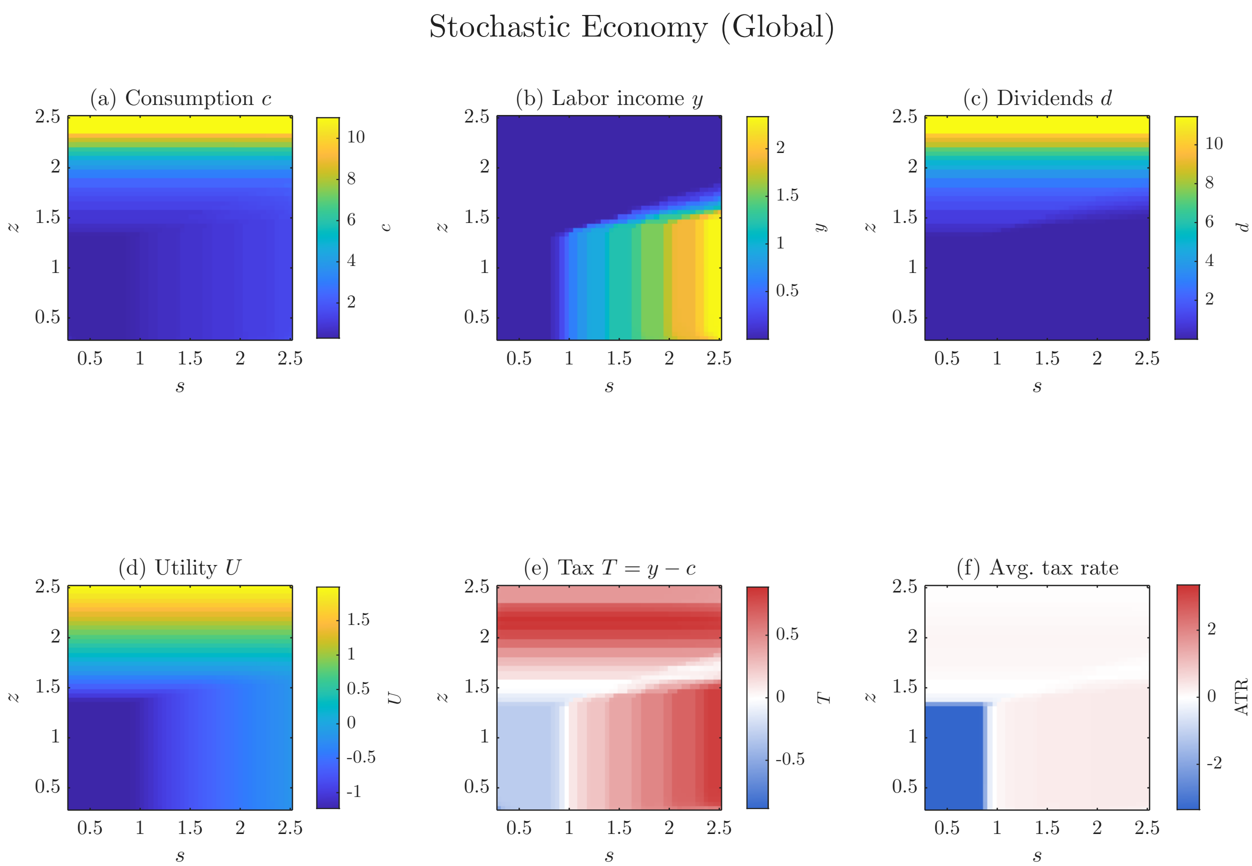
To overcome this bottleneck, we develop a *constraint generation* algorithm solved with Gurobi's barrier method. The key observation is that, at the optimum, only a small fraction of the N^2 IC constraints are binding. The algorithm starts from a relaxed LP containing only *local* IC constraints—between each type and its four adjacent neighbors on the grid—and iteratively identifies *globally violated* constraints by evaluating the full N^2 deviation-payoff matrix at each candidate solution. At each iteration, the algorithm prioritizes the most strongly violated pairs, adds them to the active set, and re-solves the LP. At convergence, the method recovers the exact global optimum of the full problem.

This makes it possible to use grid resolutions an order of magnitude finer than previously feasible, delivering smooth policy functions and reliable comparative statics. Appendix B.2 provides the full algorithmic details.

5.2 Quantitative Results

We now describe the structure of the optimal allocation in the calibrated economy and how it changes with (i) entrepreneurial risk, (ii) the scarcity of high-z entrepreneurial types, and (iii) affiliation between labor skill s and entrepreneurial ability z. Throughout, all variables are expressed in *wage units* (recall the normalization $w \equiv 1$). Taxes are reported as net tax liabilities $T(s,z) = y(s,z) - c(s,z)$, and the average tax rate is $\text{ATR}(s,z) = T(s,z)/\big(y(s,z) + d(s,z)\big)$ (with ATR $= 0$ at the inactive point). Unless otherwise stated, we report *ex ante* (pre-shock) allocations that are constant across entrepreneurial shock realizations; when we display state-by-state taxes, we do so holding fixed the same occupational choice.

Figure 2: Optimal allocations and net tax liabilities in the stochastic calibrated economy



Notes: The figure plots the optimal allocation over the two-dimensional type space (s, z) in the calibrated economy with entrepreneurial risk and baseline affiliation ($\rho = 0$). Entrepreneurial risk has two shock realizations; the plotted objects are *ex ante* allocations (and, when applicable, expectations across shock realizations). Panels display: (a) consumption c, (b) labor income y, (c) dividends d, (d) indirect utility $U(c) - V(\ell) - \mathbb{E}[V(e)]$, (e) net tax liability $T = y - c$, and (f) the average tax rate $\text{ATR} = T/(y + d)$ (defined as 0 at $(y, d) = (0, 0)$). Exclusive occupations imply $y > 0 \Rightarrow d = 0$ and $d > 0 \Rightarrow y = 0$.

Baseline stochastic economy: sorting and a tax break at the entry margin. Figure 2 summarizes the main patterns in the calibrated stochastic economy. First, the allocation exhibits sharp occupational sorting. Low entrepreneurial ability types are assigned to work (positive y, zero d), while sufficiently high z types operate firms (positive d, zero y). This sorting boundary is an *extensive margin* of entrepreneurship that depends on labor skill s: holding z fixed, higher-s agents have a better outside option in paid work and therefore require larger entrepreneurial rents to be induced into firm operation.

Second, consumption is increasing in both s and z, while the occupational choice generates a pronounced kink along the entry boundary. The key distributional object is the net tax map $T = y - c$ in panel (e). Fix s and increase z. For sufficiently high s (agents with strong labor-market outside options), taxes are *non-monotone* in z around the entrepreneurship entry margin: as z rises into the entrepreneur region, the planner reduces net tax liabilities sharply (often into negative territory), creating a localized *entrepreneurial tax break*. This pattern is the quantitative counterpart of the information-rent logic in Section 4: to prevent high-s types with moderate z from mimicking low-z types and selecting the worker contract, the planner must make the entrepreneurial contract locally attractive at the point where occupational choice is most elastic.

