Technical Appendix to *Chicago Fed Letter* No. 523: The Aftermath of the 2026 Oil Shock: Some Alternative Macroeconomic Scenarios

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This appendix describes the model we use for our scenarios. The intended audience is advanced undergraduates as well as more experienced economists.

1 Environment

We consider a New Keynesian economy with external habit formation in consumption, monopolistic competition in goods and labor markets, Rotemberg price and wage adjustment costs with backward indexation, capital accumulation with convex investment adjustment costs, variable capital utilization, and an oil shock that affects firms through the cost of the energy required to operate capital more intensively. Time is discrete, $t = 0, 1, 2, \ldots$. The economy is populated by:

(i) a representative household,

(ii) a continuum of intermediate-goods firms indexed by $i \in [0, 1]$,

(iii) a competitive final-goods producer,

(iv) a competitive labor aggregator, and

(v) a monetary authority.

The key variables are:

- installed capital K_{t-1},
- utilization U_t,
- effective capital services $U_t K_{t-1}$,
- an energy composite E_t required to sustain utilization,
- the dual real energy price index P_t^e generated by a constant elasticity of substitution (CES) aggregator over oil and nonoil energy, and
- labor demand N_t.

Now we describe the model in mathematical terms.

2 Final-goods firm

A perfectly competitive final goods producer aggregates differentiated intermediate goods according to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > 1. \tag{1}$$

Cost minimization implies the standard demand system

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \tag{2}$$

and the aggregate price index

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}. \tag{3}$$

3 Household

3.1 Preferences

The representative household chooses consumption, labor hours, bond holdings, investment, and state-contingent claims on firms so as to maximize

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right], \tag{4}$$

where $\beta \in (0,1)$ is the discount factor, $h \in [0,1)$ is external habit, $\sigma > 0$ governs curvature in utility from consumption, and $\varphi > 0$ is the inverse Frisch elasticity.

3.2 Budget constraint

In nominal terms, the household budget constraint is

$$P_t C_t + P_t I_t + B_t \leq R_{t-1} B_{t-1} + W_t N_t + \mathcal{R}_t^k + \Pi_t^f + \Pi_t^w - T_t, \tag{5}$$

where:

- B_t denotes one-period nominal bonds,
- R_t is the gross nominal interest rate,
- W_t is the nominal wage index,
- $\mathcal{R}_t^k = P_t R_t^k K_{t-1}$ denotes the income from ownership of capital,
- Π_t^f and Π_t^w are distributed profits from firms and wage adjustment, and
- T_t are lump-sum taxes or transfers.

Let Λ_t denote the multiplier on the real budget constraint. Then the first-order condition for consumption is

$$\Lambda_t = (C_t - hC_{t-1})^{-\sigma} - \beta h \, (C_{t+1} - hC_t)^{-\sigma}. \tag{6}$$

The bond Euler equation is

$$\Lambda_t = \beta \, \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}}, \qquad \Pi_t \equiv \frac{P_t}{P_{t-1}}. \tag{7}$$

3.3 Capital accumulation and investment

The household owns installed capital and chooses investment. Capital evolves according to

$$K_t = (1-\delta)K_{t-1} + \big(1 - S(X_t)\big)I_t, \qquad X_t \equiv \frac{I_t}{I_{t-1}}, \tag{8}$$

where $\delta \in (0,1)$ is the depreciation rate and

$$S(X_t) = \frac{\phi_I}{2}(X_t - 1)^2 \tag{9}$$

is the convex investment adjustment cost function. Let Q_t denote the shadow value of installed capital in units of final goods. The household chooses $\{I_t, K_t\}$, taking prices and rental rates as given. The first-order condition with respect to investment is

$$1 = Q_t\Big(1 - S(X_t) - X_t S'(X_t)\Big) + \beta\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}S'(X_{t+1})X_{t+1}^2. \tag{10}$$

The Euler equation for installed capital is

$$Q_t\Lambda_t = \beta\Lambda_{t+1}\big(R^k_{t+1} + (1-\delta)Q_{t+1}\big), \tag{11}$$

where R^k_t is the real rental rate of installed capital.