Finally, these subsidies are amplified by general equilibrium. Because entrepreneurs hire labor, a contraction in entry reduces labor demand; market clearing therefore requires a sufficiently large mass of active entrepreneurs. In equilibrium, the planner uses the tax system both to screen and to sustain entry so that labor demand is high enough to absorb workers. The resulting policy features a sharp redistribution toward marginal entrepreneurs—precisely those types who are closest to switching into paid work.

5.2.1 The role of entrepreneurial risk

Entrepreneurial risk changes the optimal policy through two channels: insurance (consumption smoothing across realizations) and screening (state-contingent distortions that relax ex ante incentive constraints). Figure 3 reports deterministic counterpart of Figure 2.

Two facts stand out. First, risk leads the planner to *smooth consumption* in the entrepreneur region, raising consumption for types that would otherwise face low realized dividends and lowering it for those with high realizations. In the deterministic economy, by contrast, there is no state to insure and the policy relies more heavily on deterministic wedges, which shows up as larger and more systematic tax breaks for entrepreneurs along the entry boundary.

Second, the stochastic economy allows the planner to make taxes more *state-contingent*. In high-dividend realizations, more likely generated by the high-productivity shock, entrepreneurs are less tempted to mimic low-z worker types because the entrepreneurial contract delivers sufficiently high resources even without large information rents. The planner can therefore impose higher taxes in the high realization with limited incentive cost. In low-dividend realizations, instead, the temptation to mimic is stronger; the optimal policy responds by expanding subsidies as z increases within the entrepreneur region, generating the blue (negative-tax) gradient visible in the state-by-state tax maps of Figure 4 (top row). In the deterministic economy taxes cannot be conditioned on realizations and thus must "front-load" rents in the single state, producing a starker and more monotone pattern of entrepreneurial tax breaks.

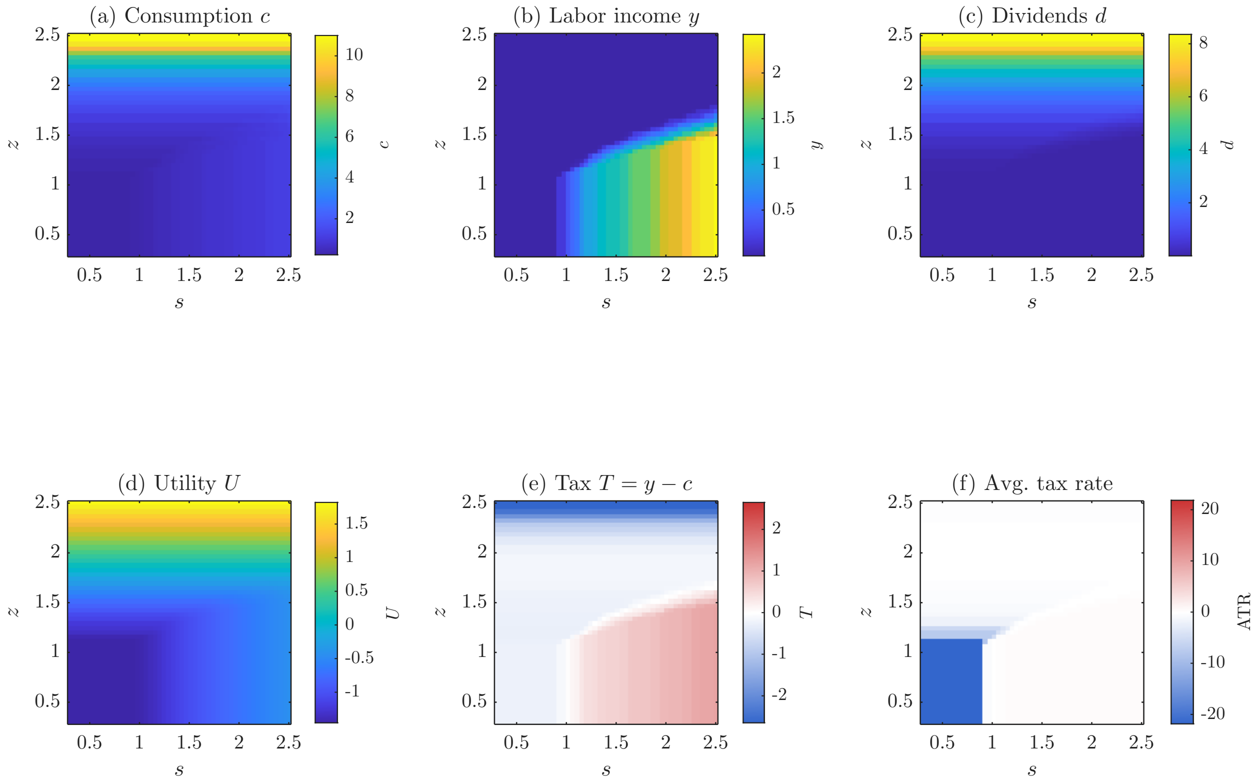
5.2.2 Scarcity of entrepreneurial types

We next reduce the dispersion of entrepreneurial ability, compressing the upper tail of z ("worsened entrepreneurial types"). Figure 4 shows how the optimal tax map changes in each realization relative to the baseline.

When entrepreneurial types are less dispersed, screening becomes harder and the mass of very productive entrepreneurs falls. Quantitatively, the planner responds by extending tax breaks deeper

Figure 3: Entrepreneurial risk versus determinism: changes in consumption, dividends, and welfare

Deterministic Economy



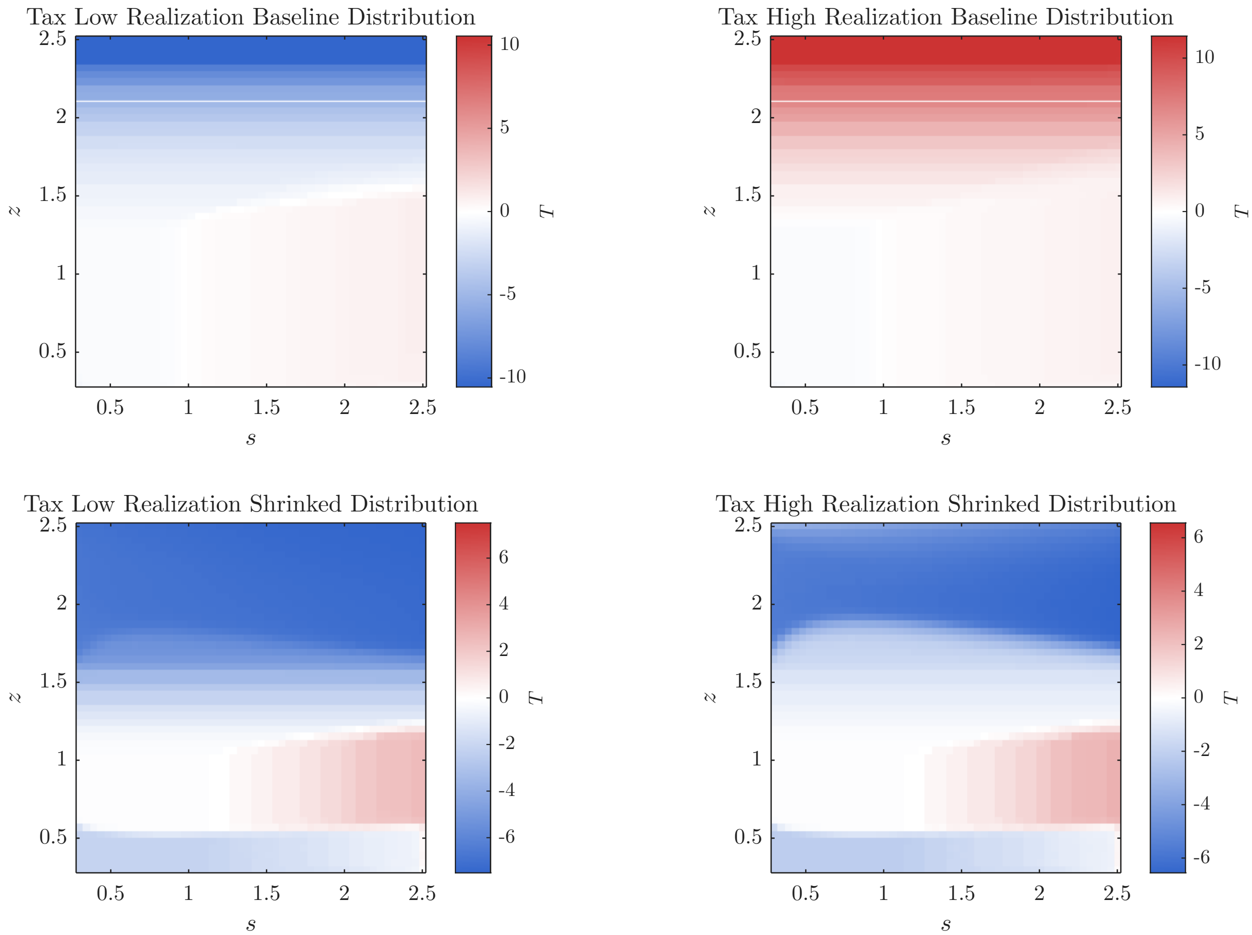
Notes: The figure plots the optimal allocation over the two-dimensional type space (s, z) in the calibrated *deterministic* economy (no entrepreneurial risk) with baseline affiliation ($\rho = 0$). The plotted objects are deterministic allocations. Panels display: (a) consumption c, (b) labor income y, (c) dividends d, (d) indirect utility $U(c) - V(\ell) - V(e)$, (e) net tax liability $T = y - c$, and (f) the average tax rate $\text{ATR} = T/(y + d)$ (defined as 0 at $(y, d) = (0, 0)$). Exclusive occupations imply $y > 0 \Rightarrow d = 0$ and $d > 0 \Rightarrow y = 0$.

into the entrepreneur region, and crucially, by doing so *even in the high realization*. In the baseline economy, high-realization taxes can be higher because incentive constraints are relatively slack in that state; once high-z types become scarce, however, sustaining entry and labor demand requires raising the attractiveness of entrepreneurship across states. This is consistent with the simple two-type logic in Section 4: when it is harder to secure sufficient entrepreneurial participation, the shadow cost of the relevant incentive constraint rises and the optimal policy tilts more resources toward entrepreneurs.

5.2.3 Affiliation between s and z: superstars versus specialized talent

Finally, we vary the correlation between labor skill and entrepreneurial ability. Positive affiliation ("superstars") increases the prevalence of types that are simultaneously high s and high z, while negative affiliation ("specialized") generates comparative advantage, with types more likely to be high in one dimension and low in the other. Figure 5 contrasts the resulting tax maps by realization.