4 Labor aggregation and wage setting

A competitive labor aggregator combines differentiated labor services $N_t(j)$ according to

$$N_t = \left(\int_0^1 N_t(j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}}, dj\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \qquad \varepsilon_w > 1. \tag{12}$$

Cost minimization implies labor demand

$$N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t, \tag{13}$$

and the aggregate wage index

$$W_t = \left(\int_0^1 W_t(j)^{1-\varepsilon_w}, dj\right)^{\frac{1}{1-\varepsilon_w}}. \tag{14}$$

Let Λ_t denote the marginal utility value of real income. The household's flexible real wage is

$$\frac{W_t^*}{P_t} = N_t^{\varphi}\Lambda_t^{-1}. \tag{15}$$

This is the real wage that would equate the marginal disutility of supplying labor to the marginal utility value of wage income in the absence of wage-adjustment frictions.

Define gross wage inflation by

$$\Pi_t^w \equiv \frac{W_t}{W_{t-1}}, \tag{16}$$

and let the indexed gross wage change be

$$\Xi_t^w \equiv \frac{\Pi_t^w}{(\Pi_{t-1}^w)^{\iota_w}}, \qquad \iota_w \in [0,1]. \tag{17}$$

The wage block is microfounded as a Rotemberg wage-setting problem. A representative wage setter chooses the nominal wage for its labor type, taking the demand schedule (13) as given, in order to maximize the present discounted value of utility-valued net returns from labor supply:

$$\max_{W_t(j)} \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[\frac{W_t(j)}{P_t} N_t(j) - \int_0^{N_t(j)} \frac{W_t^*}{P_t}, dn - \frac{\phi_W}{2} \left(\Xi_t^w(j) - 1 \right)^2 N_t \right], \tag{18}$$

where

$$\Xi_t^w(j) \equiv \frac{W_t(j)/W_{t-1}(j)}{(\Pi_{t-1}^w)^{\iota_w}}. \tag{19}$$

The first term is real wage income, the second term is the utility cost of supplying labor summarized by the target wage W_t^*/P_t, and the third term is a Rotemberg adjustment cost for changing wages relative to indexed past wage inflation.

In a symmetric equilibrium, with $W_t(j) = W_t$ for all j, the first-order condition for wage setting yields the nonlinear wage Phillips curve

$$\begin{aligned} 0 = (1-\varepsilon_w) + (\varepsilon_w - 1)\frac{W_t^*/P_t}{W_t/P_t} - \phi_W \left(\frac{\Pi_t^w}{(\Pi_{t-1}^w)^{\iota_w}} - 1 \right) \frac{\Pi_t^w}{(\Pi_{t-1}^w)^{\iota_w}} (\Pi_{t-1}^w)^{\iota_w} \\ + \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{N_{t+1}}{N_t}, \phi_W \left(\frac{\Pi_{t+1}^w}{(\Pi_t^w)^{\iota_w}} - 1 \right) \frac{\Pi_{t+1}^w}{(\Pi_t^w)^{\iota_w}} (\Pi_t^w)^{\iota_w}. \end{aligned} \tag{20}$$

This is the exact nonlinear wage-setting condition used in the code. The parameter ϕ_W is chosen so that the local slope of the nonlinear wage block matches the familiar Erceg, Henderson, and Levin (EHL) wage Phillips curve (Erceg et al., 2000) around the deterministic steady state.

Finally, the aggregate real wage evolves according to

$$\frac{W_t}{P_t} = \frac{W_{t-1}}{P_{t-1}} \frac{\Pi_t^w}{\Pi_t}. \tag{21}$$

5 Intermediate firms

5.1 Production

Each intermediate firm i produces output using total factor productivity (TFP), utilized capital services, and labor:

$$Y_t(i) = A_t \big(U_t(i) K_{t-1}(i) \big)^{\alpha_u} N_t(i)^{1-\alpha_u}, \qquad \alpha_u \in (0,1). \tag{22}$$

Aggregate productivity evolves according to

$$A_t = A_{t-1} G_t, \tag{23}$$

where G_t is exogenous gross productivity growth.