Figure 4: Scarcity of high-z types: optimal net taxes by realization

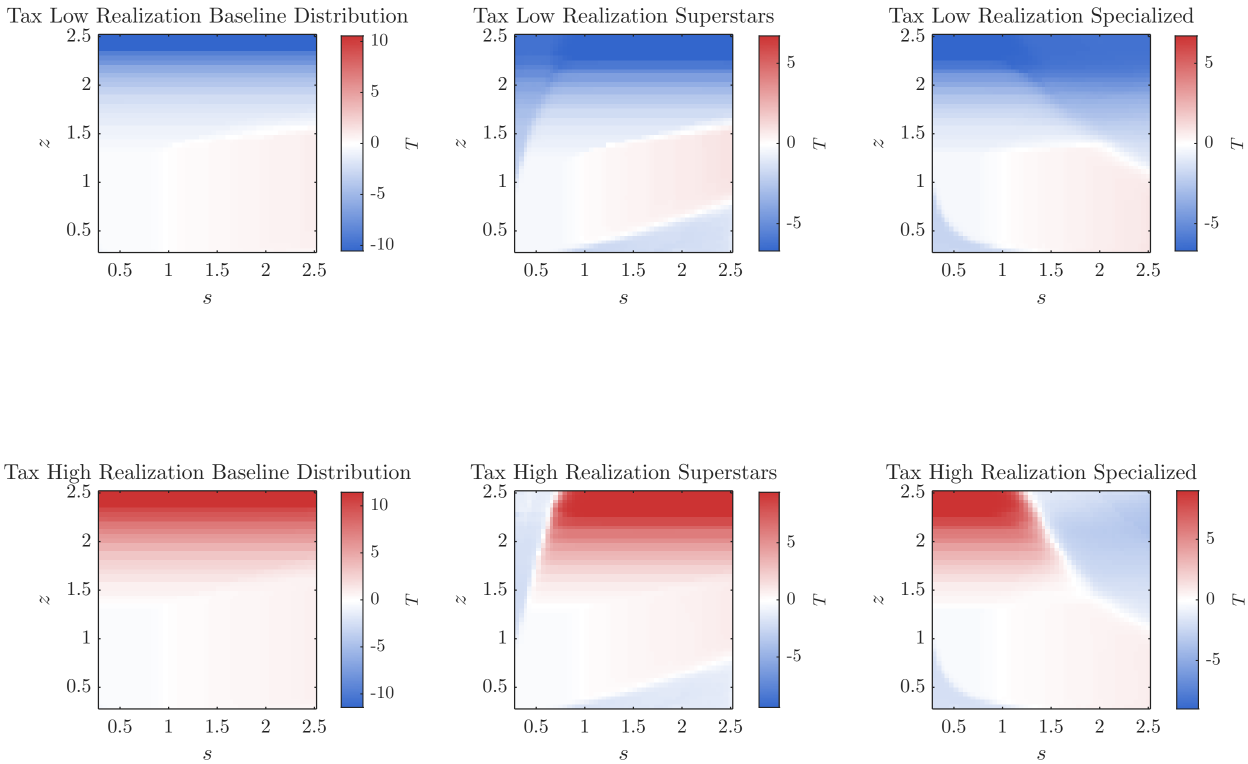


Notes: The figure reports net tax liabilities $T = y - c$ across the type space in the stochastic economy, separately by shock realization. The *top row* corresponds to the baseline entrepreneurial-type distribution; the *bottom row* corresponds to an economy with lower dispersion in entrepreneurial ability (a “shrunk” z distribution). The *left column* is the low dividend realization; the *right column* is the high dividend realization. Negative values (blue) indicate net subsidies. All variables are in wage units.

The correlation structure reshapes both the location and the intensity of entrepreneurial rents. In the superstar economy, the policy concentrates both high taxes and large subsidies in the top-right corner (high s, high z), reflecting the fact that the most productive entrepreneurs also have the strongest worker outside options. The planner therefore uses sharply state-contingent instruments: in the high realization, taxes are concentrated where realizations are high and incentive constraints are weakest, while subsidies remain targeted to types whose entry would otherwise be most fragile (including low-s, high-z entrepreneurs who generate labor demand but have limited insurance capacity).

In the specialized economy, instead, many entrepreneurs have relatively low labor skill (low s, high z). These types have weaker worker outside options, so entry is easier to sustain; the planner correspondingly shifts the tax and transfer burden toward the entrepreneur region with low s and high z, in the high and low realization respectively, while relying less on large subsidies for marginal

Figure 5: Affiliation in the joint type distribution: net taxes by realization



Notes: The figure reports net tax liabilities $T = y - c$ across the type space in the stochastic economy by shock realization, under alternative correlations between $\log s$ and $\log z$. The *top row* is the baseline distribution ($\rho = 0$). The *middle row* is the superstar economy (positive affiliation, $\rho > 0$). The *bottom row* is the specialized economy (negative affiliation, $\rho < 0$). The *left column* is the low realization; the *right column* is the high realization. Negative values (blue) indicate net subsidies. All variables are in wage units.

entrants. Overall, affiliation changes the geometry of the binding incentive constraints and therefore the distribution of entrepreneurial tax breaks across the type space.

Welfare and inequality. Across all scenarios, the agents receiving the largest entrepreneurial tax breaks also obtain higher consumption and higher indirect utility. The quantitative mechanism therefore highlights a fundamental trade-off: relaxing incentive constraints and sustaining entry requires granting rents to entrepreneurs, which raises ex post inequality even as it supports production, labor demand, and insurance. The maps in Figures 2–5 show that this trade-off is most pronounced at the extensive margin of entrepreneurship and is amplified when high-z types are scarce or when high s and high z are positively affiliated.

6 Conclusion

Entrepreneurship is both a source of volatile, unequal incomes and a driver of labor demand and aggregate performance. In an economy where individuals privately know both worker skill and entrepreneurial talent, and where policy can condition only on realized earnings and profits, optimal taxation must jointly provide insurance and preserve occupational incentives. Our analysis shows that this interaction generically produces localized subsidies to entrepreneurship at the extensive margin: information rents are not an anomaly but a second-best instrument for sustaining entry, separation, and labor-market clearing. Quantitatively, the resulting tax-and-transfer system features sharp sorting, state-contingent insurance against entrepreneurial risk, and entry incentives that are strongest where outside options are highest. More broadly, the results highlight why efficient redistribution in entrepreneurial economies cannot be designed in partial equilibrium—because the occupational allocation of talent feeds back into wages, output, and the incidence of taxation across the entire population.

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A Mathematical Appendix

A.1 Comparative statics of the endogenous-wage multiplier λ

Consider the scalar equation

$$F(\lambda; \omega_H, \alpha) = 0, \qquad \lambda \in (-\omega_H, \omega_L), \qquad \omega_L := 1 - \omega_H, \tag{49}$$

where

$$F(\lambda; \omega_H, \alpha) := \log\left(\frac{\omega_L \cdot (\omega_H + \lambda)}{\omega_H \cdot (\omega_L - \lambda)}\right) - \frac{1}{2}\left[\frac{\alpha}{\omega_H + \lambda} - \frac{1 - \alpha}{\omega_L - \lambda}\right]. \tag{50}$$

Throughout we maintain $\omega_H \in (0, 1)$, $\alpha \in (0, 1)$, and $\lambda \in (-\omega_H, \omega_L)$ so all denominators are positive.

A.1.1 Step 1: derivatives of F

Write $A := \omega_H + \lambda$ and $B := \omega_L - \lambda$. Then $A > 0$ and $B > 0$ on the admissible set. Differentiating (50) with respect to λ yields

$$F_\lambda(\lambda; \omega_H, \alpha) = \frac{1}{\omega_H + \lambda} + \frac{1}{\omega_L - \lambda} + \frac{1}{2}\left[\frac{\alpha}{(\omega_H + \lambda)^2} + \frac{1 - \alpha}{(\omega_L - \lambda)^2}\right]. \tag{51}$$

Hence,

$$F_\lambda(\lambda; \omega_H, \alpha) > 0 \qquad \text{for all } \lambda \in (-\omega_H, \omega_L). \tag{52}$$

Differentiating (50) with respect to α gives

$$F_\alpha(\lambda; \omega_H, \alpha) = -\frac{1}{2}\left[\frac{1}{\omega_H + \lambda} + \frac{1}{\omega_L - \lambda}\right] < 0. \tag{53}$$

To differentiate with respect to ω_H taking into account $\omega_L = 1 - \omega_H$, it is convenient to first compute the partials treating ω_H and ω_L as independent, and then apply the chain rule. Holding λ fixed,

$$F_{\omega_H} = \frac{1}{\omega_H + \lambda} - \frac{1}{\omega_H} + \frac{\alpha}{2(\omega_H + \lambda)^2}, \tag{54}$$

$$F_{\omega_L} = \frac{1}{\omega_L} - \frac{1}{\omega_L - \lambda} - \frac{1 - \alpha}{2(\omega_L - \lambda)^2}. \tag{55}$$

Since $\omega_L = 1 - \omega_H$, we have $d\omega_L/d\omega_H = -1$, so the total derivative of F with respect to ω_H is

$$\frac{d}{d\omega_H}F(\lambda;\omega_H,\alpha) = F_{\omega_H} - F_{\omega_L} = \underbrace{\left[\frac{1}{\omega_H+\lambda} - \frac{1}{\omega_H} - \frac{1}{\omega_L} + \frac{1}{\omega_L-\lambda}\right]}_{\text{log term}} + \underbrace{\left[\frac{\alpha}{2(\omega_H+\lambda)^2} + \frac{1-\alpha}{2(\omega_L-\lambda)^2}\right]}_{\text{RHS term}}. \tag{56}$$

A.1.2 Step 2: first derivatives via the Implicit Function Theorem

Let $\lambda^* = \lambda^*(\omega_H, \alpha)$ solve (49). By the implicit function theorem,

$$\frac{\partial \lambda^*}{\partial x} = -\left.\frac{F_x}{F_\lambda}\right|_{(\lambda^*,\omega_H,\alpha)} \qquad \text{for } x \in \{\alpha, \omega_H\}. \tag{57}$$

Derivative with respect to α. Combining (52)–(53) with (57) gives

$$\boxed{\frac{\partial \lambda^*}{\partial \alpha} = -\frac{F_\alpha}{F_\lambda} = \frac{\frac{1}{2}\left[\frac{1}{\omega_H+\lambda^*} + \frac{1}{\omega_L-\lambda^*}\right]}{\frac{1}{\omega_H+\lambda^*} + \frac{1}{\omega_L-\lambda^*} + \frac{1}{2}\left[\frac{\alpha}{(\omega_H+\lambda^*)^2} + \frac{1-\alpha}{(\omega_L-\lambda^*)^2}\right]} > 0.} \tag{58}$$

Thus, *the informational wedge indexed by λ^* is increasing in α.*

A.2 Welfare derivative with respect to z_H (endogenous wage).

Consider equilibrium welfare

$$W(z_H) = \omega_L \cdot \left[\log c^W - \tfrac{1}{2}\cdot\left(\frac{y^W}{w}\right)^2\right] + \omega_H \cdot \left[\log c^E - \tfrac{1}{2}\cdot\left(e^*(d^E; z_H, w)\right)^2\right],$$

evaluated at the closed-form allocation in the regime where (IC_H) binds and (IC_L) is slack.

Step 1: λ^* does not depend on z_H. The scalar equation pinning down λ^* is (40), which depends only on $(\omega_H, \omega_L, \alpha)$. Hence, within this regime,

$$\frac{d\lambda^*}{dz_H} = 0.$$

Step 2: the disutility terms are z_H-invariant at the optimum. Using the closed-form simplifications,

$$\left(\frac{y^W}{w}\right)^2 = \frac{1-\alpha}{\omega_L-\lambda^*}, \qquad \left(e^*(d^E; z_H, w)\right)^2 = \frac{\alpha}{\omega_H+\lambda^*},$$

so both disutility terms are constant with respect to z_H (since λ^* is constant).

Step 3: c^W and c^E scale linearly with z_H. From the wage formula (41),

$$w(\lambda^*) = z_H \cdot \Xi(\lambda^*, \omega_H, \omega_L, \alpha),$$

for some positive factor $\Xi(\cdot)$ independent of z_H. Then (42) implies

$$d^E(\lambda^*) = \frac{w(\lambda^*)}{s} \cdot \sqrt{\frac{1-\alpha}{\omega_L - \lambda^*}} = z_H \cdot \kappa(\lambda^*, \omega_H, \omega_L, \alpha),$$

so d^E is linear in z_H. Finally, (43) gives

$$c^E(\lambda^*) = (\omega_H + \lambda^*) \cdot \frac{d^E(\lambda^*)}{\alpha} = z_H \cdot A_E, \qquad c^W(\lambda^*) = \frac{\omega_L - \lambda^*}{\omega_L} \cdot \omega_H \cdot \frac{d^E(\lambda^*)}{\alpha} = z_H \cdot A_W,$$

with constants $A_E, A_W > 0$ independent of z_H.

Step 4: differentiate welfare. Because only the log terms depend on z_H and c^E, c^W are linear in z_H,

$$\frac{d}{dz_H} \log c^E(\lambda^*) = \frac{1}{z_H}, \qquad \frac{d}{dz_H} \log c^W(\lambda^*) = \frac{1}{z_H}.$$

Therefore,

$$\boxed{\frac{dW}{dz_H} = \omega_L \cdot \frac{1}{z_H} + \omega_H \cdot \frac{1}{z_H} = \frac{1}{z_H}.} \tag{59}$$

B Computational Appendix

B.1 Wage normalization: detailed derivation

This appendix derives the wage-normalized formulation used in the computational LP. The competitive wage $w > 0$ is an endogenous scalar price (the shadow price on labor market clearing). Since only relative prices matter, we can choose the wage as numeraire. Formally, we divide all *nominal* (monetary) objects by w and rewrite the model in terms of wage-normalized variables.

B.1.1 Normalization map

Define wage-normalized ("real") versions of all monetary variables:

$$\tilde{c} := \frac{c}{w}, \qquad \tilde{y} := \frac{y}{w}, \qquad \tilde{d} := \frac{d}{w}, \qquad \tilde{\kappa}_m := \frac{\kappa_m}{w} \quad (m = 1, 2), \qquad \tilde{z} := \frac{z}{w}. \tag{60}$$

We do *not* rescale real inputs and productivities measured in efficiency units: skill s, efforts (ℓ, e), labor demand n, and the shock ε are unchanged.

B.1.2 Worker income under normalization

Recall the definition of observed labor income:

$$y \;=\; w \cdot s \cdot \ell \;-\; \kappa_1 \cdot \mathbf{1}[\ell > 0]. \tag{61}$$

Divide (61) by w and use (60):

$$\tilde{y} \;=\; s \cdot \ell \;-\; \tilde{\kappa}_1 \cdot \mathbf{1}[\ell > 0]. \tag{62}$$

Because $w > 0$, the sign of y is preserved by normalization: $y > 0 \iff \tilde{y} > 0$.