5.2 Energy composite and dual price index

To operate capital more intensively, the firm must use energy. Let $O_t(i)$ denote oil input, $M_t(i)$ denote nonoil energy input, and $E_t(i)$ denote the energy composite. We assume a CES aggregator:

$$E_t(i) = \left[\omega_o^{1/\eta_e} O_t(i)^{\frac{\eta_e-1}{\eta_e}} + (1-\omega_o)^{1/\eta_e} M_t(i)^{\frac{\eta_e-1}{\eta_e}}\right]^{\frac{\eta_e}{\eta_e-1}}, \qquad \eta_e > 0. \tag{24}$$

Here $\omega_o \in (0,1)$ governs oil's share in the energy bundle and η_e is the elasticity of substitution between oil and nonoil energy. Let P_t^o denote the real oil price in units of final goods, and normalize the real price of nonoil energy to one. Then the unit-expenditure minimization problem for one unit of energy is

$$\min_{O_t(i), M_t(i)} \{P_t^o O_t(i) + M_t(i)\} \quad \text{s.t.} \quad (24). \tag{25}$$

The associated unit-cost function is the CES dual energy price index

$$P_t^e = \left[\omega_o (P_t^o)^{1-\eta_e} + (1-\omega_o)\right]^{\frac{1}{1-\eta_e}}. \tag{26}$$

Conditional on total energy demand $E_t(i)$, the cost-minimizing energy input demands are

$$O_t(i) = \omega_o \left(\frac{P_t^o}{P_t^e}\right)^{-\eta_e} E_t(i), \qquad M_t(i) = (1-\omega_o)\left(\frac{1}{P_t^e}\right)^{-\eta_e} E_t(i). \tag{27}$$

Hence the firm's total real energy bill is

$$P_t^e E_t(i) = P_t^o O_t(i) + M_t(i). \tag{28}$$

5.3 Reduced-form link from energy to utilization

Following the oil-utilization specification in the spirit of Leduc and Sill (2004), energy use is proportional to installed capital and increasing in the utilization rate:

$$\frac{E_t(i)}{K_{t-1}(i)} = \frac{U_t(i)^\zeta}{\zeta}, \qquad \zeta > 1. \tag{29}$$

Equivalently,

$$E_t(i) = K_{t-1}(i) \frac{U_t(i)^\zeta}{\zeta}. \tag{30}$$

This captures the idea not only that firms with a larger installed capital stock require more energy to operate that capital, but also that higher utilization raises energy use at an increasing marginal rate.

5.4 Static operating problem

Given a target output level $Y_t(i)$ and installed capital $K_{t-1}(i)$, the firm chooses labor and utilization to minimize real variable cost:

$$\min_{N_t(i), U_t(i)} \left\{\frac{W_t}{P_t} N_t(i) + P_t^e K_{t-1}(i) \frac{U_t(i)^\zeta}{\zeta}\right\} \quad \text{s.t.} \quad Y_t(i) = A_t \big(U_t(i) K_{t-1}(i)\big)^{\alpha_u} N_t(i)^{1-\alpha_u}. \tag{31}$$

Let $MC_t(i)$ denote the real marginal cost multiplier on the production constraint. The first-order conditions are

$$\frac{W_t}{P_t} = MC_t(i)(1-\alpha_u)\frac{Y_t(i)}{N_t(i)} \tag{32}$$

and

$$P_t^e K_{t-1}(i) U_t(i)^{\zeta-1} = MC_t(i)\alpha_u \frac{Y_t(i)}{U_t(i)}. \tag{33}$$

Equivalently,

$$P_t^e K_{t-1}(i) U_t(i)^{\zeta} = \alpha_u MC_t(i) Y_t(i). \tag{34}$$

Thus utilization is chosen so that the marginal energy cost of operating capital more intensively equals the marginal production benefit.

5.5 Rental rate of installed capital

The marginal contribution of installed capital implies the rental-rate condition

$$R_t^k(i) = MC_t(i)\alpha_u \frac{Y_t(i)}{K_{t-1}(i)}. \tag{35}$$

Under symmetry, this becomes the aggregate rental rate of capital.