Implied worker effort from a labor-income assignment. From the original mapping,

$$\ell \;=\; \frac{y + \kappa_1 \mathbf{1}[y > 0]}{w \cdot s}, \tag{63}$$

substitute $y = w\tilde{y}$ and $\kappa_1 = w\tilde{\kappa}_1$ to obtain

$$\ell \;=\; \frac{w\tilde{y} + w\tilde{\kappa}_1 \mathbf{1}[\tilde{y} > 0]}{w \cdot s} \;=\; \frac{\tilde{y} + \tilde{\kappa}_1 \mathbf{1}[\tilde{y} > 0]}{s}. \tag{64}$$

Equivalently,

$$\ell(\tilde{y}; s) := \begin{cases} \dfrac{\tilde{y} + \tilde{\kappa}_1}{s}, & \tilde{y} > 0, \\[2ex] 0, & \tilde{y} = 0. \end{cases} \tag{65}$$

B.1.3 Entrepreneurial dividends under normalization

If the agent operates a firm ($n > 0$), output is

$$q \;=\; z \cdot \varepsilon \cdot e^{\alpha} \cdot n^{1-\alpha}, \qquad \alpha \in (0, 1), \tag{66}$$

and observed dividends are

$$d \;=\; q - wn - \kappa_2 \cdot \mathbf{1}[n > 0] \;=\; z \cdot \varepsilon \cdot e^{\alpha} \cdot n^{1-\alpha} - wn - \kappa_2 \cdot \mathbf{1}[n > 0]. \tag{67}$$

Divide (67) by w and use (60):

$$\begin{aligned} \tilde{d} &= \frac{z}{w} \cdot \varepsilon \cdot e^{\alpha} \cdot n^{1-\alpha} \;-\; n \;-\; \frac{\kappa_2}{w} \cdot \mathbf{1}[n > 0] \\ &= \tilde{z} \cdot \varepsilon \cdot e^{\alpha} \cdot n^{1-\alpha} \;-\; n \;-\; \tilde{\kappa}_2 \cdot \mathbf{1}[n > 0]. \end{aligned} \tag{68}$$

Thus, the "q/w" term is absorbed into $\tilde{z} = z/w$, preserving the functional form of the dividend equation.

B.1.4 Entrepreneurial input identity and implied effort

The original identity linking observables (d) to private inputs (e, n) is

$$z \cdot \varepsilon \cdot e^{\alpha} n^{1-\alpha} \;=\; d + wn + \kappa_2, \qquad e \geq 0, \; n \geq 0. \tag{69}$$

Divide (69) by w and substitute (60):

$$\tilde{z} \cdot \varepsilon \cdot e^{\alpha} n^{1-\alpha} \;=\; \tilde{d} + n + \tilde{\kappa}_2, \qquad e \geq 0, \; n \geq 0. \tag{70}$$

Implied entrepreneurial effort as a function of labor demand. For any fixed $n > 0$, the original mapping is

$$e(n; d, z, \varepsilon, w) \;=\; \left(\frac{d + \kappa_2 + wn}{z \cdot \varepsilon \cdot n^{1-\alpha}}\right)^{1/\alpha}. \tag{71}$$

Using $d = w\tilde{d}$, $\kappa_2 = w\tilde{\kappa}_2$, $z = w\tilde{z}$ yields

$$e(n; \tilde{d}, \tilde{z}, \varepsilon) = \left(\frac{w(\tilde{d} + \tilde{\kappa}_2 + n)}{w\tilde{z} \cdot \varepsilon \cdot n^{1-\alpha}}\right)^{1/\alpha} = \left(\frac{\tilde{d} + \tilde{\kappa}_2 + n}{\tilde{z} \cdot \varepsilon \cdot n^{1-\alpha}}\right)^{1/\alpha}. \tag{72}$$

Least-effort labor demand. Under the selection rule that the agent chooses (e, n) minimizing e (equivalently $V(e)$), one obtains in the original formulation

$$n^*(d; w) \;:=\; \frac{1-\alpha}{\alpha} \cdot \frac{d + \kappa_2}{w} \qquad \text{for } d > 0. \tag{73}$$

Substituting $d = w\tilde{d}$ and $\kappa_2 = w\tilde{\kappa}_2$ gives the normalized form

$$n^*(\tilde{d}) \;:=\; \frac{1-\alpha}{\alpha} \cdot \big(\tilde{d} + \tilde{\kappa}_2\big) \qquad \text{for } \tilde{d} > 0. \tag{74}$$

Define $n^*(0) = 0$.

Implied least-effort entrepreneurial effort. Plugging (74) into (70) yields the implied least-effort entrepreneurial effort:

$$e^*(\tilde{d}; \tilde{z}, \varepsilon) := \left(\frac{\tilde{d} + \tilde{\kappa}_2 + n^*(\tilde{d})}{\tilde{z} \cdot \varepsilon \cdot \big(n^*(\tilde{d})\big)^{1-\alpha}}\right)^{1/\alpha}, \qquad e^*(0; \tilde{z}, \varepsilon) := 0. \tag{75}$$

B.1.5 Resource constraint under normalization

The aggregate resource constraint is

$$\int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon\big[c\big(y(s,z), d(s,z,\varepsilon)\big)\big] \cdot f(s,z)\, ds\, dz \;\le\; \int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon[y(s,z) + d(s,z,\varepsilon)] \cdot f(s,z)\, ds\, dz. \tag{76}$$

Divide both sides of (76) by w and use (60):

$$\int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon\Big[\tilde{c}\big(\tilde{y}(s,z), \tilde{d}(s,z,\varepsilon)\big)\Big] \cdot f(s,z)\, ds\, dz \;\le\; \int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon\Big[\tilde{y}(s,z) + \tilde{d}(s,z,\varepsilon)\Big] \cdot f(s,z)\, ds\, dz. \tag{77}$$

B.1.6 Labor market clearing under normalization

Labor market clearing in efficiency units is

$$\int_{\mathcal{S}\times\mathcal{Z}} s \cdot \ell(s,z) \cdot f(s,z) \cdot ds\, dz = \int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon\big[n^*(d(s,z,\varepsilon); w)\big] \cdot f(s,z)\, ds\, dz. \tag{78}$$

Using the normalized effort mappings (65) and (74), we can rewrite (78) equivalently as

$$\int_{\mathcal{S}\times\mathcal{Z}} s \cdot \ell(\tilde{y}(s,z);s) \cdot f(s,z)\,ds\,dz = \int_{\mathcal{S}\times\mathcal{Z}} \mathbb{E}_\varepsilon\big[n^*(\tilde{d}(s,z,\varepsilon))\big] \cdot f(s,z)\,ds\,dz, \tag{79}$$

which contains no explicit w.

B.1.7 Incentive constraints under normalization

The ex ante incentive constraints require that for all true types (s,z) and all reports $(\hat{s},\hat{z})$,

$$\begin{aligned}
&\mathbb{E}_\varepsilon\big[U\big(c(y(s,z),d(s,z,\varepsilon))\big) - V\big(e(d(s,z,\varepsilon);z,\varepsilon,w)\big)\big] - V\big(\ell(y(s,z);s,w)\big) \;\geq \\
&\mathbb{E}_\varepsilon\big[U\big(c(y(\hat{s},\hat{z}),d(\hat{s},\hat{z},\varepsilon))\big) - V\big(e(d(\hat{s},\hat{z},\varepsilon);z,\varepsilon,w)\big)\big] - V\big(\ell(y(\hat{s},\hat{z});s,w)\big).
\end{aligned} \tag{80}$$

Under the normalization (60), replace $(c,y,d,\kappa_1,\kappa_2,z)$ by $(\tilde{c},\tilde{y},\tilde{d},\tilde{\kappa}_1,\tilde{\kappa}_2,\tilde{z})$ and use the normalized implied-effort mappings (64) and (75). This yields an IC system identical in form but containing no explicit w.

B.2 Constraint Generation Algorithm

B.2.1 Notation

$N = N_1 \times N_2$	Number of types on the 2D grid
$\theta_i = (\theta_{1,i}, \theta_{2,i})$	Type i's private information (worker skill, entrepreneur ability)
ω_i	Population weight of type i
W_i	Social welfare weight of type i
K	Number of consumption grid points, $\{c_k\}_{k=1}^K$
A	Number of labor income grid points, $\{y_{1,a}\}_{a=1}^A$
B	Number of entrepreneurial dividend grid points, $\{y_{2,b}\}_{b=1}^B$
R	Number of shock states, $\{\varepsilon_r\}_{r=1}^R$ with probabilities $\{p_r\}_{r=1}^R$
$U(c_k)$	CRRA consumption utility at grid point k
$D_{i,a,b,r}$	Total effort disutility for type i at income pair (a,b) in state r
$\lambda_{i,k,r}$	Probability that type i receives consumption c_k in state r
$\mu_{i,a,b,r}$	Probability that type i produces income $(y_{1,a}, y_{2,b})$ in state r
$\mathcal{P}$	Active set of incentive-compatibility (IC) pairs

The disutility coefficients are:

$$D_{i,a,b,r} = V\left(\frac{y_{1,a} + \kappa_1 \cdot \mathbf{1}_{\{y_{1,a}>0\}}}{\theta_{1,i}}\right) + V\left(e^*_{i,b,r}\right),$$

where $V(\ell) = \frac{\ell^{1+\eta}}{1+\eta}$ is the power disutility of effort, and $e^*_{i,b,r}$ is the minimum effort required for type i to generate dividend $y_{2,b}$ in shock state r.

B.2.2 Relaxed Master Problem: LP($\mathcal{P}$)

Given an active set of IC pairs $\mathcal{P} \subseteq \{1, \ldots, N\}^2 \setminus \{(i,i)\}$, solve:

$$\max_{\lambda,\mu} \quad \sum_{i=1}^{N} \omega_i \, W_i \, \mathrm{EU}(i) \tag{81}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \lambda_{i,k,r} = 1, \qquad \forall\, i = 1, \ldots, N, \; r = 1, \ldots, R \tag{82}$$

$$\sum_{(a,b)\in\mathcal{A}} \mu_{i,a,b,r} = 1, \qquad \forall\, i = 1, \ldots, N, \; r = 1, \ldots, R \tag{83}$$

$$\mathrm{EU}(i) \geq \mathrm{DevUtil}(i,j), \qquad \forall\, (i,j) \in \mathcal{P} \tag{84}$$

$$\sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{k=1}^{K} \lambda_{i,k,r}\, c_k \;\leq\; \sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{A}} \mu_{i,a,b,r}(y_{1,a} + y_{2,b}) \tag{85}$$

$$\lambda_{i,k,r} \geq 0, \quad \mu_{i,a,b,r} \geq 0, \qquad \forall\, i, k, a, b, r \tag{86}$$

where the set of allowed income pairs is $\mathcal{A} = \{(a,b) : \mathrm{not}(y_{1,a} > 0 \text{ and } y_{2,b} > 0)\}$ (occupational exclusivity), and the expected utilities are:

$$\mathrm{EU}(i) = \sum_{r=1}^{R} p_r \left[\sum_{k=1}^{K} \lambda_{i,k,r}\, U(c_k) \;-\; \sum_{(a,b)\in\mathcal{A}} \mu_{i,a,b,r}\, D_{i,a,b,r} \right], \tag{87}$$

$$\mathrm{DevUtil}(i,j) = \sum_{r=1}^{R} p_r \left[\sum_{k=1}^{K} \lambda_{j,k,r}\, U(c_k) \;-\; \sum_{(a,b)\in\mathcal{A}} \mu_{j,a,b,r}\, D_{i,a,b,r} \right]. \tag{88}$$

Note: DevUtil(i,j) uses type j's lottery (λ_j, μ_j) but type i's disutility D_i; it represents the payoff type i would obtain by mimicking type j.
Optionally, a labor market clearing constraint is added (general equilibrium):

$$\sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{A}} \mu_{i,a,b,r}\, L^s_{i,a} \;=\; \sum_{i=1}^{N} \omega_i \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{A}} \mu_{i,a,b,r}\, L^d_{i,b,r}, \tag{89}$$

where $L^s_{i,a}$ is labor supply and $L^d_{i,b,r}$ is equilibrium labor demand from type i's firm at dividend $y_{2,b}$ in state r.

B.2.3 Local IC Pair Initialization

Types are indexed on an $N_1 \times N_2$ grid where $i = (i_2 - 1)N_1 + i_1$ for $i_1 = 1, \ldots, N_1$ and $i_2 = 1, \ldots, N_2$. The local neighbor pairs are:

Algorithm 1 BuildLocalICPairs(N, N_1, N_2)

Input: Grid dimensions N_1, N_2 with $N = N_1 \times N_2$

Output: Set of adjacent-neighbor IC pairs $\mathcal{P}_0$

$\mathcal{P}_0 \leftarrow \emptyset$
for $i = 1, \ldots, N$ **do**
if $i \bmod N_1 \neq 0$ **then** ▷ Neighbor to the right in θ_1
$\mathcal{P}_0 \leftarrow \mathcal{P}_0 \cup \{(i, i+1)\}$
end if
if $i \bmod N_1 \neq 1$ **then** ▷ Neighbor to the left in θ_1
$\mathcal{P}_0 \leftarrow \mathcal{P}_0 \cup \{(i, i-1)\}$
end if
if $i \leq N - N_1$ **then** ▷ Neighbor above in θ_2
$\mathcal{P}_0 \leftarrow \mathcal{P}_0 \cup \{(i, i+N_1)\}$
end if
if $i > N_1$ **then** ▷ Neighbor below in θ_2
$\mathcal{P}_0 \leftarrow \mathcal{P}_0 \cup \{(i, i-N_1)\}$
end if
end for
return $\mathcal{P}_0$

B.2.4 IC Violation Computation

Algorithm 2 ComputeICViolations($\lambda^*, \mu^*, U, D, p, N$)

Input: Optimal marginals λ^*, μ^* from the LP; utilities $U(c_k)$; disutilities $D_{i,a,b,r}$; shock probabilities p_r