5.6 Dynamic dividend maximization and price setting

Given demand (2), firm i chooses its price, labor, utilization, and energy inputs to maximize the present discounted value of real dividends:

$$\max_{\{P_t(i),N_t(i),U_t(i),O_t(i),M_t(i)\}_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t(i), \tag{36}$$

where real dividends are

$$D_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} N_t(i) - P_t^o O_t(i) - M_t(i) - \frac{\phi_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)\Pi_{t-1}^{\iota_p}} - 1\right)^2 Y_t. \tag{37}$$

The firm is subject to production (22), the energy composite (24), the reduced-form energy requirement (30), and demand (2). After solving out the static cost-minimization problem, the remaining intertemporal choice is price setting under Rotemberg adjustment costs. In a symmetric equilibrium, the corresponding nonlinear price Phillips curve can be written as

$$\begin{aligned} 0 = {} & (1-\varepsilon) + (\varepsilon - 1) MC_t \\ & - \phi_P \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p}} - 1\right) \frac{\Pi_t}{\Pi_{t-1}^{\iota_p}} \Pi_{t-1}^{\iota_p} \\ & + \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \phi_P \left(\frac{\Pi_{t+1}}{\Pi_t^{\iota_p}} - 1\right) \frac{\Pi_{t+1}}{\Pi_t^{\iota_p}} \Pi_t^{\iota_p}. \end{aligned} \tag{38}$$

6 Natural allocation

The natural allocation is the flexible-price, flexible-wage counterpart of the economy. It satisfies the same household optimality conditions, the same capital-accumulation block, the same production function, and the same energy-utilization relation, but without nominal adjustment costs and with constant markups. In particular,

$$MC_t^n = \bar{MC} = \frac{\varepsilon - 1}{\varepsilon}, \tag{39}$$

$$Y_t^n = A_t (U_t^n K_{t-1}^n)^{\alpha_u} (N_t^n)^{1-\alpha_u}, \tag{40}$$

$$\frac{W_t^n}{P_t} = MC_t^n (1 - \alpha_u) \frac{Y_t^n}{N_t^n}, \tag{41}$$

$$P_t^e K_{t-1}^n (U_t^n)^\zeta = \alpha_u MC_t^n Y_t^n, \tag{42}$$

$$R_t^{k,n} = MC_t^n \alpha_u \frac{Y_t^n}{K_{t-1}^n}. \tag{43}$$

The natural real interest rate is

$$R_t^n = \frac{\Lambda_t^n}{\beta \Lambda_{t+1}^n}, \tag{44}$$

and the output gap is

$$\widetilde{y}_t \equiv \log\left(\frac{Y_t}{Y_t^n}\right). \tag{45}$$

7 Monetary policy

The central bank follows a Taylor rule with interest rate smoothing. Written in normalized form, the rule is

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_x} \left(\frac{R_t^n}{\bar{R}^n}\right)^{\chi_r}\right]^{1-\rho_R} e^{\varepsilon_t^R}, \tag{46}$$

where $\rho_R \in [0, 1)$ governs smoothing, ϕ_π and ϕ_x are policy coefficients, and χ_r is either zero or ϕ_r depending on whether the natural rate enters the rule. In the oil-shock scenarios considered here, the monetary policy shock is set to zero, $\varepsilon_t^R = 0$.

8 Resource constraint

At the level of the nonlinear code, the aggregate resource constraint is

$$Y_t + \mathcal{T}_t^{oil} = C_t + I_t + P_t^e E_t + \mathcal{AC}_t^p + \mathcal{AC}_t^w, \tag{47}$$

where

$$\mathcal{T}_t^{oil} = s_{\text{oilnet}} \bar{Y} \left(P_t^o - 1\right) \tag{48}$$

is the reduced-form oil export rebate,

$$\mathcal{AC}_t^p = \frac{\phi_P}{2} \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p}} - 1\right)^2 Y_t \tag{49}$$