Output: Violation matrix $\mathbf{V} \in \mathcal{R}^{N \times N}$, maximum violation, list of violated pairs

for $i = 1, \ldots, N$ **do** ▷ Compute expected consumption utility
  for $r = 1, \ldots, R$ **do**
    $\text{ConsUtil}(i, r) \leftarrow \sum_{k=1}^{K} \lambda^*_{i,k,r} U(c_k)$
  end for
  $\text{ECons}(i) \leftarrow \sum_{r=1}^{R} p_r \cdot \text{ConsUtil}(i, r)$
end for

for $i = 1, \ldots, N$ **do** ▷ Compute truthful disutility
  $\text{TruthDis}(i) \leftarrow \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{A}} \mu^*_{i,a,b,r} D_{i,a,b,r}$
end for

for $i = 1, \ldots, N$ **do** ▷ Truthful expected utility
  $\text{EU}(i) \leftarrow \text{ECons}(i) - \text{TruthDis}(i)$
end for

for $i = 1, \ldots, N$ **do** ▷ Compute cross-disutility matrix
  for $j = 1, \ldots, N$ **do**
    $\text{DevDis}(i, j) \leftarrow \sum_{r=1}^{R} p_r \sum_{(a,b)\in\mathcal{A}} \mu^*_{j,a,b,r} D_{i,a,b,r}$
  end for
end for

for $i = 1, \ldots, N$ **do** ▷ Compute violation matrix
  for $j = 1, \ldots, N,\ j \neq i$ **do**
    $\mathbf{V}(i, j) \leftarrow \left[\text{ECons}(j) - \text{DevDis}(i, j)\right] - \text{EU}(i)$ ▷ $= \text{DevUtil}(i, j) - \text{EU}(i)$
  end for
end for

$v^{\max} \leftarrow \max_{i \neq j} \mathbf{V}(i, j)$
$\text{ViolPairs} \leftarrow \{(i, j) : \mathbf{V}(i, j) > \tau,\ i \neq j\}$
return $\mathbf{V}$, $v^{\max}$, ViolPairs

Algorithm 3 Constraint Generation for the Mirrlees Optimal Tax LP

Input: Type grid $\{\theta_i\}_{i=1}^{N}$ on $N_1 \times N_2$; weights ω_i, W_i; grids $\{c_k\}$, $\{y_{1,a}\}$, $\{y_{2,b}\}$; utility $U(\cdot)$; disutility $D_{i,a,b,r}$; shocks $(\varepsilon_r, p_r)_{r=1}^{R}$; tolerance $\tau > 0$; max iterations T; max pairs per iteration M

Output: Optimal allocations (λ^*, μ^*); social welfare SW*; active IC pair set $\mathcal{P}^*$

Phase 1: Initialization

$\mathcal{P} \leftarrow \textsc{BuildLocalICPairs}(N, N_1, N_2)$ ▷ $|\mathcal{P}| \approx 4N$ local pairs

Phase 2: Constraint Generation Loop

for $t = 1, 2, \ldots, T$ **do**

  Step 2a: Solve Relaxed Master Problem

  $(\lambda^*, \mu^*, \mathrm{SW}) \leftarrow$ Solve LP($\mathcal{P}$) via AMPL/CPLEX ▷ Eqs. (81)–(86)

  if LP infeasible or solver failure **then**

    return failure

  end if

  Step 2b: Separation Oracle — Check All N^2 IC Constraints

  $(\mathbf{V}, v^{\max}, \mathrm{ViolPairs}) \leftarrow \textsc{ComputeICViolations}(\lambda^*, \mu^*, U, D, p, N)$

  Step 2c: Filter Active-Set Artifacts

  $$\mathbf{V}_{\text{out}}(i,j) \leftarrow \begin{cases} \mathbf{V}(i,j) & \text{if } (i,j) \notin \mathcal{P} \text{ and } i \neq j \\ -\infty & \text{otherwise} \end{cases}$$

  $v_{\text{out}}^{\max} \leftarrow \max_{i,j} \mathbf{V}_{\text{out}}(i,j)$

  $\mathrm{NewViol} \leftarrow \{(i,j) \in \mathrm{ViolPairs} : (i,j) \notin \mathcal{P}\}$

  Step 2d: Sort by Violation Magnitude

  Sort NewViol in decreasing order of $\mathbf{V}(i,j)$

  Step 2e: Convergence Check

  if $v_{\text{out}}^{\max} \leq \tau$ **then**

    return $(\lambda^*, \mu^*, \mathrm{SW}, \mathcal{P})$ ▷ Converged: all IC constraints satisfied

  end if

  Step 2f: Augment Active Set

  $n_{\text{add}} \leftarrow \min(|\mathrm{NewViol}|, M)$

  $\mathcal{P} \leftarrow \mathcal{P} \cup \mathrm{NewViol}[1 : n_{\text{add}}]$ ▷ Add most-violated pairs

  Remove duplicates from $\mathcal{P}$

end for

return $(\lambda^*, \mu^*, \mathrm{SW}, \mathcal{P})$ with warning: did not converge in T iterations

B.2.5 Main Algorithm: Constraint Generation

B.2.6 Remarks

1. **Initialization.** The algorithm starts with only $|\mathcal{P}_0| \approx 4N$ local IC constraints (adjacent neighbors on the 2D type grid), rather than the full $N^2 - N$ pairwise constraints. For $N = 2{,}500$ (a 50×50 grid), this reduces the initial constraint count from $\sim 6.25 \times 10^6$ to $\sim 10{,}000$.

2. **Separation oracle.** At each iteration, the full $N \times N$ violation matrix is computed in MATLAB via vectorized matrix operations (the cross-disutility matrix $\mathrm{DevDis}(i,j) = \sum_r p_r \, \mathbf{D}_r \, \mathbf{M}_r^\top$ is computed as a matrix product), making the separation step efficient despite its $O(N^2)$ scaling.

3. **Convergence criterion.** Only violations *outside* the current active set $\mathcal{P}$ determine convergence. Violations on active-set constraints (which should be zero in exact arithmetic) may be nonzero due to LP solver numerical tolerances and are treated as artifacts.

4. **Capping.** At most M new pairs are added per iteration (sorted by violation magnitude in descending order) to prevent the LP size from growing too rapidly.

5. **Marginal lotteries.** The LP uses *marginal* lotteries $\lambda_{i,k,r}$ (consumption) and $\mu_{i,a,b,r}$ (income) rather than the joint lottery $x_{i,k,a,b,r}$. This is valid because the IC constraints and the objective depend on consumption and income separately through their marginals, and reduces the number of decision variables from $O(N \cdot K \cdot A \cdot B \cdot R)$ to $O(N \cdot (K + A \cdot B) \cdot R)$.

6. **Occupational exclusivity.** The set of allowed income pairs $\mathcal{A}$ excludes combinations where both $y_{1,a} > 0$ and $y_{2,b} > 0$, enforcing that agents either work or run a firm but not both simultaneously.

7. **Solver.** The LP is formulated as an AMPL model and dispatched to CPLEX (barrier method, `lpmethod=4`) or Gurobi (`method=2, crossover=0`) for solution.