is the Rotemberg price-adjustment cost, and

$$\mathcal{A}C_t^w = \frac{\phi_W}{2}\left(\frac{\Pi_t^w}{(\Pi_{t-1}^w)^{\iota_w}} - 1\right)^2 N_t \tag{50}$$

is the Rotemberg wage-adjustment cost. So the resource constraint is

$$Y_t + s_{\text{oilnet}}\bar{Y}\left(P_t^o - 1\right) = C_t + I_t + P_t^e E_t + \frac{\phi_P}{2}\left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_p}} - 1\right)^2 Y_t + \frac{\phi_W}{2}\left(\frac{\Pi_t^w}{(\Pi_{t-1}^w)^{\iota_w}} - 1\right)^2 N_t. \tag{51}$$

9 Equilibrium and solution

An equilibrium is defined as the solution to the problem of all agents optimizing taking prices and aggregate quantities as given and all but one market clearing. In the labor market, hours worked are demand-determined. In the scenario analysis, there are a few types of shocks we consider, each with slightly different solution mehods:

- When the scenario being considered is a deterministic perfect-foresight transition following an MIT shock, we proceed as follows. The economy starts from its initial steady state. At date zero, agents learn the full future path of the exogenous disturbances and policy assumptions used in the scenario, as described in the main article. The equilibrium path is then obtained by solving the resulting stacked system of nonlinear equilibrium equations, subject to convergence to the terminal steady state far in the future. The computation therefore uses the nonlinear equilibrium conditions directly, rather than a local linear approximation around the initial steady state. This procedure is used for the yellow path in figures 2-4, the red path in figure 3, and the perfect-foresight portion of other paths where applicable.

- When the scenario being considered involves a sequence of more than one surprise, we proceed as follows. For each MIT shock at time t, we first solve the perfect-foresight model as above, starting at time t and assuming no further shocks occur. Second, starting with the state of the economy at time t, we add the (initially unexpected) shock at time $t+1$ and solve the model for period $t+1$. We continue with this procedure for all future (initially unexpected) MIT shocks.

10 Parameters Summary

Table 1: Parameter Calibration

Parameter	Symbol	Value
Preferences		
Discount factor	β	0.99
Risk aversion	σ	1.0
Inverse Frisch elasticity	φ	0.5
External habit	h	0.75
Steady-state hours	$\bar{N}$	0.33
Production		
Capital share in production	α_u	0.4
Depreciation rate	δ	0.025
Energy		
Energy substitution elasticity	η_e	0.5
Oil share of GDP (target)		0.025
Energy share of GDP (target)		0.056
Price setting		
Goods elasticity of substitution	ε	6.0
Rotemberg price adjustment cost	ϕ_P	50.0
Price indexation	ι_p	0.5
Wage setting		
Labor elasticity of substitution	ε_w	6.0
Calvo wage parameter (target)*	θ_w	0.75
Wage indexation	ι_w	0.5
Investment		
Investment adjustment cost	ϕ_I	2.0
Labor adjustment		
Hiring adjustment cost	ψ_n	0.0
Monetary policy		
Interest rate smoothing	ρ_R	0.7
Inflation response	ϕ_π	2.0
Output gap response	ϕ_x	0.5
*Exogenous processes ***		
Oil price persistence (Fig2, "Disruption continues")	ρ_{po}	0.85
Oil price persistence [realized] (Fig2, "Disruption continues, rolling surprises")	ρ_{po}	0.85
Oil price persistence [expected] (Fig2, "Disruption continues, rolling surprises")	ρ_{po}	0.7
Oil price persistence (Fig3, "Oil price permanently elevated at +15% ")	ρ_{po}	0.55
Oil price persistence (Fig4, "1979-80 shock")	ρ_{po}	0.92
Oil trade		
Net oil export share	s_{oilnet}	0.0015

* The model uses a Rotemburg adjustment cost for wages, but this is implemented by matching the slope of the wage Phillips curve in the linearized Calvo model.

** The exogenous oil price decay function, where applicable, is given by $P_t^o = \rho_{po} \cdot P_{t-1}^o + \varepsilon_t^o$, where ε_t^o is the oil price shock in period t

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